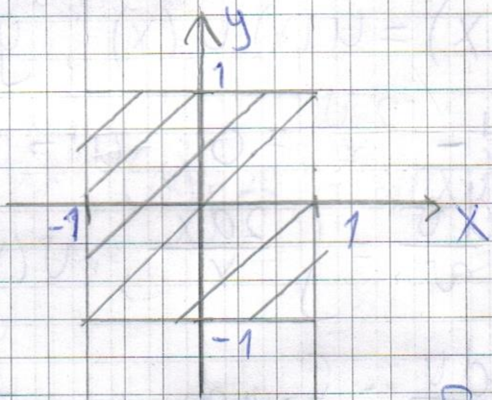
КР по ФНП Корсаков Макар ИУ1-21

вариант 7

№1

$z = 2\sqrt{1-x^2} - \sqrt{1-y^2}$; линии уровня через $(\frac{\sqrt{3}}{2}; 0)$

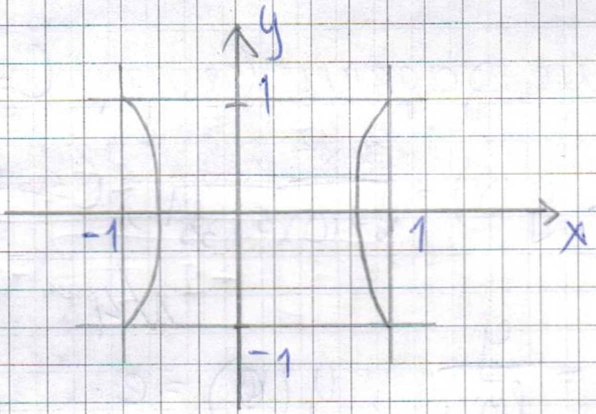
1) $\begin{cases} 1-x^2 \geq 0 \\ 1-y^2 \geq 0 \end{cases} \Leftrightarrow \begin{cases} -1 \leq x \leq 1 \\ -1 \leq y \leq 1 \end{cases}$



2) $C = 2\sqrt{1-x^2} - \sqrt{1-y^2}$, т.к. линия проходит через $(\frac{\sqrt{3}}{2}; 0)$

~~$C = 2\sqrt{1-x^2}$~~ $C = 2\sqrt{1-(\frac{\sqrt{3}}{2})^2} - \sqrt{1-0^2} = 2\sqrt{1-\frac{3}{4}} - 1 = 0$

$2\sqrt{1-x^2} - \sqrt{1-y^2} = 0$



№2

$u = xy^2z^3 + x^2 + y^2 + z^2$; $M(1; 1, 1)$; $\ell = i + j + k$

1) $\frac{du}{dx} = (xy^2z^3 + x^2 + y^2 + z^2)'_x = y^2z^3 + 2x$

$\left.\frac{du}{dx}\right|_M = 1^2 + 1^3 + 2 \cdot 1 = 3$

$$\frac{du}{dy} = (xy^2z^3 + x^2 + y^2 + z^2)'_y = 2xyz^3 + 2y$$

$$\left.\frac{du}{dy}\right|_M = 2 \cdot 1 \cdot 1 \cdot 1^3 + 2 \cdot 1 = 4$$

$$\frac{du}{dz} = (xy^2z^3 + x^2 + y^2 + z^2)'_z = 3xy^2z^2 + 2z$$

$$\left.\frac{du}{dz}\right|_M = 3 \cdot 1 \cdot 1^2 \cdot 1^2 + 2 \cdot 1 = 5.$$

$$\operatorname{grad} u = \left(\frac{du}{dx}; \frac{du}{dy}; \frac{du}{dz}\right) = (3; 4; 5)$$

2) $|a| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$; $a_0 = \left(\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}\right)$

$$\left.\frac{du}{dl}\right| = \left.\frac{du}{dx}\right|_M \cdot \cos\alpha + \left.\frac{du}{dy}\right|_M \cdot \cos\beta + \left.\frac{du}{dz}\right|_M \cdot \cos\gamma$$

$$\left.\frac{du}{dl}\right| = \left.\frac{du}{dx}\right|_M \cdot \cos\alpha + \left.\frac{du}{dy}\right|_M \cdot \cos\beta + \left.\frac{du}{dz}\right|_M \cdot \cos\gamma =$$

$$= 3 \cdot \frac{1}{\sqrt{3}} + 4 \cdot \frac{1}{\sqrt{3}} + 5 \cdot \frac{1}{\sqrt{3}} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$$

Ответ: $\operatorname{grad} u = (3; 4; 5)$; $\left.\frac{du}{dl}\right| = 4\sqrt{3}$.

