№ 3

a) $x^2 - 7y^2 - 6xy - 6\sqrt{10}x + 2\sqrt{10}y + 42 = 0$

$$A = \begin{pmatrix} 1 & -3 \\ -3 & -7 \end{pmatrix}$$

$$\begin{vmatrix} 1-\lambda & -3 \\ -3 & -7-\lambda \end{vmatrix} = (1-\lambda)(-7-\lambda) - 9 = 0$$

$\lambda^2 + 6\lambda - 16 = 0$

$D_1 = 9 + 16 = 25$

$$\lambda = \frac{-3 \pm \sqrt{25}}{1} = \left[\begin{array}{l} -8 \\ 2 \end{array}\right.$$

1) $\lambda = -8$

$$\begin{pmatrix} 9 & -3 \\ -3 & 1 \end{pmatrix} \xrightarrow{II + \frac{1}{3}I} \begin{pmatrix} 9 & -3 \\ 0 & 0 \end{pmatrix} \Rightarrow Rg = 1$$

x_1 – баз

y_2 – свобд.

$$\begin{cases} 9x_* - 3y = 0 \\ y = c \end{cases} \Leftrightarrow \begin{cases} x = \frac{1}{3}c \\ y = c \end{cases} \quad \overline{X}_1 = \begin{pmatrix} \frac{1}{3}c \\ c \end{pmatrix}$$

$$\vec{f}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad \vec{e}_1 = \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

2) $\lambda = 2$

$$\begin{pmatrix} -1 & -3 \\ -3 & -9 \end{pmatrix} \underset{\sim}{\text{II} - 3\text{I}} \begin{pmatrix} -1 & -3 \\ 0 & 0 \end{pmatrix} \Rightarrow Rg = 1$$

x – баз.

y – своб.

$$\begin{cases} -x - 3y = 0 \\ y = c \end{cases} \Leftrightarrow \begin{cases} x = -3c \\ y = c \end{cases} \Rightarrow \overline{X}_2 = \begin{pmatrix} -3c \\ c \end{pmatrix}$$

$$\vec{f}_2 = \begin{pmatrix} 3 \\ -1 \end{pmatrix} ; \vec{e}_2 = \frac{1}{\sqrt{10}} \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$U = \frac{1}{\sqrt{10}} \begin{pmatrix} 1 & 3 \\ 3 & -1 \end{pmatrix}$$

$$\bar{X} = \Gamma \bar{X}' \Rightarrow \begin{cases} x = \frac{1}{\sqrt{10}}(x' + 3y') \\ y = \frac{1}{\sqrt{10}}(3x' - y') \end{cases}$$

