1

9.88 $y' = y(y^3\cos x + \operatorname{tg} x)$

$y' - y\underbrace{\operatorname{tg} x}_{p(x)} = y^4 \underbrace{\cos x}_{q(x)}$ ур-ние Бернулли

Метод Бернулли: $y(x) = u(x)\cdot v(x)$

$y' = u'v + uv'$

$u'v + uv' - \operatorname{tg}x \cdot u \cdot v = u^4 v^4 \cos x$

$v(\underbrace{u' - u\cdot \operatorname{tg}x}_{0}) = u^4 v^4 \cos x - uv'$

$\frac{du}{dx} = u \operatorname{tg} x$ д.у. с раз. пер.

$\frac{du}{u} = \operatorname{tg}x\, dx \qquad u \neq 0.$

$\int \frac{du}{u} = \int \frac{\sin x}{\cos x} dx$

$\ln|u| = -\int \frac{d(\cos x)}{\cos x}$

$\ln|u| = -\ln|\cos x| + c, \ \forall c$

$|u| = |\cos x|^{-1} \cdot e^c, \quad e^c = c_1$

$u = \frac{c_1}{\cos x}, \quad \forall c_1 \neq 0.$

$c_1 = 1 \Rightarrow u = \frac{1}{\cos x}$

$u^3 v^4 \cos x = \cancel{u} v'$ ДУ с разд. перем.

$$\frac{dv}{dx} = \frac{1}{\cos^3 x} v^4 \cdot \cos x = \frac{1}{\cos^2 x} \cdot v^4$$

$$\frac{dv}{v^4} = \frac{1}{\cos^2 x} dx \quad , v \neq 0.$$

$$-\frac{1}{3v^3} = \int \frac{dv}{v^4}$$

$$\int \frac{1}{\cos^2 x} dx = tg x$$

$$-\frac{1}{3v^3} = tg x + C$$

$$3v^3 = -\frac{1}{tg x + C}$$

$$v^3 = -\frac{1}{3(tg x + C)}$$

$$v = -\sqrt[3]{\frac{1}{3(tg x + C)}}$$

+

$$\boxed{y = -\frac{1}{\cos x} \cdot \frac{1}{\sqrt[3]{3(tg x + C)}}}$$ ← Ответ.

При $u = 0$, $y = 0$
$v = 0$, $y = 0$ } проверим $y = 0$ пот. решение или нет

$(0)' - 0 = 0 \Rightarrow 0 = 0$ $\boxed{y = 0}$ - пот. решение

9.92

$xy' + y = 2x^2 y \ln y\, y'$

$xy' - 2x^2 y \ln y\, y' = -y$

$xy'(1 - 2xy\ln y) = -y$

$y' = -\dfrac{y}{x(1-2xy\ln y)}$

$x' = -\dfrac{x(1-2xy\ln y)}{y}$

$x' = -\dfrac{1}{y}x + 2x^2\ln y$

$x' + \dfrac{1}{y}x = 2x^2\ln y$ Бернулли

$x(y) = u(y)\cdot v(y)$, $x = x(y)$ – ? $n=2$

$u'v + v'u + \dfrac{uv}{y} = 2u^2v^2\ln y$

$v\big(\underbrace{u' + \dfrac{u}{y}}_{=0}\big) + v'u = 2u^2v^2\ln y$

$u' + \dfrac{u}{y} = 0$ ДУ с раз. пер.

$\dfrac{du}{dy} = -\dfrac{u}{y}$

$\dfrac{du}{u} = -\dfrac{dy}{y}$

$\ln|u| = -\ln|y| + C_1$, $\forall C$

$u = C_1\cdot y^{-1}$, $\forall C_1 \neq 0$

$C_1 = 1$

$u = \dfrac{1}{y}$

$2u^{\cancel{2}}v^2\ln y - v'\cancel{u} = 0$

$v' = 2uv^2\ln y$

$\dfrac{dv}{dy} = 2uv^2\ln y$

$\dfrac{dv}{v^2} = 2u\ln y\, dy$

$\dfrac{dv}{v^2} = 2\cdot\dfrac{1}{y}\ln y\, dy$ ДУ с раз. пер.

$-\dfrac{1}{v} = 2\int \ln y\, d(\ln y)$

$-\dfrac{1}{v} = 2\cdot\dfrac{\ln^2 y}{2} + C$, $\forall C$.

$v = -\dfrac{1}{\ln^2 y + C}$

$x = u\cdot v$

$x = \dfrac{1}{y}\cdot\left(-\dfrac{1}{\ln^2 y + C}\right)$

Ответ.

