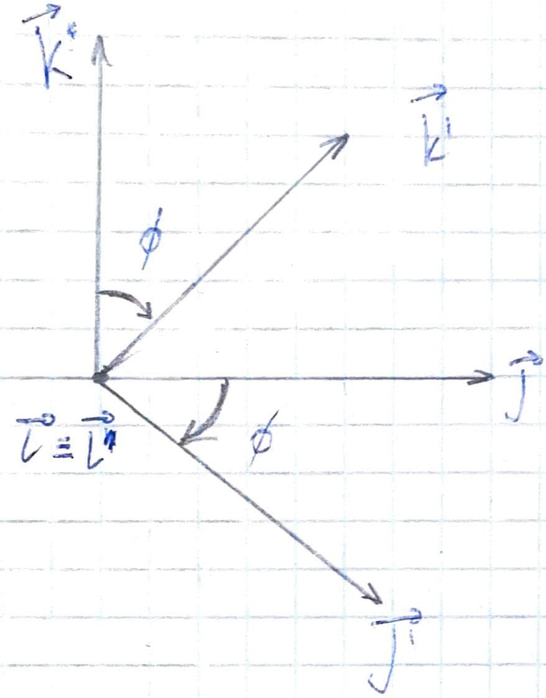
N 1

1) $\phi = -45^\circ$




$\{\vec{i}; \vec{j}; \vec{k}\} \xrightarrow{T_1} \{\vec{i'}; \vec{j'}; \vec{k'}\}$

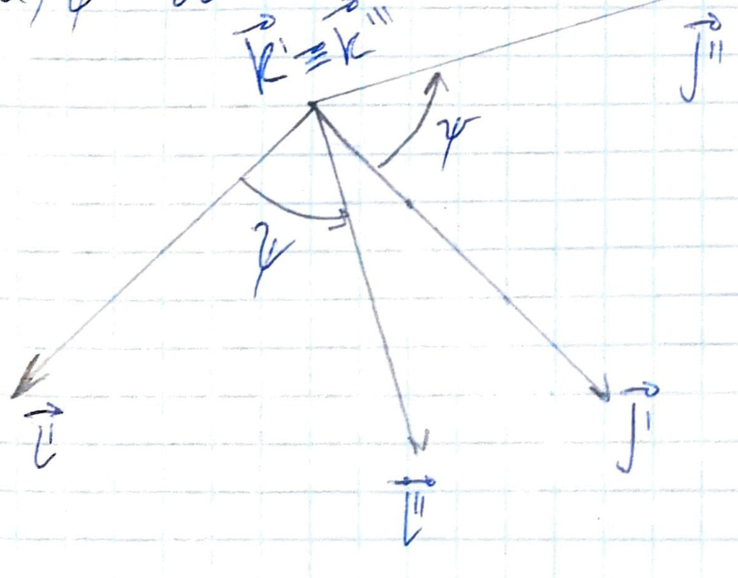
$\xrightarrow{T_2} \{\vec{i''}; \vec{j''}; \vec{k''}\}$

$\{\vec{i}; \vec{j}; \vec{k}\} \xrightarrow{T} \{\vec{i''}; \vec{j''}; \vec{k''}\}$

$T = T_1 \cdot T_2$

$$T_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

2) $\psi = 60^\circ$




$$T_2 = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T = T_1 \times T_2 = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{6}}{4} & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

№2

$$\vec{p} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}_{\{\vec{a}\}} \qquad \vec{q} = \begin{pmatrix} 3 \\ 3 \\ 3 \\ 1 \end{pmatrix}_{\{\vec{a}\}}$$

$$\vec{a}_1 = \begin{pmatrix} 1 \\ -2 \\ 3 \\ -2 \end{pmatrix} \quad \vec{a}_2 = \begin{pmatrix} 1 \\ 5 \\ -1 \\ 3 \end{pmatrix} \quad \vec{a}_3 = \begin{pmatrix} -4 \\ -3 \\ 2 \\ -5 \end{pmatrix} \quad \vec{a}_4 = \begin{pmatrix} -4 \\ -4 \\ 0 \\ 2 \end{pmatrix}$$

