

№3 по 1А

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вариант 7  
№3

$$a) 6x^2 + 3y^2 - 4xy + 4\sqrt{5}x + 8\sqrt{5}y + 22 = 0$$

$W_{K_2} =$

нб. форма:  $f = 6x^2 + 3y^2 - 4xy$

$$A = \begin{pmatrix} 6 & -2 \\ -2 & 3 \end{pmatrix}$$

$$\begin{vmatrix} 6-\lambda & -2 \\ -2 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 9\lambda + 14 = 0, \lambda_1 = 2, \lambda_2 = 7$$

1)  $\lambda_1 = 2$

$$(A - \lambda_1 E) \bar{u} = 0 \Leftrightarrow \begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix} \sim \begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix} \Rightarrow 2u_1 - u_2 = 0$$

$u_1 = \frac{c_1}{2}$ ,  $u_1$  - базис,  $u_2$  - свобод.  $u_2 = c_1 \Rightarrow$

$$\Rightarrow \bar{u} = c_1 \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}, \text{ пусть } \bar{a}_1 = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}, |\bar{a}_1| = \frac{\sqrt{5}}{2} \\ c_1 = \frac{2}{\sqrt{5}} \Rightarrow$$

$$\Rightarrow \bar{u} = \frac{2}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$2) \lambda_2 = 7$$

$$(A - \lambda_2 E) \bar{v} = 0 \Leftrightarrow \begin{pmatrix} -1 & -2 \\ -2 & -4 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -2 \\ -2 & -4 \end{pmatrix} \sim \begin{pmatrix} -1 & -2 \\ 0 & 0 \end{pmatrix} \Rightarrow \begin{aligned} -v_1 - 2v_2 &= 0 \\ v_1 &= -2v_2 \end{aligned}$$

$v_1$  - базис,  $v_2$  - свободная,  $v_1 = -2v_2$   
 $v_2 = c_2$

$$\bar{v} = c_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \text{ пусть } \bar{a}_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, |\bar{a}_2| = \sqrt{5}$$

$$c_2 = \frac{1}{\sqrt{5}}; \quad \bar{v} = \frac{1}{\sqrt{5}} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

В базисе  $\bar{u}, \bar{v}$  кв. форма  $f = 2x_1^2 + 7y_1$

Матрица перехода:

$$T = \begin{pmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$$

$$\det T = 1.$$

Замена переменных:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

$$\begin{cases} x = \frac{1}{\sqrt{5}} x_1 - \frac{2}{\sqrt{5}} y_1 \\ y = \frac{2}{\sqrt{5}} x_1 + \frac{1}{\sqrt{5}} y_1 \end{cases} \Rightarrow$$

$$\Rightarrow 2x_1^2 + 7y_1^2 + 4\sqrt{5} \left( \frac{1}{\sqrt{5}} x_1 - \frac{2}{\sqrt{5}} y_1 \right) + 8\sqrt{5} \left( \frac{2}{\sqrt{5}} x_1 + \frac{1}{\sqrt{5}} y_1 \right) +$$