№3

$$\exists \lim_{\substack{x \to 0 \\ y \to 0}} \ln\left(\frac{1-y^2}{1+x^2}\right) - ?$$

$$\lim_{\substack{x \to 0 \\ y \to 0}} \ln\left(\frac{1-y^2}{1+x^2}\right) = \lim_{x \to 0} \ln\left(\frac{1-0^2}{1+x^2}\right) = \lim_{x \to 0} \ln\left(\frac{1}{1+x^2}\right) = \ln 1 = 0.$$

$$\lim_{\substack{x \to 0 \\ y \to 0}} \ln\left(\frac{1-y^2}{1+x^2}\right) = \lim_{y \to 0} \ln\left(\frac{1-y^2}{1+0^2}\right) = \lim_{y \to 0} \ln(1-y^2) = \ln 1 = 0$$

Значит, предел существует так как не зависит от способа приближения

№4

$u = x^2 y + \arcsin\frac{y}{z}$

1) $\frac{du}{dx} = (x^2 \cdot y + \arcsin\frac{y}{z})'_x = 2xy$

$\frac{du}{dy} = (x^2 \cdot y + \arcsin\frac{y}{z})'_y = x^2 + \frac{1}{\sqrt{1-\frac{y^2}{z^2}}} \cdot \frac{1}{z} = x^2 + \frac{1}{\sqrt{z^2-y^2}}$

$\frac{du}{dz} (x^2 \cdot y + \arcsin\frac{y}{z})'_z = \frac{1}{\sqrt{1-\frac{y^2}{z^2}}} \cdot \left(-\frac{y}{z^2}\right) = -\frac{y}{z\sqrt{z^2-y^2}}$

2) $\frac{d^2u}{dx^2} = (2xy)'_x = 2y$

$\frac{d^2u}{dy^2} = \left(x^2 + \frac{1}{\sqrt{z^2-y^2}}\right)'_y = -\frac{1}{2}(z^2-y^2)^{-\frac{3}{2}} \cdot (-2y) = \frac{y}{\sqrt{(z^2-y^2)^3}}$

$\frac{d^2u}{dz^2} = \left(-\frac{y}{z\sqrt{z^2-y^2}}\right)'_z = -y \cdot \left(-\frac{1}{2}\right)(z^4-y^2z^2)^{-\frac{3}{2}} \cdot (4z^3-2y^2z) =$

$= \cancel{y} \frac{y(2z^3-yz)}{z^3\sqrt{(z^2-y^2)^3}} = \frac{y(2z^2-y^2)}{z^2\sqrt{\cancel{z}(z^2-y^2)^3}}$

$\frac{d^2u}{dx\,dy} = (2xy)' = 2x$, $\frac{d^2u}{dx\,dz} = (2xy)' = 0$

$\frac{d^2u}{dy\,dx} = \left(x^2 + \frac{1}{\sqrt{z^2-y^2}}\right)' = 2x$

$\frac{d^2u}{dy\,dz} = \left(x^2 + \frac{1}{\sqrt{z^2-y^2}}\right)' = -\frac{1}{2}(z^2-y^2)^{-\frac{3}{2}} \cdot (2z) = \frac{-z}{\sqrt{(z^2-y^2)^3}}$

$$\frac{d^2y}{dz\,dx}=\left(-\frac{y}{2\sqrt{z^2-y^2}}\right)=0$$

$$\frac{d^2y}{dz\,dy}=\left(-\frac{y}{2\sqrt{z^2-y^2}}\right)'=-\frac{1}{2\sqrt{z^2-y^2}}+\left(-\frac{y}{2}\right)\left(-\frac{1}{2}\right)(z^2-y^2)^{-}$$

$$\cdot(-2y)=-\frac{1}{2\sqrt{z^2-y^2}}-\frac{y^2}{2\sqrt{(z^2-y^2)^3}}$$