Ортогонализация

$$\vec{g}_1 = \vec{a}_1$$

$$\vec{g}_2 = \vec{a}_2 - \frac{(\vec{a}_2, \vec{a}_1)}{(\vec{a}_1, \vec{a}_1)} \vec{a}_1 = \left\{ \begin{aligned} &(\vec{a}_2; \vec{a}_1) = 1 + (-10) - 3 - 6 = -18 \\ &(\vec{a}_1; \vec{a}_1) = 1 + 4 + 9 + 4 = 18 \\ &\vec{a}_2 + \vec{a}_1 = \{2; 3; 2; 1\} \end{aligned} \right.$$

$$= \{2; 3; 2; 1\}$$

$$\vec{g_3} = \vec{a_3} - \frac{(\vec{a_3};\vec{a_1})}{(\vec{a_1};\vec{a_1})}\vec{a_1} - \frac{(\vec{a_3};\vec{g_2})}{(\vec{g_2};\vec{g_2})}\vec{g_2} =$$

$$= \begin{cases} (\vec{a_3};\vec{a_1}) = -4+6+6+10=18 \\ (\vec{a_3};\vec{g_2}) = -8-9+4-5=-18 \\ (\vec{g_2};\vec{g_2}) = 4+9+4+1=18 \\ \vec{a_3}-\vec{a_1}+\vec{g_2} = \{-3;2;1;-2\} \end{cases} = \{-3;2;1;-2\}$$

$$\vec{g_4} = \vec{a_4} - \frac{(\vec{a_4};\vec{a_1})}{(\vec{a_1};\vec{a_1})}\vec{a_1} - \frac{(\vec{a_4};\vec{g_2})}{(\vec{g_2};\vec{g_2})}\vec{g_2} - \frac{(\vec{a_4};\vec{g_3})}{(\vec{g_3};\vec{g_3})}\vec{g_3} =$$

$$= \begin{cases} (\vec{a_4};\vec{a_1}) = -4+8+0-4=0 \\ (\vec{a_4};\vec{g_2}) = -8-12+0+2=-18 \\ (\vec{a_4};\vec{g_3}) = +12-8+0-4=\cancel{24}\ 0 \\ (\vec{g_3};\vec{g_3}) = 9+4+1+4=18 \end{cases} =$$

$$= \{-2;-1;2;3\}$$

$$\vec{b_1} = \frac{\vec{a_1}}{\sqrt{18}} = \begin{pmatrix} \frac{1}{\sqrt{18}} \\ \frac{-2}{\sqrt{18}} \\ \frac{3}{\sqrt{18}} \\ -\frac{2}{\sqrt{18}} \end{pmatrix} \qquad \vec{b_2} = \frac{\vec{g_2}}{\sqrt{18}} = \begin{pmatrix} \frac{2}{\sqrt{18}} \\ \frac{3}{\sqrt{18}} \\ \frac{2}{\sqrt{18}} \\ \frac{1}{\sqrt{18}} \end{pmatrix}$$

$$\vec{b}_3 = \begin{pmatrix} -\frac{3}{\sqrt{18}} \\ \frac{2}{\sqrt{18}} \\ \frac{1}{\sqrt{18}} \\ -\frac{2}{\sqrt{18}} \end{pmatrix} \qquad \vec{b}_4 = \begin{pmatrix} -\frac{2}{\sqrt{18}} \\ -\frac{1}{\sqrt{18}} \\ \frac{2}{\sqrt{18}} \\ \frac{3}{\sqrt{18}} \end{pmatrix}$$