$$+ 22 = 0$$

$$2x_1^2 + 7y_1^2 + 20x_1 + 22 = 0$$

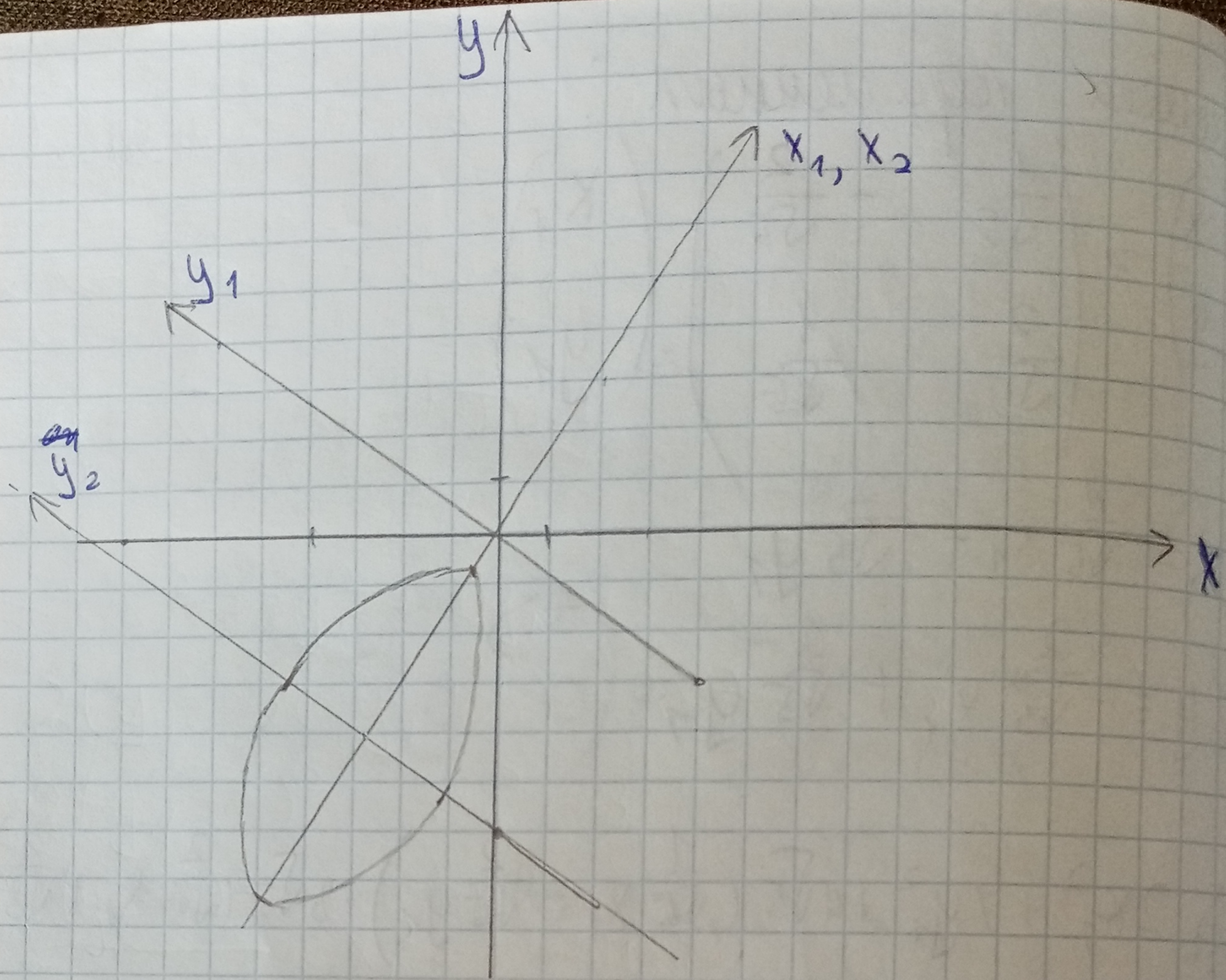
$$2(x_1^2 + 10x_1 + 25) - 28 + 7y_1^2 = 0$$

$$2(x_1 + 5)^2 + 7y_1^2 = 28$$

$$\frac{(x_1 + 5)^2}{14} + \frac{y_1^2}{4} = 1$$

$$\begin{cases} x_2 = x + 5 \\ y_1 = y_2 \end{cases} \Rightarrow \frac{x_2^2}{14} + \frac{y_2^2}{4} = 1 - \text{эллипс}$$

$$a = \sqrt{14}; \quad b = 2.$$



$$8) -7x^2 + y^2 - 6xy + 2\sqrt{10}x - 6\sqrt{10}y + 42 = 0$$

$$f = -7x^2 + y^2 - 6xy$$

$$A = \begin{pmatrix} -7 & -3 \\ -3 & 1 \end{pmatrix}; \begin{vmatrix} -7-\lambda & -3 \\ -3 & 1-\lambda \end{vmatrix} = 0$$

$$\lambda_1 = -8; \lambda_2 = 2$$

$$1) \lambda_1 = -8$$

$$(A - \lambda_1 E) \bar{u} = 0 \Leftrightarrow \begin{pmatrix} 1 & -3 \\ -3 & 9 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 \\ -3 & 9 \end{pmatrix} \sim$$

