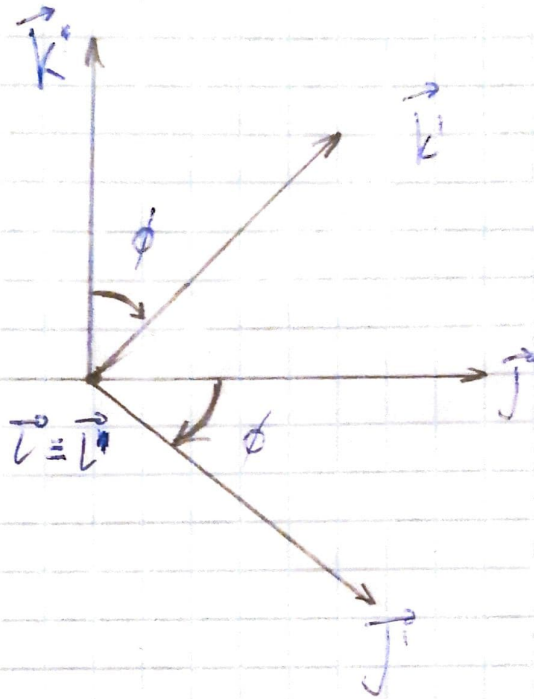


N1

1) $\phi = -45^\circ$



$$\{\vec{i}, \vec{j}, \vec{k}\} \xrightarrow{T_1} \{\vec{i}', \vec{j}', \vec{k}'\}$$

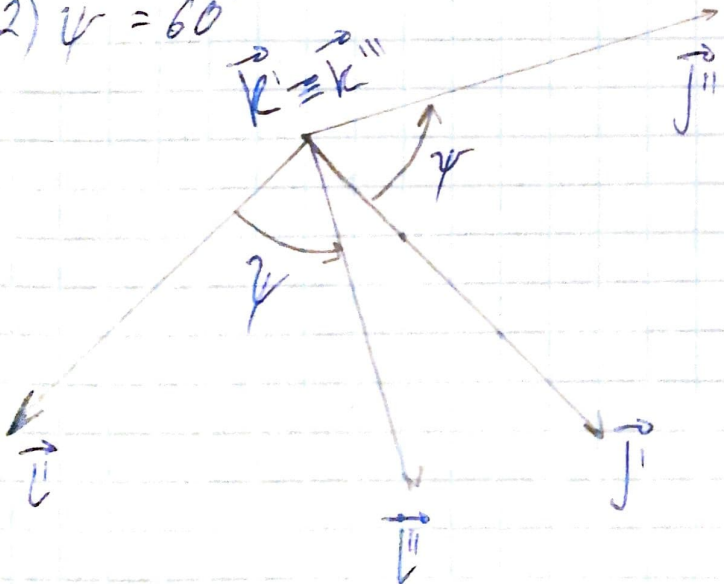
$$\xrightarrow{T_2} \{\vec{i}'', \vec{j}'', \vec{k}''\}$$

$$\{\vec{i}, \vec{j}, \vec{k}\} \xrightarrow{T} \{\vec{i}'', \vec{j}'', \vec{k}''\}$$

$$T = T_1 \cdot T_2$$

$$T_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

2) $\psi = 60^\circ$



$$T_2 = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T = T_1 \times T_2 = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{6}}{4} & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

√2

$$\vec{p} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} \quad \vec{q} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 1 \end{bmatrix}$$

$\{ \vec{a} \}$
 $\{ \vec{a} \}$

$$\vec{a}_1 = \begin{pmatrix} 1 \\ -2 \\ 3 \\ -2 \end{pmatrix} \quad \vec{a}_2 = \begin{pmatrix} 1 \\ 5 \\ -1 \\ 3 \end{pmatrix} \quad \vec{a}_3 = \begin{pmatrix} -4 \\ -3 \\ 2 \\ -5 \end{pmatrix} \quad \vec{a}_4 = \begin{pmatrix} -4 \\ -4 \\ 0 \\ 2 \end{pmatrix}$$