$$T_{A\to B} = A^{-1}B$$

$$\cancel{A} \left(\begin{array}{cccc|cccc} 1 & 1 & -4 & -4 & 1 & 0 & 0 & 0 \\ -2 & 5 & -3 & -4 & 0 & 1 & 0 & 0 \\ 3 & -1 & 2 & 0 & 0 & 0 & 1 & 0 \\ -2 & 3 & 5 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \quad \mathrm{II} - \mathrm{IV}$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & -4 & -4 & 1 & 0 & 0 & 0 \\ 0 & 2 & -8 & -6 & 0 & 1 & 0 & -1 \\ 3 & -1 & 2 & 0 & 0 & 0 & 1 & 0 \\ -2 & 3 & 5 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \quad \tfrac{1}{2}\mathrm{II}, \mathrm{III}, \mathrm{IV}$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & -4 & -4 & 1 & 0 & 0 & 0 \\ 0 & 1 & -4 & -3 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{3}{2} & -\frac{1}{2} & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ -1 & +\frac{3}{2} & \frac{5}{2} & 1 & 0 & 0 & 0 & \frac{1}{2} \end{array}\right) \quad \mathrm{I} - \mathrm{II}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & -4 & -3 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{3}{2} & \frac{1}{2} & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ -1 & \frac{3}{2} & \frac{5}{2} & 1 & 0 & 0 & 0 & \frac{1}{2} \end{array}\right)$$

$\underline{\mathrm{III}} + \frac{3}{2}\underline{\mathrm{I}}$

$\underline{\mathrm{IV}} + \underline{\mathrm{I}}$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & -4 & -3 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & 0 & -\frac{3}{2} & \frac{3}{2} & -\frac{3}{4} & \frac{1}{2} & \frac{3}{4} \\ 0 & \frac{3}{2} & \frac{5}{2} & 0 & 1 & -\frac{1}{2} & 0 & 1 \end{array}\right)$$

$\underline{\mathrm{III}} + \frac{1}{2}\underline{\mathrm{II}}$

$\underline{\mathrm{IV}} - \frac{3}{2}\underline{\mathrm{II}}$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & -4 & -3 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & -2 & -3 & \frac{3}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{17}{2} & \frac{9}{2} & 1 & -\frac{5}{4} & 0 & \frac{7}{4} \end{array}\right)$$

$\underline{\mathrm{III}} \cdot \left(\frac{1}{-2}\right)$

$\underline{\mathrm{IV}} - \frac{17}{2}\underline{\mathrm{III}}$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & -4 & -3 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{3}{2} & -\frac{3}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 0 & -21 & \frac{59}{8} & -\frac{27}{8} & \frac{17}{8} & \frac{31}{8} \end{array}\right)$$

$\underline{\mathrm{IV}} \cdot \left(-\frac{1}{21}\right)$

$\underline{\mathrm{I}} + \overline{\mathrm{IV}}$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & -4 & -3 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{3}{2} & -\frac{3}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 0 & 1 & -\frac{59}{168} & \frac{9}{56} & -\frac{17}{168} & -\frac{31}{168} \end{array}\right)$$

$III + \frac{3}{2} IV$

$II + 4 III$

$II + 3 IV$

$$\begin{pmatrix} -\frac{29}{18} & \frac{10}{9} & \frac{17}{18} & -1 \\ -\frac{11}{6} & \frac{4}{3} & \frac{5}{6} & -1 \\ \frac{3}{2} & -1 & -\frac{1}{2} & 1 \\ -\frac{47}{18} & \frac{29}{18} & \frac{17}{18} & -\frac{3}{2} \end{pmatrix} = A^{-1}$$

$$T_{A \to B} = A^{-1} B$$

$$\begin{pmatrix} -\frac{29}{18} & \frac{10}{9} & \frac{17}{18} & -1 \\ -\frac{11}{6} & \frac{4}{3} & \frac{5}{6} & -1 \\ \frac{3}{2} & -1 & -\frac{1}{2} & 1 \\ -\frac{47}{18} & \frac{29}{18} & \frac{17}{18} & -\frac{3}{2} \end{pmatrix} \times \begin{pmatrix} \frac{1}{\sqrt{18}} & \frac{2}{\sqrt{18}} & -\frac{3}{\sqrt{18}} & -\frac{2}{\sqrt{18}} \\ -\frac{2}{\sqrt{18}} & \frac{3}{\sqrt{18}} & \frac{2}{\sqrt{18}} & -\frac{1}{\sqrt{18}} \\ \frac{3}{\sqrt{18}} & \frac{2}{\sqrt{18}} & \frac{1}{\sqrt{18}} & \frac{2}{\sqrt{18}} \\ -\frac{2}{\sqrt{18}} & \frac{1}{\sqrt{18}} & -\frac{2}{\sqrt{18}} & \frac{3}{\sqrt{18}} \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{29\cdot 1}{18\sqrt{18}} + \frac{10\cdot 2}{9\sqrt{18}} + \frac{17\cdot 3}{18\sqrt{18}} + \frac{2}{\sqrt{18}} & -\frac{29\cdot 2}{18\sqrt{18}} + \frac{10\cdot 3}{9\sqrt{18}} + \frac{17\cdot 2}{18\sqrt{18}} - \frac{1}{\sqrt{18}} & \cdots \\ -\frac{11}{6\sqrt{18}} - \frac{8}{3\sqrt{18}} + \frac{15}{6\sqrt{18}} + \frac{2}{\sqrt{18}} & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{18}} & \frac{1}{\sqrt{18}} & 0 & \frac{1}{\sqrt{18}} \\ 0 & \frac{1}{\sqrt{18}} & \frac{1}{\sqrt{18}} & \frac{1}{\sqrt{18}} \\ 0 & 0 & \frac{1}{\sqrt{18}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{18}} \end{pmatrix}$$