$$\bar{u} = C_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$2) \lambda_2 = 2$$

$$(A - \lambda_2 E)$$

$$\begin{pmatrix} -9 & -3 \\ -3 & -1 \end{pmatrix}$$

~~$$C_2 = \frac{3}{\sqrt{10}}$$~~

$$C_2 = \frac{3}{\sqrt{10}}$$

$$\bar{v} = \frac{3}{\sqrt{10}}$$

В базисе

направ

$$1 = \begin{pmatrix} 3 \\ \sqrt{10} \\ 1 \\ \sqrt{10} \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 \\ -3 & 9 \end{pmatrix} \sim \begin{pmatrix} 1 & -3 \\ 0 & 0 \end{pmatrix} \Rightarrow u_1 - 3u_2 = 0$$

$u_1$ -базис,  
 $u_2$ -свобод.

$$u_1 = 3c_1 \quad u_2 = c_1$$

$$\bar{u} = c_1 \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \text{ пусть } \bar{a}_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, |\bar{a}_1| = \sqrt{10}, c_1 = \frac{1}{\sqrt{10}}$$

$$\bar{u} = \frac{1}{\sqrt{10}} \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$2) \lambda_2 = 2$$

$$(A - \lambda_2 E) \bar{v} = 0 \Rightarrow \begin{pmatrix} -9 & -3 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -9 & -3 \\ -3 & 1 \end{pmatrix} \sim \begin{pmatrix} 3 & 1 \\ 0 & 0 \end{pmatrix} \Rightarrow 3v_1 + v_2 = 0; \quad v_1\text{-базис.}$$

$v_2$ -свобод.

$$v_1 = -\frac{1}{3}c_2; \quad v_2 = c_2$$

$$\bar{v} = c_2 \begin{pmatrix} -\frac{1}{3} \\ 1 \end{pmatrix}, \text{ пусть } \bar{a}_2 = \begin{pmatrix} -\frac{1}{3} \\ 1 \end{pmatrix}, |\bar{a}_2| = \frac{\sqrt{10}}{3}$$

$$c_2 = \frac{3}{\sqrt{10}}$$

$$\bar{v} = \frac{3}{\sqrt{10}} \begin{pmatrix} -\frac{1}{3} \\ 1 \end{pmatrix}$$

В базисе  $\bar{u}, \bar{v}$   $f = -8x^2 + 2y^2$

Матрица перехода:

$$T = \begin{pmatrix} \frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \end{pmatrix} \quad \det T = 1$$

$$3y + 4z = 0$$

$$\begin{vmatrix} 3 & 4 \\ 1 & 1 \end{vmatrix} = 0$$

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Замена переменных :

$$\begin{cases} x = \frac{3}{\sqrt{10}} x_1 - \frac{1}{\sqrt{10}} y_1 \\ y = \frac{1}{\sqrt{10}} x_1 + \frac{3}{\sqrt{10}} y_1 \end{cases}$$

