1) $(y^2-1)\,y'' = 2y(y')^2;\ y(0)=2;\ y'(0)=3$

$$\begin{cases} y'=p \\ y''=p'\cdot p \end{cases} \Rightarrow (y^2-1)\cancel{p}p' = 2yp^{\cancel{2}};\ p=0$$

$$(y^2-1)\frac{dp}{dy} = 2yp;\ y^2-1=0$$

$$\int\frac{dp}{p} = \int\frac{2y\,dy}{y^2-1}$$

$$\ln|p| = \ln|y^2-1| + C_1$$

$p = (y^2-1)C_2$, где $C_2 = \pm e^{C_1}$

$y' = (y^2-1)\,C_2 \Rightarrow C_2 = \frac{y'}{y^2-1} = \frac{3}{4-1} = 1 \Rightarrow y' = (y^2-1)$

$$\frac{dy}{dx} = (y^2-1)$$

$$\int\frac{dy}{y^2-1} = \int dx$$

$$\frac{\ln|y-1| - \ln|y+1|}{2} = x\,\cancel{C_2} + C_3$$

$\cancel{y=3}$.

$$\frac{\ln 1 - \ln 3}{2} = C_3$$

$$-\frac{\ln 3}{2} = C_3$$

Ответ: $\frac{\ln|y-1| - \ln|y+1|}{2} = x - \frac{\ln 3}{2}$

2) $xy'' - y' = x^2 e^x$

$\begin{cases} y' = p \\ y'' = p' \end{cases} \Rightarrow xp' - p = x^2 e^x \quad ; x = 0$

$p' - \frac{p}{x} = x e^x \; (1)$ ЛНДУ

$p' - \frac{p}{x} = 0$

$\frac{dp}{dx} = \frac{p}{x}$

$\frac{dp}{d} = \frac{dx}{x} \; ; p = 0$

$\ln|p| = \ln|x| + C_1$

$p = x C_2$, где $C_2 = \pm e^{C_1}$

Пусть C_2 - ф-ция:

$p = x C_2(x) \; (2)$

$(2) \to (1)$:

$\cancel{C_2} + x C_2' - \frac{\cancel{x C_2}}{\cancel{x}} = x e^x$

$\cancel{x} C_2' = \cancel{x} e^x \; , x = 0$

$C_2' = e^x \Rightarrow C_2 = e^x + C_3$

$y' = 0 \Rightarrow y'' = 0 ; y = C$

~~$xy'' = x^2 e^x$~~

~~$y'' = e^x$~~

~~$y = e^x + C_5$~~

$0 \neq x^2 e^x$

не является

$x = 0$:

$-y' = 0$

$y' = 0$

$\int dy = \int 0 dx$

$y = 0 + C_6 = C_6$

$y' = x e^x + C_3$

$\frac{dy}{dx} = (x e^x + x C_3)$

$\int dy = \int (x e^x + x C_3) dx$

$y = (x - 1) e^x + \frac{C_3 x^2}{2} + C_4$

$y' = 0$

Ответ: $y=(x-1)e^x+\frac{C_3x^2}{2}+C_4$; $\cancel{y=e^x+C_5}$; $y=C_6$.

N 3

$$y'''-3y''+4y'-2y=(3\cos x+2x\sin x)e^x+e^{-4x}+x^3e^x+x^2-e^x$$

$$f_1=(3\cos x+2x\sin x)e^x$$

$$\lambda^3-3\lambda^2+4\lambda-2=0$$

$$\lambda_1=1$$

$$\begin{array}{l|l} \lambda^3-3\lambda^2+4\lambda-2 & \lambda-1 \\ \lambda^3-\lambda^2 & \lambda^2-2\lambda+2 \\ \hline -2\lambda^2+4\lambda & \\ -2\lambda^2+2\lambda & \\ \hline 2\lambda-2 & \\ 2\lambda-2 & \\ \hline 0 & \end{array}$$

$\lambda^2-2\lambda+\lambda=0$

$D=4-4=0$

$\lambda_{2,3}=\frac{2}{2}=1$

$f_1=(3\cos x+2x\sin x)e^x$, $\alpha=1$ $\cancel{=\lambda_2=\lambda_3=\lambda_1\Rightarrow r=3}$