Ортонормализация

$$\vec{q}_1 = \vec{a}_1$$

$$\vec{q}_2 = \vec{a}_2 - \frac{(\vec{a}_2, \vec{a}_1)}{(\vec{a}_1, \vec{a}_1)} \vec{a}_1 = \left\{ \begin{array}{l} (\vec{a}_2, \vec{a}_1) = 1 + (-10) - 3 - 6 = -18 \\ (\vec{a}_1, \vec{a}_1) = 1 + 4 + 9 + 4 = 18 \\ \vec{a}_2 + \vec{a}_1 = \{2; 3; 2; 1\} \end{array} \right.$$

$$= \{2; 3; 2; 1\}$$

$$\vec{g}_3 = \vec{a}_3 - \frac{(\vec{a}_3; \vec{a}_1)}{(\vec{a}_1; \vec{a}_1)} \vec{a}_1 - \frac{(\vec{a}_3; \vec{g}_2)}{(\vec{g}_2; \vec{g}_2)} \vec{g}_2 =$$

$$= \left\{ \begin{array}{l} (\vec{a}_3; \vec{a}_1) = -4 + 6 + 6 + 10 = 18 \\ (\vec{a}_3; \vec{g}_2) = -8 - 9 + 4 - 5 = -18 \\ (\vec{g}_2; \vec{g}_2) = 4 + 9 + 4 + 1 = 18 \\ \vec{a}_3 - \vec{a}_1 + \vec{g}_2 = \{-3; 2; 1; -2\} \end{array} \right\} = \{-3; 2; 1; -2\}$$

$$\vec{g}_4 = \vec{a}_4 - \frac{(\vec{a}_4; \vec{a}_1)}{(\vec{a}_1; \vec{a}_1)} \vec{a}_1 - \frac{(\vec{a}_4; \vec{g}_2)}{(\vec{g}_2; \vec{g}_2)} \vec{g}_2 - \frac{(\vec{a}_4; \vec{g}_3)}{(\vec{g}_3; \vec{g}_3)} \vec{g}_3 =$$

$$= \left\{ \begin{array}{l} (\vec{a}_4; \vec{a}_1) = -4 + 8 + 0 - 4 = 0 \\ (\vec{a}_4; \vec{g}_2) = -8 - 12 + 0 + 2 = -18 \\ (\vec{a}_4; \vec{g}_3) = +12 - 8 + 0 - 4 = 0 \\ (\vec{g}_3; \vec{g}_3) = 9 + 4 + 1 + 4 = 18 \end{array} \right\} =$$

$$= \{-2; -1; 2; 3\}$$

$$\vec{b}_1 = \frac{\vec{a}_1}{\sqrt{18}} = \begin{pmatrix} \frac{1}{\sqrt{18}} \\ \frac{-2}{\sqrt{18}} \\ \frac{3}{\sqrt{18}} \\ \frac{-2}{\sqrt{18}} \end{pmatrix} \quad \vec{b}_2 = \frac{\vec{g}_2}{\sqrt{18}} = \begin{pmatrix} \frac{2}{\sqrt{18}} \\ \frac{3}{\sqrt{18}} \\ \frac{2}{\sqrt{18}} \\ \frac{1}{\sqrt{18}} \end{pmatrix}$$

$$\vec{b}_3 = \begin{pmatrix} -\frac{3}{\sqrt{18}} \\ \frac{2}{\sqrt{18}} \\ \frac{1}{\sqrt{18}} \\ -\frac{2}{\sqrt{18}} \end{pmatrix}$$

$$\vec{b}_4 = \begin{pmatrix} -\frac{2}{\sqrt{18}} \\ -\frac{1}{\sqrt{18}} \\ \frac{2}{\sqrt{18}} \\ \frac{3}{\sqrt{18}} \end{pmatrix}$$