$$-8x'^2 + 2y'^2 - \cancel{6x'} - 18y' + \cancel{6x'} - 2y' + 42 = 0$$

$$-8x'^2 + 2y'^2 - 20y' + 42 = 0 \,/\, :(-2)$$

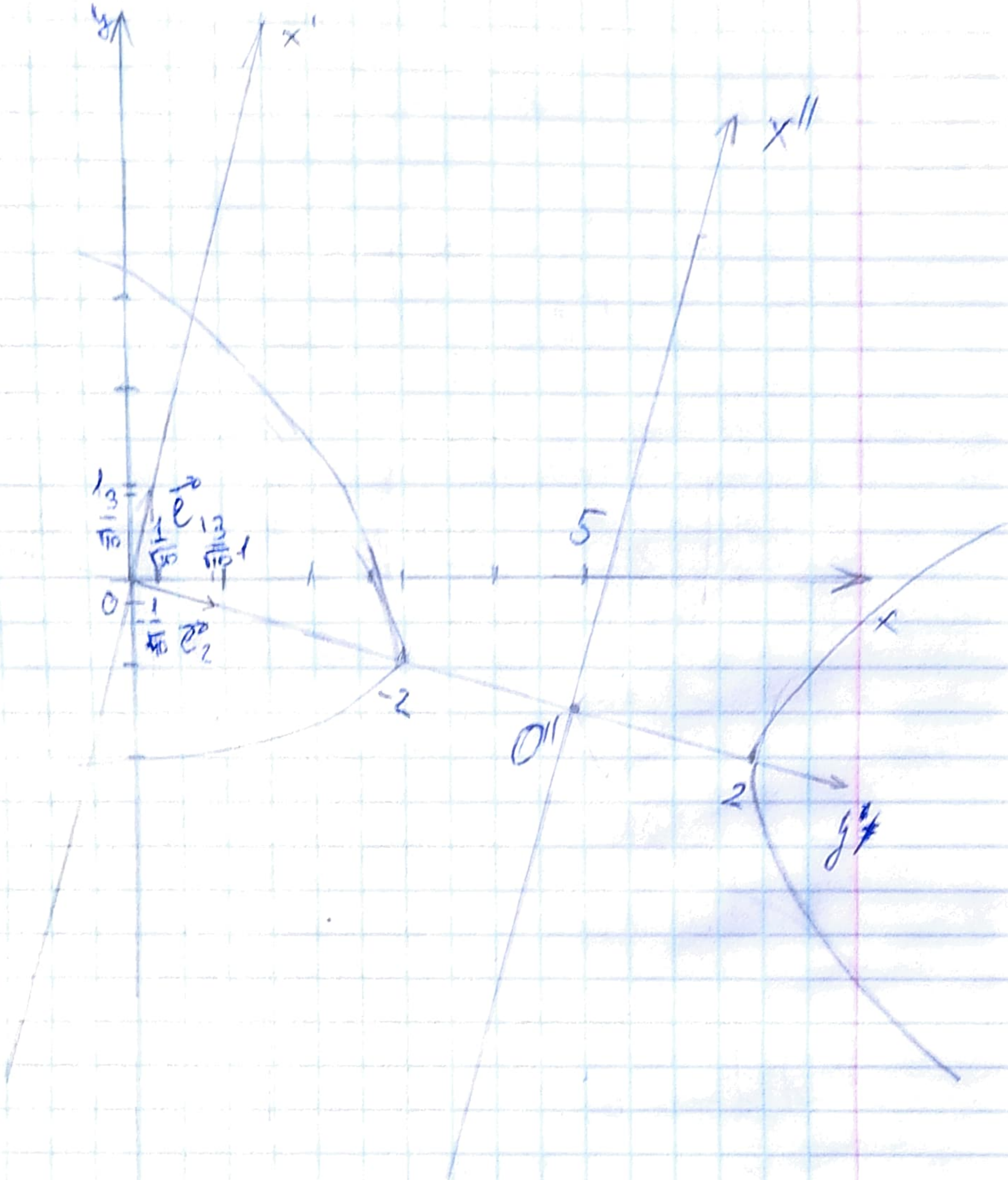
$$4x'^2 - y'^2 + 10y' - 21 = 0$$

$$4x'^2 - (y'-5)^2 + 4 = 0$$

$$x'^2 - \frac{(y'-5)^2}{4} = -1$$

$$\frac{(y'-5)^2}{4} - x'^2 = 1 \text{ - гипербола}$$

$$\begin{cases} x'' = x' \\ y'' = y' - 5 \end{cases}$$



y
x'
x''
5
-2
O''
2

б) $4xy - 5x^2 - 2y^2 - 4\sqrt{5}x + 4\sqrt{5}y - 4 = 0$

$$A = \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix}$$

$$\begin{vmatrix} -5-\lambda & 2 \\ 2 & -2-\lambda \end{vmatrix} = 0 = (-5-\lambda)(-2-\lambda) - 4 = 0$$

$$\lambda^2 + 7\lambda + 6 = 0$$

$$D = 49 - 24 = 25$$

$$\lambda = \frac{-7 \pm 5}{2} = \left[\begin{array}{l} -6 \\ -1 \end{array}\right.$$

1) $\lambda = -6$

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \overset{II-2I}{\sim} \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix} \Rightarrow Rg = 1$$

x - баз

y - своб.