$\beta=1$ $\quad\alpha\pm\beta i\neq\lambda_{1,2,3}\Rightarrow r=0$

$$y_{ч.н}=e^x\left((Kx+L)\cos x+(Mx+N)\sin x\right)x^0$$

$f_2=x^2=x^2\cdot e^{0x}\Rightarrow\alpha=0\neq\lambda_{1,2,3}\Rightarrow r=0$

$$y_{ч.н_2}=e^{0x}\cdot(Ax^2+Bx+C)x^0=(Ax^2+Bx+C)$$

$f_3 = e^{-4x} \Rightarrow \alpha = -4 \neq \lambda_{1,2,3} \Rightarrow r = 0$

$y_{3чн} = e^{-4x} \cdot Dx^0 = De^{-4x}$

$f_4 = x^3e^x - e^x = e^x(x^3 - 1), \Rightarrow \alpha = 1 = \lambda_1 = \lambda_2 = \lambda_3 \Rightarrow r = 3$

$y_{4чн} = e^x(Ex^3 + Fx^2 + Gx + H)x^3$

$y_{чн} = e^x((Kx + L)\cos x + (Mx + N)\sin x) + Ax^2 + Bx + C + De^{-4x} +$
$+ e^x(Ex^3 + Fx^2 + Gx + H)x^3$

N4

$y'' - 8y' + 16y = 16\cos 4x - 1$, при $x = 0, y = -\frac{1}{16}; y' = 0$

$\lambda^2 - 8\lambda + 16 = 0$

$D_1 = 16 - 16 = 0$

$\lambda_{1,2} = \frac{4}{1} = 4$ ФСР ЛОДУ $= \{e^{4x}; xe^{4x}\}$

$y_{oo} = C_1 e^{4x} + xC_2 e^{4x}$

$f_1 = 16e^{0x}\cos 4x \Rightarrow \alpha = 0; \beta = 4 \cdot \pm 4i \neq \lambda_{1,2} \Rightarrow r = 0$

$Q(x) = 16 \Rightarrow n = 0$

$y_{1чн} = (A\cos 4x + B\sin 4x)e^{0x} \cdot x^0 = A\cos 4x + B\sin 4x$

$f_2 = -1e^{0x} \Rightarrow \alpha = 0 \neq \lambda_{1,2} \Rightarrow r = 0$

$P(x) = -1 \Rightarrow n = 0$

$y_{2чн} = Ce^{0x^0} = C$

$y_{чн} = A\cos 4x + B\sin 4x + C$

$y'_{чн} = -4A\overset{\sin}{\cancel{\cos}}4x + 4B\cos 4x$

$y''_{чн} = -16B\sin 4x - 16A\cos 4x$

$-16B\sin 4x - 16A\cos 4x + 32A\sin 4x - 32B\cos 4x +$

$+16A\cos 4 + 16B\sin 4t + C = 16\cos 4x - 1$

$$\begin{cases} \cancel{-16B} + 32A + \cancel{16B} = 0 \\ \cancel{-16A} - 32B + \cancel{16A} = 16 \\ C = -1 \end{cases} \Leftrightarrow \begin{cases} A = 0 \\ B = -\frac{1}{2} \\ C = -1 \end{cases}$$

$y_{чн} = -\frac{1}{2}\sin 4x - 1$

$y_{он} = C_1 e^{4x} + xC_2 e^{4x} - \frac{1}{2}\sin 4x - 1$

$-\frac{1}{16} = C_1 + 1 \Rightarrow C_1 = \frac{15}{16}$

$y'_{он} = 4C_1 e^{4x} + \cancel{4xC_2 e^{4x}} - \frac{1}{2}\cdot 4\cos 4x = 0$

~~$4C_1 + 0 = 0 \Rightarrow C_1 = 0$~~

~~Ответ: $y=$~~ $y'_{он} = 4C_1e^{4x} + C_2e^{4x} + C_2xe^{4x} - \frac{1}{2}\cdot 4\cos 4x = 0$