$$\vec{\Gamma}_{A \rightarrow B} = A^{-1}B$$

$$\vec{A} \left(\begin{array}{cccc|cccc} 1 & 1 & -4 & -4 & 0 & 0 & 0 & 0 \\ -2 & 5 & -3 & -4 & 0 & 1 & 0 & 0 \\ 3 & -1 & 2 & 0 & 0 & 0 & 1 & 0 \\ -2 & 3 & 5 & 2 & 0 & 0 & 0 & 1 \end{array} \right) \quad \underline{\underline{II-N}}$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & -4 & -4 & 1 & 0 & 0 & 0 \\ 0 & 2 & -8 & -6 & 0 & 1 & 0 & -1 \\ 3 & -1 & 2 & 0 & 0 & 0 & 1 & 0 \\ -2 & 3 & 5 & 2 & 0 & 0 & 0 & 1 \end{array} \right) \quad \underline{\underline{\frac{1}{2}II, III, IV}}$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & -4 & -4 & 1 & 0 & 0 & 0 \\ 0 & 1 & -4 & -3 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{3}{2} & -\frac{1}{2} & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ -1 & \frac{3}{2} & \frac{5}{2} & 1 & 0 & 0 & 0 & \frac{1}{2} \end{array} \right) \quad \underline{\underline{I-II}}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & -4 & -3 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ -1 & \frac{3}{2} & \frac{5}{2} & 1 & 0 & 0 & 0 & \frac{1}{2} \end{array} \right) \quad \begin{array}{l} \text{III} + \frac{3}{2}\text{I} \\ \text{IV} + \text{I} \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & -4 & -3 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & 0 & -\frac{3}{2} & \frac{3}{2} & -\frac{3}{4} & \frac{1}{2} & \frac{3}{4} \\ 0 & \frac{3}{2} & \frac{5}{2} & 0 & 1 & -\frac{1}{2} & 0 & 1 \end{array} \right) \quad \begin{array}{l} \text{III} + \frac{1}{2}\text{II} \\ \text{IV} - \frac{3}{2}\text{II} \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & -4 & -3 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & -2 & -3 & \frac{3}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{17}{2} & \frac{9}{2} & 1 & -\frac{5}{4} & 0 & \frac{7}{4} \end{array} \right) \quad \begin{array}{l} \text{III} \cdot (-\frac{1}{2}) \\ \text{IV} - \frac{17}{2}\text{III} \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & -4 & -3 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{3}{2} & -\frac{5}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 0 & -21 & \frac{59}{8} & -\frac{27}{8} & \frac{17}{8} & \frac{31}{8} \end{array} \right) \quad \begin{array}{l} \text{IV} \cdot (-\frac{1}{21}) \\ \text{I} + \text{IV} \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & -4 & -3 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{3}{2} & -\frac{3}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 0 & 1 & -\frac{59}{168} & \frac{9}{56} & -\frac{17}{168} & \frac{31}{168} \end{array} \right)$$

$$\begin{array}{l} \text{III} + \frac{3}{2}\text{IV} \\ \text{II} + 4\text{IV} \\ \text{II} + 3\text{IV} \end{array}$$

$$\left(\begin{array}{cccc} -\frac{29}{18} & \frac{10}{9} & \frac{17}{18} & -1 \\ -\frac{11}{6} & \frac{4}{3} & \frac{5}{6} & -1 \\ \frac{3}{2} & -1 & -\frac{1}{2} & 1 \\ -\frac{47}{12} & \frac{29}{18} & \frac{17}{18} & -\frac{3}{2} \end{array} \right) = A^{-1}$$

$$\nabla_{A \rightarrow B} = A^{-1}B$$

$$\left(\begin{array}{cccc} -\frac{29}{18} & \frac{10}{9} & \frac{17}{18} & -1 \\ -\frac{11}{6} & \frac{4}{3} & \frac{5}{6} & -1 \\ \frac{3}{2} & -1 & -\frac{1}{2} & 1 \\ -\frac{47}{12} & \frac{29}{18} & \frac{17}{18} & -\frac{3}{2} \end{array} \right) \times \left(\begin{array}{cccc} \frac{1}{\sqrt{18}} & \frac{2}{\sqrt{18}} & -\frac{3}{\sqrt{18}} & -\frac{2}{\sqrt{18}} \\ -\frac{2}{\sqrt{18}} & \frac{3}{\sqrt{18}} & \frac{2}{\sqrt{18}} & \frac{1}{\sqrt{18}} \\ \frac{3}{\sqrt{18}} & \frac{2}{\sqrt{18}} & \frac{1}{\sqrt{18}} & \frac{2}{\sqrt{18}} \\ -\frac{2}{\sqrt{18}} & \frac{1}{\sqrt{18}} & -\frac{2}{\sqrt{18}} & \frac{3}{\sqrt{18}} \end{array} \right) =$$

