$$\begin{cases} 2y'\operatorname{ctg}x - 4y = -y^2\sin 2x, \\ y(0) = 1 \end{cases}$$

Задача Коши

$2y'\operatorname{ctg}x - 4y = -y^2\sin 2x$ ОДЗ $\sin x \neq 0$.

$x \neq \pi n, n \in \mathbb{Z}$

$$y' - \frac{4y}{2\operatorname{ctg}x} = -\frac{y^2 \cdot 2\sin x \cdot \cos x}{2\operatorname{ctg}x}$$

$$y' - 2\operatorname{tg}x \cdot y = -\frac{y^2 \cdot \cancel{2}\sin x \cdot \cancel{\cos x} \cdot \sin x}{\cancel{2}\cancel{\cos x}} = -\sin^2 x \cdot y^2$$

$$y' - \underbrace{2\operatorname{tg}x}_{p(x)} \cdot y = \underbrace{-\sin^2 x}_{f(x)} \cdot y^2$$ Бернулли $\boxed{m=2}$

Метод Бернулли: $y = u \cdot v$

$u'v + v'u - 2\operatorname{tg}x \cdot u \cdot v = -\sin^2 x \cdot u^2 v^2$

$$v(\underbrace{u' - 2\operatorname{tg}x \cdot u}_{=0}) = -\sin^2 x \cdot u^2 \cdot v^2 - v'u \quad (*)$$

$u' - 2\operatorname{tg}x \cdot u = 0$

$$\frac{du}{dx} = 2 \cdot \frac{\sin x}{\cos x} \cdot u$$

$$\frac{du}{u} = 2 \cdot \frac{\sin x}{\cos x} dx$$ – д.у с разд. перем.

$$\int \frac{du}{u} = -2\int \frac{d(\cos x)}{\cos x}$$

$$\ln|u| = -2\ln|\cos x| + C, \quad \forall C$$

$$e^{\ln|u|} = e^{-2\ln|\cos x| + c}, \quad \forall c$$

$$|u| = e^{-2\ln|\cos x|} \cdot e^{c}, \quad \forall c$$

$$|u| = |\cos x|^{-2} \cdot c_1, \quad \forall c_1 \neq 0, \quad c_1 = e^{c}$$

$$u = \frac{c_1}{\cos^2 x}, \quad c_1 = 1.$$

$$u = \frac{1}{\cos^2 x} \longrightarrow \text{в } (*)$$

$$v' u + \sin^2 x \, u^2 \cdot v^2 = 0$$

$$v' + \sin^2 x \cdot u \cdot v^2 = 0$$

$$\frac{dv}{du} = -\sin^2 x \cdot u \cdot v^2 = -\sin^2 x \cdot \frac{1}{\cos^2 x} \cdot v^2 \Rightarrow \text{ДУ с разд. пер.}$$

$$\frac{dv}{v^2} = -\frac{\sin^2 x}{\cos^2 x} dx$$

$$\int \frac{dv}{v^2} = -\int \frac{\sin^2 x}{\cos^2 x} dx = -\int \frac{1-\cos^2 x}{\cos^2 x} dx = -\int \frac{dx}{\cos^2 x} + \int dx$$

$$-\frac{1}{v} = -\operatorname{tg} x + x + c$$

$$v = -\frac{1}{-\operatorname{tg} x + x + c}$$

$$y = u \cdot v = -\frac{1}{\cos^2 x} \cdot \left(\frac{1}{-\operatorname{tg} x + x + c}\right) \text{ Общее решение}$$

$x=0, \ y=1$

$$1 = -\frac{1}{\cos^2 0} \cdot \left(\frac{1}{-\operatorname{tg} 0 + 0 + C}\right) = -1 \cdot \frac{1}{C} = 1$$

$C = -1$

$$\boxed{y = -\frac{1}{\cos^2 x} \cdot \left(\frac{1}{-\operatorname{tg} x + x - 1}\right)}$$

Окончательный ответ.