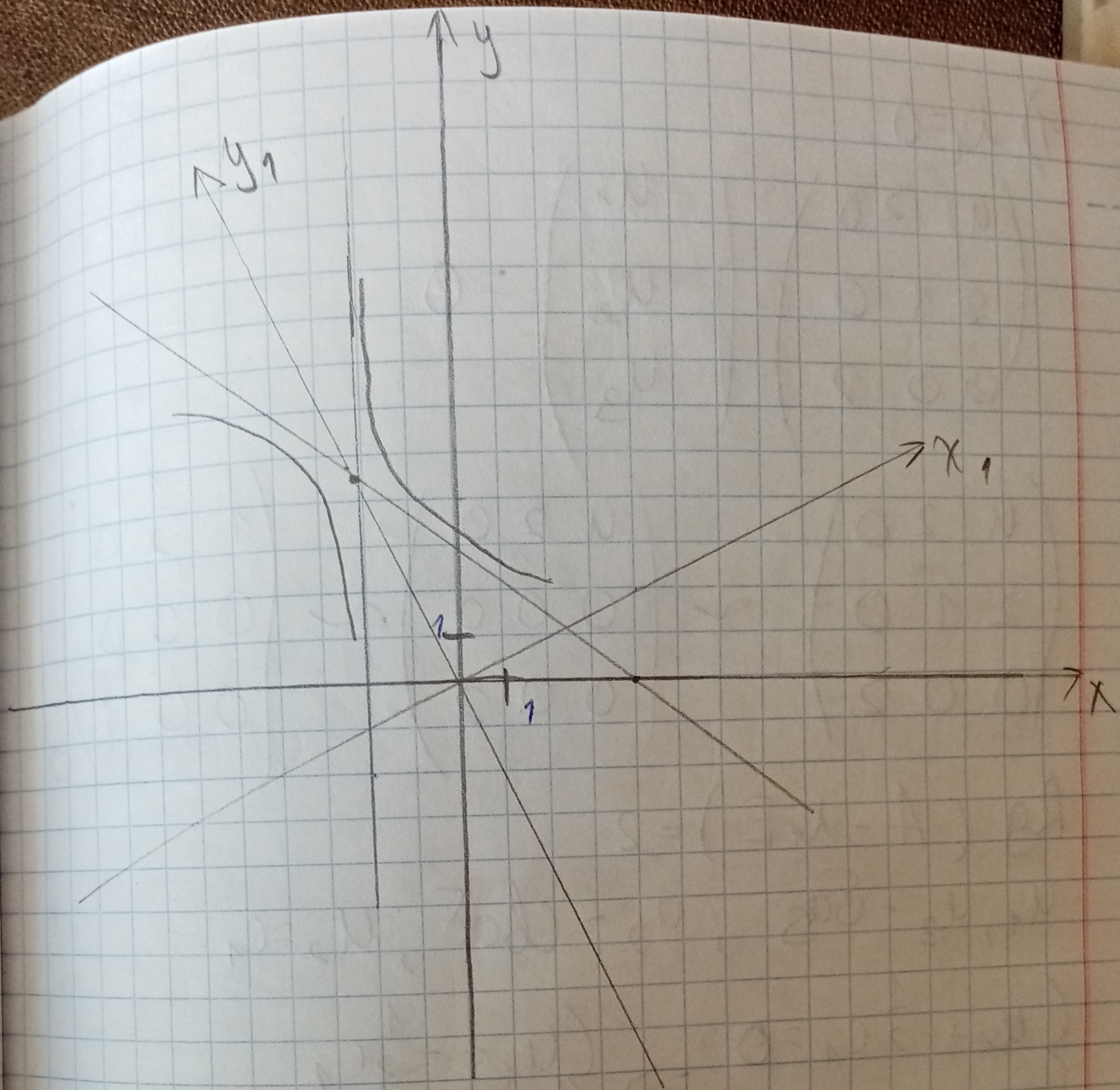
$$-8x_1^2 + 2y_1^2 - 20y_1 + 42 = 0$$

$$2(y_1^2 - 20y_1 + 25) - 8x_1^2 - 8 = 0$$

$$2(y_1 - 5)^2 - 8x_1^2 = 8$$

$$\frac{(y_1 - 5)^2}{4} - x_1^2 = 1 \text{ - гиперболою}$$

$$\begin{cases} x_2 = x_1 \\ y_2 = y_1 - 5 \end{cases} \Rightarrow \frac{y_2^2}{4} - \frac{x_2^2}{1} = 1$$



$$c) 4x^2 + y^2 + 2z^2 + uxy + 2\sqrt{5}x - 4\sqrt{5}y - 42 - 18 = 0$$

$$f = 4x^2 + y^2 + 2z^2 + uxy$$

$$A = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\begin{vmatrix} 4-\lambda & 2 & 0 \\ 2 & 1-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{vmatrix} = 0$$

$$d_1 = 0$$

$$d_2 = 2$$

$$d_3 = 5$$

$$1) \lambda_1 = 0$$

$$\begin{pmatrix} 4 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} 4 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 4 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\text{Rg}(A - \lambda_1 E) = 2$$

$$u_1, u_3 - \text{базис}; u_2 - \text{свобод}; u_2 = c_1$$

$$\begin{cases} u_1 + \frac{1}{2}c_1 = 0 \\ u_3 = 0 \end{cases} \Rightarrow \begin{cases} u_1 = -\frac{1}{2}c_1 \\ u_3 = 0 \end{cases}$$

$$\bar{u} = \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix} c_1; \text{ пусть } \bar{a}_1 = \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix}$$

$$\bar{u} = \frac{2}{\sqrt{5}} \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix}$$