995

$$\begin{cases} y\,dx + \left(x - \frac{1}{2}x^3 y\right)dy = 0, \\ y\left(\frac{1}{2}\right) = 1 \end{cases}$$

Задача Коши

$$y\,dx + \left(x - \frac{1}{2}x^3 y\right)dy = 0$$

$$y\,dx + x\left(1 - \frac{1}{2}x^2 y\right)dy = 0.$$

$$y\,dx = x\left(\frac{1}{2}x^2 y - 1\right)dy$$

$$x\left(\frac{1}{2}x^2 y - 1\right)dy = y\,dx$$

$$\frac{dy}{dx} = \frac{y}{x\left(\frac{1}{2}x^2 y - 1\right)}$$

$$y' = \frac{y}{x\left(\frac{1}{2}x^2 y - 1\right)}$$

$$x' = \frac{x\left(\frac{1}{2}x^2 y - 1\right)}{y}$$

$$x' = -\frac{1}{y}x + \frac{1}{2}x^3$$

$$x' + \underbrace{\frac{1}{y}}_{p(y)}x = \underbrace{\frac{1}{2}}_{f(y)}x^3$$

Бернулли

$$x(y) = u(y)\cdot v(y)$$

$$x' = u'v + uv'$$

$$\underline{u'v} + uv' + \underline{\frac{1}{y}\cdot u\cdot v} = \frac{1}{2}\cdot u^3\cdot v^3$$

$$v\underbrace{\left(u' + \frac{1}{y}\cdot u\right)}_{=0} = \frac{1}{2}u^3\cdot v^3 - uv'$$

$$\frac{du}{dy} = -\frac{u}{y}$$ Д.У с раз. пер

$$\frac{du}{u} = -\frac{dy}{y}$$

$$\ln|u| = -\ln|y| + c, \ \forall c$$

$$u = \frac{c_1}{y}, \ \forall c_1 \neq 0$$

$$u = \frac{1}{y}, \quad c_1 = 1$$

$$\cancel{\frac{1}{2}x^3 - \frac{1}{y}v' = 0}$$

$$\frac{1}{2}u^3v^3 - uv' = 0$$

$$\frac{1}{2}u^2v^3 = v'$$

$$\frac{1}{2y^2}v^3 = v'$$

$$\frac{dv}{dy} = \frac{1}{2y^2}v^3$$ Д.У с.

$$\frac{dv}{v^3} = \frac{dy}{2y^2}$$

$$-\frac{1}{2v^2} = \frac{1}{2}\cdot -\frac{1}{y} + C$$

$$-\frac{1}{v^2} = -\frac{1}{y} + 2C$$

$$\frac{1}{v^2} = \frac{1}{y} - 2C$$

$$v^2 = \frac{1}{\frac{1}{y} - 2C}$$

$$x^2 = u^2 \cdot v^2 = \frac{1}{y^2} \cdot \frac{1}{\frac{1}{y} - 2C} = \frac{1}{y^{\cancel{2}}} \cdot \frac{\cancel{y}}{1 - 2Cy} =$$

$$= \frac{1}{y(1-2Cy)}$$

Нач. условие: $x = \frac{1}{2}$, $y = 1$

$$\frac{1}{4} = \frac{1}{1 \cdot (1 - 2C)} \Rightarrow 1 - 2C = 4$$

$$2C = 1 - 4 = -3$$

$$C = -\frac{3}{2}$$

$$x^2 = \frac{1}{y(1 + \cancel{2} \cdot \frac{3}{\cancel{2}} \cdot y)} = \frac{1}{y(1+3y)}$$

$$x^2 y \cdot (1 + 3y) = 1$$ Ответ.

+

9.91

$$y' = \frac{x(x^2+y^2-1)}{2y\cdot(x^2-1)} = \frac{x(x^2-1)+xy^2}{2y(x^2-1)} = \frac{x}{2y} + \frac{xy}{2(x^2-1)}$$

$$y' - \frac{x}{2(x^2-1)}y = \frac{x}{2}\cdot y^{-1}$$ Бернулли $m = -1$

$y(x) = y - ?$

$y = uv$

$$u'v + uv' - \frac{x\cdot u\cdot v}{2(x^2-1)} = \frac{x}{2}\cdot\frac{1}{u\cdot v}$$

$$v\left(u' - \frac{xu}{2(x^2-1)}\right) = \frac{x}{2}\cdot\frac{1}{u\cdot v} - uv'$$

$$\frac{du}{dx} = \frac{xu}{2(x^2-1)}$$

$$\frac{du}{u} = \frac{x\,dx}{2(x^2-1)} = \frac{d(x^2-1)}{4(x^2-1)}$$

$$\ln|u| = \frac{1}{4}\ln|x^2-1| + C, \quad \forall C.$$

$$u = (x^2-1)^{\frac{1}{4}}\cdot C_1, \quad \forall C_1 > 0,$$

$C_1 = 1.$ $\quad u = \sqrt[4]{x^2-1}$

$$uv' = \frac{x}{2uv}$$

$$v' = \frac{x}{2u^2 v}$$

$$\frac{dv}{dx} = \frac{x}{2\sqrt{x^2-1}\cdot v}$$

$$v\,dv = \frac{x}{2\sqrt{x^2-1}}\,dx$$

$$v\,dv = \frac{1}{4}(x^2-1)^{-\frac{1}{2}}\,d(x^2-1)$$

$$\frac{v^2}{2} = \frac{1}{4}\cdot 2(x^2-1)^{\frac{1}{2}} + C$$

$$v^2 = 2\cdot 2\cdot\frac{1}{4}(x^2-1)^{\frac{1}{2}} + C$$

$$v^2 = \sqrt{x^2-1} + C, \quad \forall C$$

$$u^2 = \sqrt{x^2-1}$$

$$v^2 = \left(\sqrt{x^2-1} + C\right), \quad \forall C.$$

$$y^2 = u^2 v^2 = \sqrt{x^2-1}\left(\sqrt{x^2-1} + C\right) = x^2 - 1 + C\sqrt{x^2-1}$$

Ответ

+