$$\begin{cases} x + 2y = 0 \\ y = c \end{cases} \Leftrightarrow \begin{cases} x = -2c \\ y = c \end{cases} \quad X_1 = \begin{pmatrix} -2c \\ c \end{pmatrix}$$

$\vec{f}_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$; $\vec{e}_1 = \frac{1}{\sqrt{5}}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

2) $\lambda = -1$

$$\begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix} \underset{\text{II}+2\text{I}}{\overset{\text{I} \leftrightarrow \text{II}}{\sim}} \begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix} \Rightarrow Rg = 1$$

x - баз.

y - своб.

$$\begin{cases} 2x = y \\ y = c \end{cases} \Leftrightarrow \begin{cases} x = \frac{1}{2}c \\ y = c \end{cases} \quad \overline{X}_2 = \begin{pmatrix} \frac{1}{2}c \\ c \end{pmatrix}$$

$\vec{f}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$; $\vec{e}_2 = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$U = \frac{1}{\sqrt{5}}\begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} \quad \overline{X} = U\overline{X}' \quad \begin{cases} x = \frac{1}{\sqrt{5}}(-2x' + y') \\ y = \frac{1}{\sqrt{5}}(x' + 2y') \end{cases}$$

$$\cancel{4x} - 6x'^2 - y'^2 + 8x' - 4y' + 4x' + 8y' - 4 = 0$$

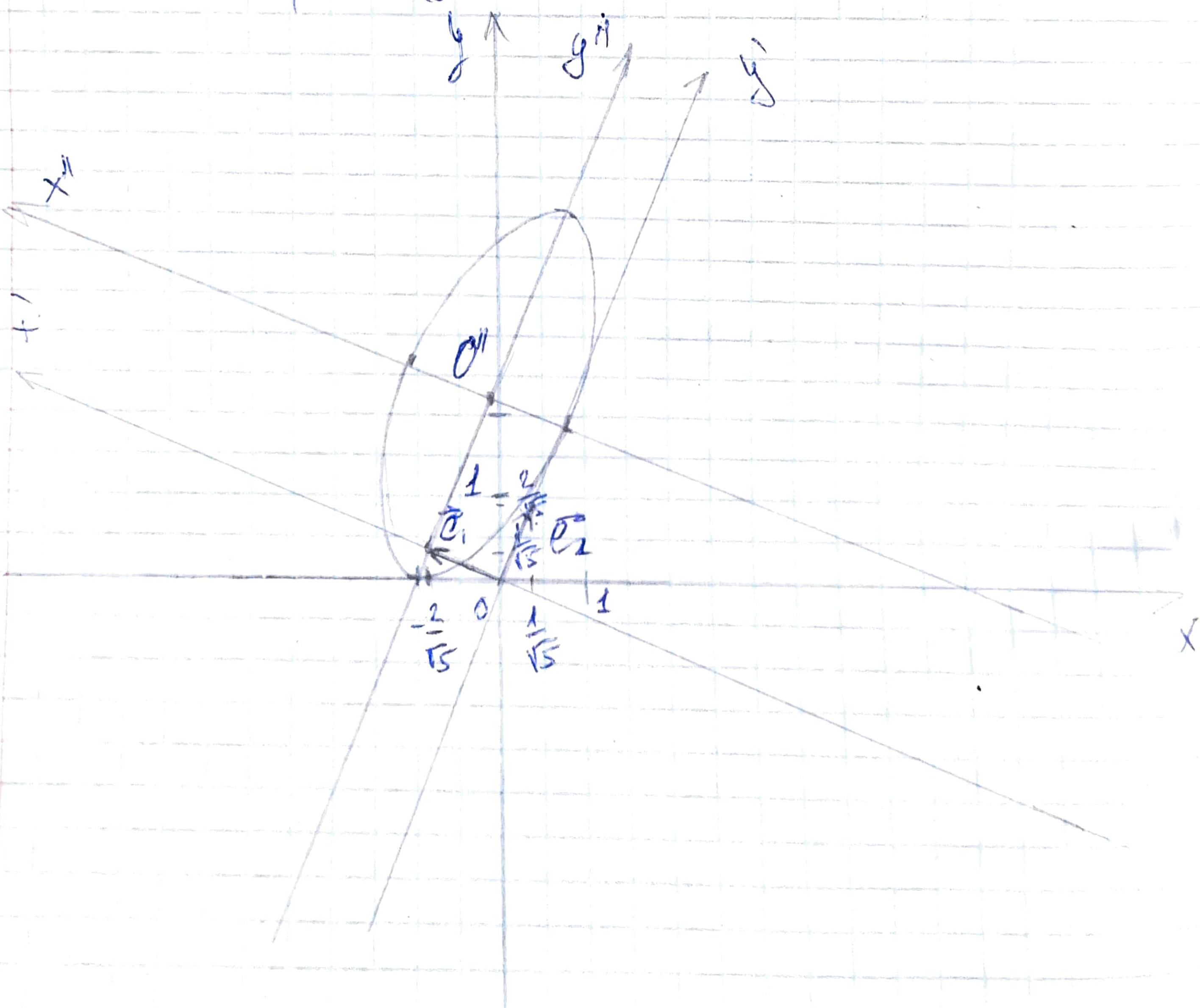
$$-6x'^2 - y'^2 + 12x' + 4y' - 4 = 0$$

$$-6(x'^2-2x'+1)+6-(y'^2-4y'+4)+4-4=0$$

$$\frac{(x'-1)^2}{1}+\frac{(y'-2)^2}{6}=1 \text{ - эллипс}$$

$$\begin{cases} x''=x'-1 \\ y''=y'-2 \end{cases}$$

$$\frac{x''^2}{1}+\frac{y''^2}{6}=1$$