2)  $\lambda_2 = 2$

$$\begin{pmatrix} 2 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{rg}(A - \lambda_2 E) = 2$$

$v_1, v_2$  - базис,  $v_3$  - свободный,  $v_3 = c_2$

$$\begin{cases} v_1 + v_2 = 0 \\ v_1 - v_2 \end{cases} \Rightarrow \begin{cases} v_1 = 0 \\ v_3 = 0 \end{cases}$$

$$|a_1| = \frac{\sqrt{5}}{2}$$

$$c_1 = \frac{2}{\sqrt{5}}$$

$$v = \begin{pmatrix} 0 \\ 0 \\ c_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} c_2, \text{ пусть } a_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, |a_2| = 1,$$

$$c_2 = 1.$$

$$\frac{1}{\sqrt{5}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$3) \lambda_3 = 5$$

$$\begin{pmatrix} -1 & 2 & 0 \\ 2 & -4 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 2 & 0 \\ 2 & -4 & 0 \\ 0 & 0 & -3 \end{pmatrix} \sim \begin{pmatrix} -1 & 2 & 0 \\ 0 & 0 & -3 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} -1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{rg}(A - \lambda_3 E) = 2$$

$w_1, w_3$  - базис,  $w_2$  - свободен,  $w_2 = c_3$

$$\begin{cases} -w_1 + 2c_3 = 0 \\ -w_3 = 0 \end{cases} \Rightarrow \begin{cases} w_1 = 2c_3 \\ w_3 = 0 \end{cases}$$

$$\overline{w} = c_3 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \text{ пусть } \overline{a}_3 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, |\overline{a}| = \sqrt{5}, c_3 = \frac{1}{\sqrt{5}}$$

$$w = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

Матрица

$$T = \begin{pmatrix} -\frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{5}} \\ 0 \end{pmatrix}$$

Замена

$$\begin{cases} x = -\frac{1}{\sqrt{3}} \\ y = \frac{2}{\sqrt{5}} \\ z = y_1 \end{cases}$$

$$2y_1^2 + 3$$

$$2(y_1^2 -$$

$$2(y_1 - 1)$$

$$\frac{(y_1 - 1)^2}{5}$$

$$5$$

$$x_2 = x$$

$$y_2 = y$$

$$z_1 = z$$

$$\begin{pmatrix} -1 & 20 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$W_2 = C_3$$

$$|\bar{a}| = \sqrt{5}$$

$$C_3 = \frac{1}{\sqrt{5}}$$

Матрица перехода:

$$T = \begin{pmatrix} -\frac{1}{\sqrt{5}} & 0 & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}} \\ 0 & 1 & 0 \end{pmatrix}$$

$$\det T = 1$$

Замена переменных:

$$x = -\frac{1}{\sqrt{5}}x_1 + \frac{2}{\sqrt{5}}z_1$$

$$y = \frac{2}{\sqrt{5}}x_1 + \frac{1}{\sqrt{5}}z_1$$

$$z = y_1$$

$$2y_1^2 + 5z_1^2 - 10x_1 - 4y_1 - 18 = 0$$

$$2(y_1^2 - 2y_1 + 1) + 5z_1^2 - 10x_1 - 20 = 0$$

$$2(y_1 - 1)^2 + 5z_1^2 = 10x_1 + 20$$

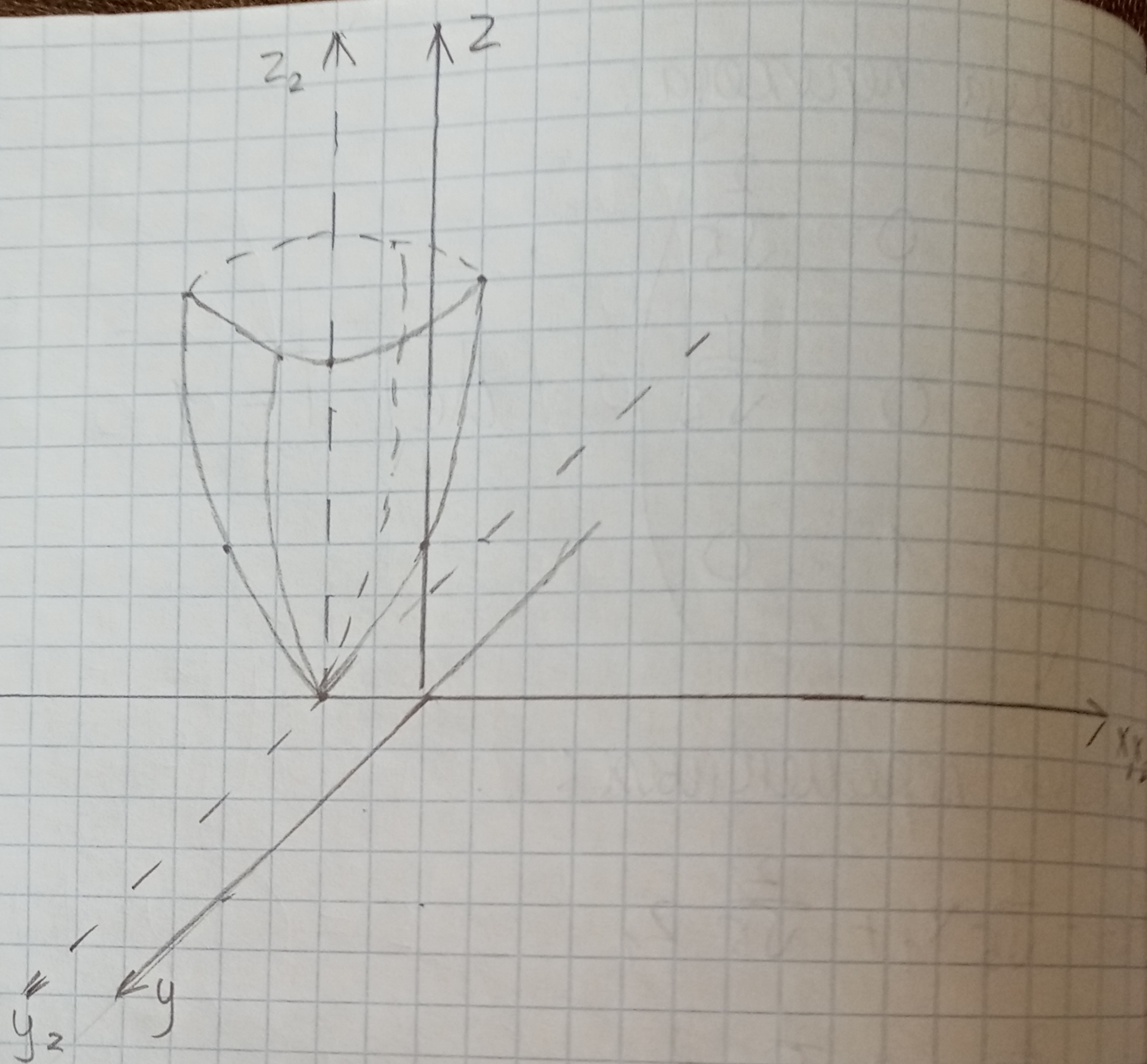
$$\frac{(y_1 - 1)^2}{5} + \frac{z_1^2}{2} = x_1 + 2$$

$$x_2 = x_1 + 2$$

$$\frac{y_2^2}{5} + \frac{z_2^2}{2} = x_2$$

$$\begin{cases} y_2 = y_1 - 1 \\ z_2 = z_1 \end{cases}$$

- эллиптический параболоид



d)  $5x^2 + 3y^2 + 7z^2 + 8xy - 8xz - 10x - 20y - 20z$   
 $f = 5x^2 + 3y^2 + 7z^2 + 8xy - 8xz$