$$= \begin{pmatrix} \frac{-29 \cdot 1}{18\sqrt{18}} + \frac{10 \cdot 2}{9\sqrt{18}} + \frac{17 \cdot 3}{18\sqrt{18}} + \frac{2}{\sqrt{18}} & \frac{-29 \cdot 2}{18\sqrt{18}} + \frac{10 \cdot 3}{9\sqrt{18}} + \frac{17 \cdot 2}{18\sqrt{18}} + \frac{1}{\sqrt{18}} \\ \frac{-11}{6\sqrt{18}} - \frac{8}{3\sqrt{18}} + \frac{15}{6\sqrt{18}} + \frac{2}{\sqrt{18}} & \dots \\ \dots & \dots \\ \dots & \dots \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{18}} & \frac{1}{\sqrt{18}} & 0 & \frac{1}{\sqrt{18}} \\ 0 & \frac{1}{\sqrt{18}} & \frac{1}{\sqrt{18}} & \frac{1}{\sqrt{18}} \\ 0 & 0 & \frac{1}{\sqrt{18}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{18}} \end{pmatrix}$$

$$T_{B \rightarrow A} = \begin{pmatrix} \sqrt{18} & -\sqrt{18} & \sqrt{18} & 0 \\ 0 & \sqrt{18} & -\sqrt{18} & -\sqrt{18} \\ 0 & 0 & \sqrt{18} & 0 \\ 0 & 0 & 0 & \sqrt{18} \end{pmatrix}$$

$$\vec{p}^1 = T_{A \rightarrow B} \vec{p}^0 = \begin{pmatrix} \frac{1}{\sqrt{18}} & \frac{1}{\sqrt{18}} & 0 & \frac{1}{\sqrt{18}} \\ 0 & \frac{1}{\sqrt{18}} & \frac{1}{\sqrt{18}} & \frac{1}{\sqrt{18}} \\ 0 & 0 & \frac{1}{\sqrt{18}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{18}} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{84}{\sqrt{18}} \\ \frac{9}{\sqrt{18}} \\ \frac{1}{\sqrt{18}} \\ \frac{2}{\sqrt{18}} \end{pmatrix}$$

$$\vec{p}' = \mathbb{T}_{B \rightarrow A} \vec{p} = \begin{pmatrix} \sqrt{18} & -\sqrt{18} & \sqrt{18} & 0 \\ 0 & \sqrt{18} & -\sqrt{18} & -\sqrt{18} \\ 0 & 0 & \sqrt{18} & 0 \\ 0 & 0 & 0 & \sqrt{18} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \sqrt{18} \\ -2\sqrt{18} \\ \sqrt{18} \\ 2\sqrt{18} \end{pmatrix}$$

$$\vec{q}' = \mathbb{T}_{B \rightarrow A} \vec{q} = \begin{pmatrix} \sqrt{18} & -\sqrt{18} & \sqrt{18} & 0 \\ 0 & \sqrt{18} & -\sqrt{18} & -\sqrt{18} \\ 0 & 0 & \sqrt{18} & 0 \\ 6 & 0 & 0 & \sqrt{18} \end{pmatrix} \begin{pmatrix} 3 \\ 3 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3\sqrt{18} \\ -\sqrt{18} \\ 3\sqrt{18} \\ \sqrt{18} \end{pmatrix}$$

$$\begin{aligned} (\vec{p}'; \vec{q}') &= \sqrt{18} \cdot 3 \cdot 18 + 2 \cdot 18 + 3 \cdot 18 + 2 \cdot 18 = \\ &= 10 \cdot 18 = 180 \end{aligned}$$

$$\cos(\widehat{\vec{p}'; \vec{q}'}) = \frac{(\vec{p}'; \vec{q}')}{|\vec{p}'| \cdot |\vec{q}'|} = \left| \begin{array}{l} |\vec{p}'|^2 = 18 + 4 \cdot 18 + 18 + 2 \cdot 18 = \\ = 180 \\ |\vec{q}'|^2 = 9 \cdot 18 - 18 + 9 \cdot 18 + 18 = \\ = 324 \end{array} \right|$$

$$= \frac{180}{\sqrt{180} \cdot \sqrt{324}} = \frac{1}{324} \Rightarrow (\widehat{\vec{p}'; \vec{q}'}) = \arccos\left(\frac{1}{324}\right)$$