б) $7x^2+5y^2+3z^2-8xy+8yz-20x-10y-20z+45=0$

$$A=\begin{pmatrix} 7 & -4 & 0 \\ -4 & 5 & 4 \\ 0 & 4 & 3 \end{pmatrix}$$

$$\begin{vmatrix} 7-\lambda & -4 & 0 \\ -4 & 5-\lambda & 4 \\ 0 & 4 & 3-\lambda \end{vmatrix} = (7-\lambda)\begin{vmatrix} 5-\lambda & 4 \\ 4 & 3-\lambda \end{vmatrix} + 4\begin{vmatrix} -4 & 4 \\ 0 & 3-\lambda \end{vmatrix} =$$

$$=0$$

$$(7-\lambda)((5-\lambda)(3-\lambda)-16)+4(-4(3-\lambda))=0$$

$$7\lambda^2-56\lambda-7-\lambda^3+8\lambda^2+\lambda+48+16\lambda=0$$

$$-\lambda^3+55\lambda^2-39\lambda-55=0$$

$$\lambda=-1$$

$$\begin{array}{r|l} -\lambda^3+15\lambda^2-39\lambda-55 & \lambda+1 \\ \cline{2-2} -\ \underline{-\lambda^3-\lambda^2} & -\lambda^2+16\lambda-55 \\ 16\lambda^2-39\lambda & \\ -\ \underline{16\lambda^2+16\lambda} & \\ \underline{-55\lambda-55} & \\ 0 & \end{array}$$

$(\lambda+1)(-\lambda^2+16\lambda+55)=0$

$-\lambda^2+16\lambda \mp 55=0$

$D_1=64-55=9$

$\lambda=\frac{-8\pm 3}{-1}=\left[\begin{array}{l}5\\11\end{array}\right.$

1) $\lambda=-1$

$A=\begin{pmatrix}8 & -4 & 0\\ -4 & 6 & 4\\ 0 & 4 & 4\end{pmatrix} \underset{(-1)I}{\overset{II\leftrightarrow I}{\sim}} \begin{pmatrix}4 & -6 & -4\\ 8 & -4 & 0\\ 0 & 4 & 4\end{pmatrix} \underset{III-2I}{\overset{II\leftrightarrow III}{\sim}}$

$\begin{pmatrix}4 & -6 & -4\\ 0 & 4 & 4\\ 0 & 8 & 8\end{pmatrix} \overset{III-2II}{\sim} \begin{pmatrix}4 & -6 & -4\\ 0 & 4 & 4\\ 0 & 0 & 0\end{pmatrix} \quad Rg\,A=2$

x, y – бз

z – своб.