$$A = \begin{pmatrix} 5 & 4 & -4 \\ 4 & 3 & 0 \\ -4 & 0 & 7 \end{pmatrix}$$

$$\begin{vmatrix} 5-\lambda & 4 & -4 \\ 4 & 3-\lambda & 0 \\ -4 & 0 & 7-\lambda \end{vmatrix} = 0; \quad \lambda_1 = -1; \quad \lambda_2 = 5; \quad \lambda_3 = 1$$

$$1) \lambda = 1$$

$$\begin{pmatrix} 6 & 4 \\ 4 & 4 \\ -4 & 0 \end{pmatrix}$$

$$2) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$u_1, u_3$$

$$\begin{cases} u_1 + \frac{2}{3} \\ 2u_3 + \end{cases}$$

$$\bar{u} = c_1$$

$$\bar{u} = \frac{2}{3}$$

$$1) \lambda = 1 \quad \begin{pmatrix} 6 & 4 & -4 \\ 4 & 4 & 0 \\ -4 & 0 & 8 \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{2}{3} & -\frac{2}{3} \\ 4 & 4 & 0 \\ -4 & 0 & 8 \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{2}{3} & -\frac{2}{3} \\ 0 & \frac{4}{3} & \frac{8}{3} \\ 0 & \frac{8}{3} & \frac{16}{3} \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & \frac{2}{3} & -\frac{2}{3} \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{rg}(A - \lambda_1 E) = 2,$$

$$u_1, u_3 - \text{базис}; \quad u_2 = 0, \quad u_2 = c_1$$

$$\begin{cases} u_1 + \frac{2}{3}c_1 - \frac{2}{3}u_3 = 0 \\ 2u_3 + c_1 = 0 \end{cases} \Rightarrow \begin{cases} u_1 = -c_1 \\ u_3 = -\frac{1}{2}c_1 \end{cases}$$

$$\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} \sim \frac{2}{3} \begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix}$$

$$\frac{6}{4}$$

$$\bar{u} = c_1 \begin{pmatrix} -1 \\ 1 \\ -\frac{1}{2} \end{pmatrix}, \quad \text{нормы } \bar{a}_1 = \begin{pmatrix} -1 \\ 1 \\ -\frac{1}{2} \end{pmatrix}, \quad |\bar{a}_1| = \frac{3}{2}$$

$$\bar{u} = \frac{2}{3} \begin{pmatrix} -1 \\ 1 \\ -\frac{1}{2} \end{pmatrix}$$

$$5; \quad \mathcal{R}_3 = 11$$

$$2) \lambda_2 = 5$$

$$\begin{pmatrix} 0 & 4 & -4 \\ 4 & -2 & 0 \\ -4 & 0 & 2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 4 & -4 \\ 4 & -2 & 0 \\ -4 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 4 & -4 \\ -4 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{rg}(A - \lambda_2 E) = 2$$

$$v_1, v_3 - \text{базис}, v_2 - \text{об.}, v_2 = c_2$$

$$\begin{cases} v_1 - \frac{1}{2} v_2 = 0 \\ c_2 - v_3 = 0 \end{cases} \begin{cases} v_1 = \frac{1}{2} c_2 \\ c_2 = v_3 \end{cases}$$

$$\vec{v} = c_2 \begin{pmatrix} \frac{1}{2} \\ 2 \\ 1 \end{pmatrix}, \text{нормь } \vec{a}_2 = \begin{pmatrix} \frac{1}{2} \\ 1 \\ 1 \end{pmatrix}, |\vec{a}| = \frac{3}{2}, c_2 = \frac{2}{3}$$