$4C_1 + C_2 - 2 = 0; \; C_2 = 2 - 4\frac{15}{16} = -\frac{7}{4}$

Ответ: $\frac{15}{16}e^{4x} - x\frac{7}{4}e^{4x} - \frac{1}{2}\sin 4x - 1$

N5

$xy'' - (2x+1)y' + (x+1)y = 8x^3e^x, \; y_1 = e^x$

$$y_2 = y_1 \cdot \int \frac{e^{-\int p_1(x)dx}}{y_1^2}dx \; ; \; p_1 = (-2x-1)$$

$$-\int p_1(x)dx = -\int -(2x+1)dx = \int (2x)dx + \int 1dx =$$

$$= \frac{2x^2}{2} + x = x^2 + x$$

$$y_2 = e^x \int \frac{e^{x^2+x}}{e^{2x}}dx = e^x \int e^{x^2+x-2x}dx = e^x \int e^{x^2-x}dx$$

$$y'' - \frac{2x+1}{x}y' + \frac{x+1}{x}y = 8x^2e^x, \; y_1$$

$$p_1 = \frac{-2x-1}{x} \; ; \; -\int p_1 dx = \int \frac{2x+1}{x}dx = \ln|x| + 2x + C$$

$$y_2 = e^x \int \frac{e^{\ln|x|+2x}}{e^{2x}}dx = e^x \int \frac{|x|\cdot \cancel{e^{2x}}}{\cancel{e^{2x}}}dx = e^x \int \pm x dx =$$

$$= \pm e^x \frac{x^2}{2} + C_1 = \pm \frac{x^2e^x}{2}$$

$$y_{oo} = C_2e^x \pm C_3\frac{x^2e^x}{2} = C_2e^x + C_4\frac{x^2e^x}{2}$$

$$y_{он} = C_2(x)e^x + C_4(x)\frac{x^2e^x}{2}$$

$$\begin{cases} C_2' \cancel{e^x} + C_4' \frac{x^2 \cancel{e^x}}{2} = 0 \\ C_2' \cdot \cancel{e^x} + C_4' \frac{2x\cancel{e^x} + \cancel{e^x} \cdot x^2}{2} = 8x^2 \cancel{e^x} \end{cases}$$

$$- \begin{cases} C_2' + C_4' \cdot \frac{2x + x^2}{2} = 8x^2 \quad (1) \\ C_2' + C_4' \frac{x^2}{2} = 0 \quad (2) \end{cases}$$

$$C_4' \frac{2x + x^2}{2} - C_4' \frac{x^2}{2} = 8x^2$$

$$C_4' \left(\frac{\cancel{2}x + \cancel{x^2} - \cancel{x^2}}{\cancel{2}}\right) = 8x^2$$

$$C_4' x = 8x^2 \ (3) \ / \ \boxed{x = 0}$$

$$\frac{dC_4}{dx} = 8x$$

$$\int dC_4 = \int 8x\,dx$$

$$C_4 = \overset{4}{\cancel{8}} \frac{x^2}{\cancel{2}} + \tilde{C}_1$$

$(3) \to (2)$

$$C_2' + \frac{\overset{4}{\cancel{8}}x \cdot x^2}{\cancel{2}} = 0$$

$$C_2' = -4x^3$$

$$\int dC_2 = -4\int x^3 dx$$

$$C_2 = \frac{-4x^4}{4} + \tilde{C}_2 = -x^4 + \tilde{C}_2$$

$$y_{он} = (-x^4 + \tilde{C}_2)e^x + (4x^2 + \tilde{C}_1)\frac{x^2e^x}{2}$$ – общее решение ЛНДУ.

$x = 0$:

$$-y' + y = 0$$

$$\frac{dy}{dx} = y$$

$$\frac{dy}{y} = dx$$

$$\ln|y| = x + \tilde{C}_3$$

$$y = e^x\tilde{C}_4, \ \forall \tilde{C}_4$$

Ответ: $y_{он} = (-x^4 + \tilde{C}_2)e^x + (4x^2 + \tilde{C}_1)\frac{x^2e^x}{2}$; $e^x\tilde{C}_4$.