$\begin{cases}4x-6y-4c\\ 4y=-4c\\ z=c\end{cases} \Longleftrightarrow \begin{cases}x=-\frac{1}{2}c\\ y=-c\\ z=c\end{cases}$

$\cancel{\overline{X}_1=\begin{pmatrix}-\frac{1}{2}c\\ -c\end{pmatrix}}$

$$\overline{X}_1 = \begin{pmatrix} -\frac{1}{2}c \\ -c \\ c \end{pmatrix} \quad \vec{f}_1 = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}; \quad \vec{e}_1 = \frac{1}{3}\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

2) $\lambda = 5$

$$A = \begin{pmatrix} 2 & -4 & 0 \\ -4 & 0 & 4 \\ 0 & 4 & -2 \end{pmatrix} \overset{II \leftrightarrow III}{\sim} \begin{pmatrix} 2 & -4 & 0 \\ 6 & 4 & -2 \\ -4 & 0 & 4 \end{pmatrix} \overset{III+2I}{\sim} \begin{pmatrix} 2 & -4 & 0 \\ 0 & 4 & -2 \\ 0 & -8 & 4 \end{pmatrix}$$

$$\overset{III+2II}{\sim} \begin{pmatrix} 2 & -4 & 0 \\ 0 & 4 & -2 \\ 0 & 0 & 0 \end{pmatrix} \quad Rg\,A = 2$$

x, y – бз.

z – своб.

$$\begin{cases} 2x - 4y = 0 \\ 4y = 2c \\ z = c \end{cases} \Leftrightarrow \begin{cases} x = c \\ y = \frac{1}{2}c \\ z = c \end{cases} \quad \overline{X}_2 = \begin{pmatrix} c \\ \frac{1}{2}c \\ c \end{pmatrix}$$

$$\vec{f}_2 = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}; \quad \vec{e}_2 = \frac{1}{3}\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

3) $\lambda = 11$

$$A = \begin{pmatrix} -4 & -4 & 0 \\ -4 & -6 & 4 \\ 0 & 4 & -8 \end{pmatrix} \underbrace{\overset{II - I}{\sim}}_{III + 2II} \begin{pmatrix} -4 & -4 & 0 \\ 0 & -2 & 4 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow Rg\,A = 2$$

x, y – бз.

z – свбд

$$\begin{cases} -4x = 4y \\ 4z = 2y \\ z = c \end{cases} \Leftrightarrow \begin{cases} x = -y \\ y = 2z \\ z = c \end{cases} \Leftrightarrow \begin{cases} x = -2c \\ y = 2c \\ z = c \end{cases} \quad \overline{X}_3 = \begin{pmatrix} -2c \\ 2c \\ c \end{pmatrix}$$

$$\vec{f}_3 = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} ; \quad \vec{e}_3 = \frac{1}{3}\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$$

$$U = \frac{1}{3}\begin{pmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ -2 & 2 & 1 \end{pmatrix}$$

$$\overline{X} = U\overline{X}' \Rightarrow \begin{cases} x = \frac{1}{3}(x' + 2y' - 2z') \\ y = \frac{1}{3}(2x' + y' + 2z') \\ z = \frac{1}{3}(-2x' + 2y' + z') \end{cases}$$

$$-x'^2 - 5y'^2 + 11z'^2 - \frac{20}{3}x' - \frac{40}{3}y' + \frac{40}{3}z' - \frac{20}{3}x' - \frac{10}{3}y' - \frac{20}{3}z' +$$