$$\vec{v} = \frac{2}{3} \begin{pmatrix} \frac{1}{2} \\ 1 \\ 1 \end{pmatrix}$$

$$3) \lambda_3 = 11$$

$$\begin{pmatrix} -6 & 4 & -4 \\ -4 & -8 & 0 \\ -4 & 0 & -4 \end{pmatrix}$$

$$\begin{pmatrix} -6 & 4 & -4 \\ 4 & -8 & 0 \\ -4 & 0 & -4 \end{pmatrix}$$

$$\text{rg}(A - \lambda_3 E)$$

$$w_1, w_2 - \text{базис}$$

$$\begin{cases} w_1 - \frac{2}{3} w_2 \\ w_2 + \frac{1}{2} c_3 \end{cases}$$

$$\begin{cases} w_1 = -\frac{2}{6} c_3 \\ w_2 = -\frac{1}{2} c_3 \end{cases}$$

$$\vec{w} = \begin{pmatrix} -1 \\ -\frac{1}{2} \\ -1 \end{pmatrix} c_3$$

$$\vec{w} = \frac{2}{3} \begin{pmatrix} -1 \\ -\frac{1}{2} \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} -\frac{1}{2} & 0 \\ 1 & -1 \\ 0 & 0 \end{pmatrix}$$

$$3) \lambda_3 = 11$$

$$\begin{pmatrix} -6 & 4 & -4 \\ -4 & -8 & 0 \\ -4 & 0 & -4 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -6 & 4 & -4 \\ 4 & -8 & 0 \\ -4 & 0 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & -\frac{2}{3} & \frac{2}{3} \\ 0 & -\frac{16}{3} & -\frac{8}{3} \\ 0 & -\frac{8}{3} & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & -\frac{2}{3} & \frac{2}{3} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{rg}(A - \lambda_3 E) = 2$$

$w_1, w_2$  - базис,  $w_3$  - об.  $w_3 = c_3$

$$\begin{cases} w_1 - \frac{2}{3}w_2 + \frac{2}{3}c_3 = 0 \\ w_2 + \frac{1}{2}c_3 = 0 \end{cases}$$

$$\begin{cases} w_1 = -\frac{2}{3}w_2 + \frac{2}{3}c_3 \\ w_2 = -\frac{1}{2}c_3 \end{cases} \Rightarrow \begin{cases} w_1 = -c_3 \\ w_2 = -\frac{1}{2}c_3 \end{cases}$$

$$\begin{cases} w_1 = -\frac{2}{3}(-\frac{1}{2}c_3) + \frac{2}{3}c_3 \\ w_2 = -\frac{1}{2}c_3 \end{cases} \Rightarrow \begin{cases} w_1 = -c_3 \\ w_2 = -\frac{1}{2}c_3 \end{cases}$$

$$\bar{w} = \begin{pmatrix} -1 \\ -\frac{1}{2} \\ -1 \end{pmatrix} c_3, \text{ выберем } \bar{a}_3 = \begin{pmatrix} -1 \\ -\frac{1}{2} \\ -1 \end{pmatrix}, |\bar{a}_3| = \frac{3}{2}, c_3 = \frac{2}{3}$$

$$\bar{w} = \frac{2}{3} \begin{pmatrix} -1 \\ -\frac{1}{2} \\ -1 \end{pmatrix}$$

Матрица перехода

$$T = \begin{pmatrix} -\frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{pmatrix} \quad \det T = -1$$

Замена переменных:

$$\begin{cases} x = -\frac{2}{3}x_1 + \frac{1}{3}y_1 - \frac{2}{3}z_1 \\ y = \frac{2}{3}x_1 + \frac{2}{3}y_1 - \frac{1}{3}z_1 \\ z = -\frac{1}{3}x_1 + \frac{2}{3}y_1 + \frac{2}{3}z_1 \end{cases}$$

$$-x_1^2 + 5y_1^2 + 11z_1^2 - 10x - 20y - 20z + 45 = 0$$

$$-x_1^2 + 5y_1^2 + 11z_1^2 - 30y + 45 = 0$$

$$-x_1^2 + 5(y_1^2 - 6y + 9) + 11z_1^2 = 0$$

$$-x_1^2 + 5(y_1 - 3)^2 + 11z_1^2 = 0$$

$$-\frac{x_1^2}{11} + \frac{5(y_1 - 3)^2}{11} + z_1^2 = 0$$

$$\begin{cases} x_1 = x_2 \\ y_1 - 3 = y_2 \\ z_1 = z_2 \end{cases}$$

$$-\frac{x_2^2}{11} + \frac{5y_2^2}{11} + z_2^2 = 0$$

- конус 2-го порядка