$$T_{B\to A} = \begin{pmatrix} \sqrt{18} & -\sqrt{18} & \sqrt{18} & 0 \\ 0 & \sqrt{18} & -\sqrt{18} & -\sqrt{18} \\ 0 & 0 & \sqrt{18} & 0 \\ 0 & 0 & 0 & \sqrt{18} \end{pmatrix}$$

$$\vec{p}\,' = T_{A\to B}\,\vec{p} = \begin{pmatrix} \frac{1}{\sqrt{18}} & \frac{1}{\sqrt{18}} & 0 & \frac{1}{\sqrt{18}} \\ 0 & \frac{1}{\sqrt{18}} & \frac{1}{\sqrt{18}} & \frac{1}{\sqrt{18}} \\ 0 & 0 & \frac{1}{\sqrt{18}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{18}} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{4}{\sqrt{18}} \\ \frac{4}{\sqrt{18}} \\ \frac{1}{\sqrt{18}} \\ \frac{2}{\sqrt{18}} \end{pmatrix}$$

$$\vec{p}' = T_{(B\to A)}\,\vec{p} = \begin{pmatrix} \sqrt{18} & -\sqrt{18} & \sqrt{18} & 0 \\ 0 & \sqrt{18} & -\sqrt{18} & -\sqrt{18} \\ 0 & 0 & \sqrt{18} & 0 \\ 0 & 0 & 0 & \sqrt{18} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1\sqrt{18} \\ -2\sqrt{18} \\ \sqrt{18} \\ 2\sqrt{18} \end{pmatrix}$$

$$\vec{q}' = T_{B\to A}\,\vec{q} = \begin{pmatrix} \sqrt{18} & -\sqrt{18} & \sqrt{18} & 0 \\ 0 & \sqrt{18} & -\sqrt{18} & -\sqrt{18} \\ 0 & 0 & \sqrt{18} & 0 \\ 6 & 0 & 0 & \sqrt{18} \end{pmatrix} \begin{pmatrix} 3 \\ 3 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3\sqrt{18} \\ -\sqrt{18} \\ 3\sqrt{18} \\ \sqrt{18} \end{pmatrix}$$

$$(\vec{p}'; \vec{q}') = \cancel{4\sqrt{18}}\; 3\cdot 18 + 2\cdot 18 + 3\cdot 18 + 2\cdot 18 =$$

$$= 10\cdot 18 = 180$$

$$\cos\widehat{(\vec{p}'; \vec{q}')} = \frac{(\vec{p}'; \vec{q}')}{|\vec{p}'|\cdot|\vec{q}'|} = \left| \begin{array}{l} |\vec{p}'| = 18 + 4\cdot 18 + 18 + 2\cdot 18 = \\ \quad = 180 \\ |\vec{q}'| = 9\cdot 18 - 18 + 9\cdot 18 + 18 = \\ \quad = 324 \end{array} \right|$$

$$= \frac{180}{\cancel{180}\cdot 324} = \frac{1}{324} \Rightarrow \widehat{(\vec{p}'; \vec{q}')} = \operatorname{acos}\left(\frac{1}{324}\right)$$