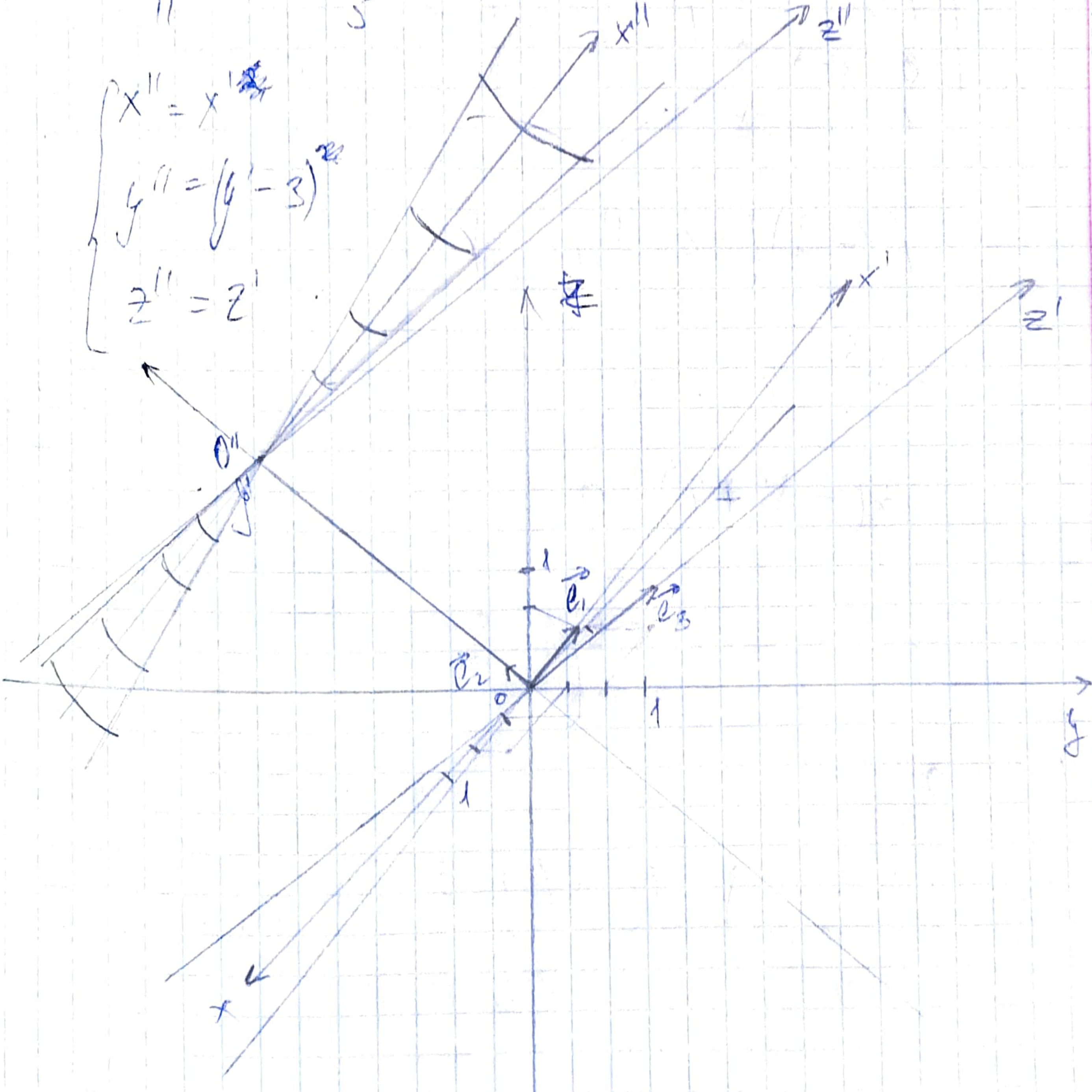
$$+ \frac{40}{3}x' - \frac{40}{3}y' - \frac{20}{3}z' + 45 = 0$$

$$-x'^2 + 5y'^2 + 11z'^2 - 30y' + 45 = 0$$

$$+\frac{x'^2}{5} - \frac{(y'-3)^2}{1} - \frac{11z'^2}{5} = 0$$

$$\frac{z'^2}{\frac{1}{11}} + \frac{(y'-3)^2}{\frac{1}{5}} = \frac{x'^2}{1} \text{ - конус}$$

$$\begin{cases} x'' = x' \\ y'' = (y'-3) \\ z'' = z' \end{cases}$$



d) $x^2-2y^2+z^2+2zx-4x\sqrt{2}+4y+4z\sqrt{2}-2=0$

$$A=\begin{pmatrix}1&0&1\\0&-2&0\\1&0&1\end{pmatrix}$$

$$\begin{vmatrix}1-\lambda&0&1\\0&-2-\lambda&0\\1&0&1-\lambda\end{vmatrix}=(1-\lambda)\begin{vmatrix}-2-\lambda&0\\0&1-\lambda\end{vmatrix}+\begin{vmatrix}0&-2-\lambda\\1&0\end{vmatrix}=0$$

$(1-\lambda)(-2-\lambda)(1-\lambda)+2+\lambda=0$

$-\lambda^3+4\lambda=0$

$\lambda(-\lambda^2+4)=0$

$\lambda=0 \qquad \lambda=\pm 2$

1) $\lambda=0$

$$A=\begin{pmatrix}1&0&1\\0&-2&0\\1&0&1\end{pmatrix}\xrightarrow{\text{III}-\text{I}}\begin{pmatrix}1&0&1\\0&-2&0\\0&0&0\end{pmatrix}\quad Rg\,A=2$$

x, y - бз

z = свбз

$$\begin{cases} x + z = 0 \\ -2y = 0 \\ z = c \end{cases} \Longleftrightarrow \begin{cases} x = -c \\ y = 0 \\ z = c \end{cases} \quad \bar{X}_1 = \begin{pmatrix} -c \\ 0 \\ c \end{pmatrix}$$

$$\vec{f}_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}; \quad \vec{e}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

2) $\lambda = 2$

$$A = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -4 & 0 \\ 1 & 0 & -1 \end{pmatrix} \underset{\sim}{\overline{III + I}} \begin{pmatrix} -1 & 0 & 1 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad Rg\, A = 2$$

$$\begin{cases} -x + z = 0 \\ -4y = 0 \\ z = c \end{cases} \Longleftrightarrow \begin{cases} x = c \\ y = 0 \\ z = c \end{cases} \quad \bar{X}_2 = \begin{pmatrix} c \\ 0 \\ c \end{pmatrix}$$

$$\vec{f}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}; \quad \vec{e}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

3) $\lambda = -2$

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 3 \end{pmatrix} \underset{II \leftrightarrow III}{\overset{III \leftrightarrow I}{\sim}} \begin{pmatrix} 1 & 0 & 3 \\ 3 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \overset{II - 3I}{\sim} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 0 & -8 \\ 0 & 0 & 0 \end{pmatrix} \text{Rg} A$$

$x,\ \cancel{z}\, y$ – б.з

$\cancel{z}\, y$ – своб

$$\begin{cases} x + 3z = 0 \\ -8z = 0 \\ \cancel{z}\, y = c \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y = c \\ z = 0 \end{cases} \Rightarrow \overline{X}_3 = \begin{pmatrix} 0 \\ c \\ 0 \end{pmatrix}$$

$$\vec{f}_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix};\quad \vec{e}_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{cases} x = \frac{1}{\sqrt{2}}(-x' + z') \\ y = y' \\ z = \frac{1}{\sqrt{2}}(x' + z') \end{cases}$$

$$-2y'^2 + 2z'^2 + 4x' - \cancel{4z'} + 4y' + 4x' + \cancel{4z'} - 2 = 0$$

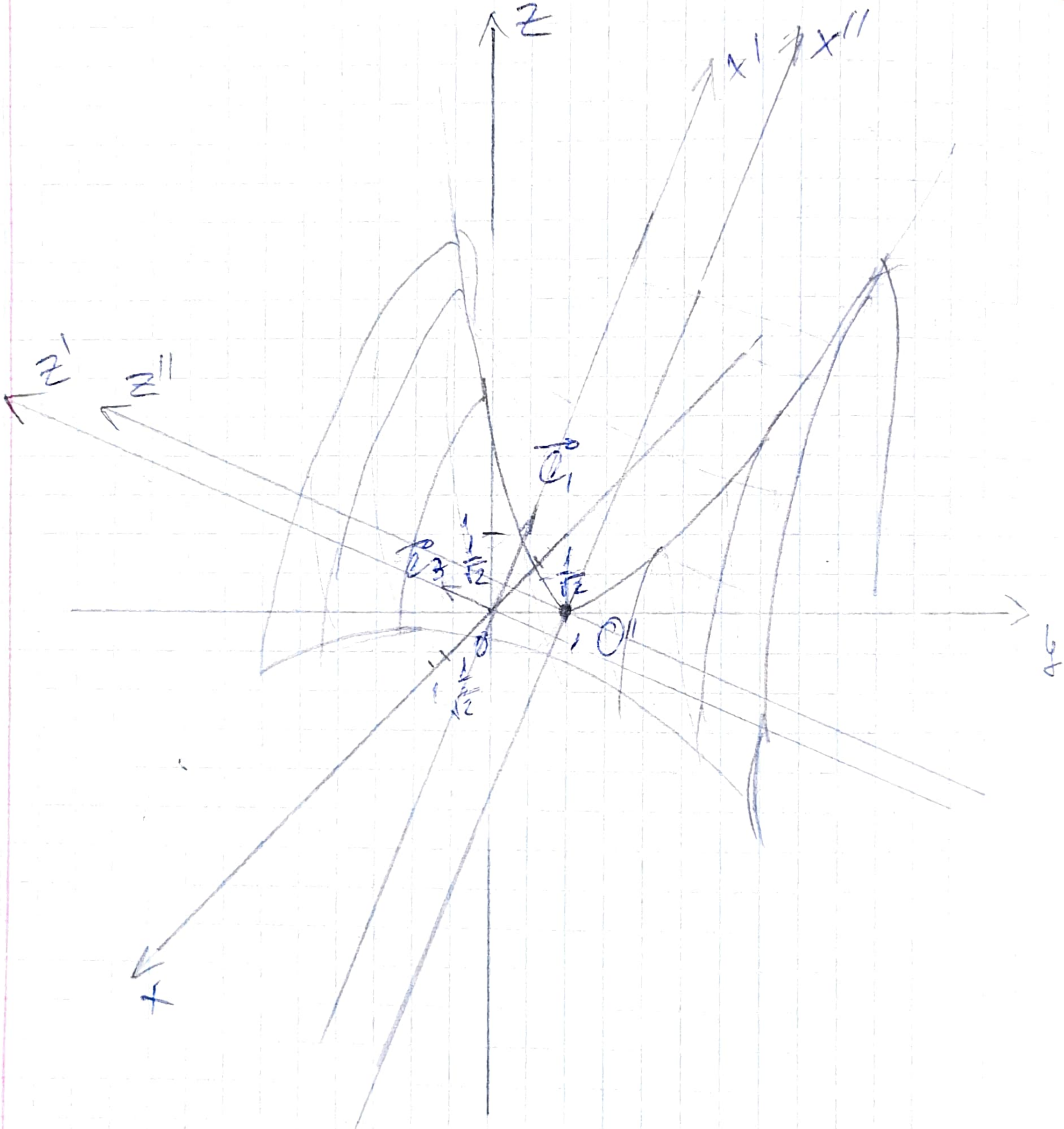
$$-2y'^2 + 2z'^2 + 8x' + 4y' - 2 = 0$$

$$-2(y'^2 - 2y' + 1) + z'^2 = -8x'$$

$$-2(y'-1)^2 + z'^2 = -8x' \quad \Big| :(-2)$$

$$\frac{(y'-1)^2}{2} - \frac{z'^2}{4} = 2x'$$ — гиперболический параболоид

$$\begin{cases} x'' = x' \\ y'' = y' - 1 \\ z'' = z' \end{cases}$$



z
x'
x''
z'
z''
$\vec{e}_1$
$\vec{e}_3$
$\frac{1}{\sqrt{2}}$
$\frac{1}{\sqrt{2}}$
$\frac{1}{\sqrt{2}}$
0
1
O''
y
x