ТЕТРАДЬ

для КР №2 по ЛАиФИП

студента группы ИХ1-21

учени____________класса____________

____________школы____________

Кузимова

Леонида

Варіант 8

№1

$$C = \frac{x+y+z-1}{x+y+z+1}$$

$(x_0; y_0; z_0) = (0; 3; 0)$

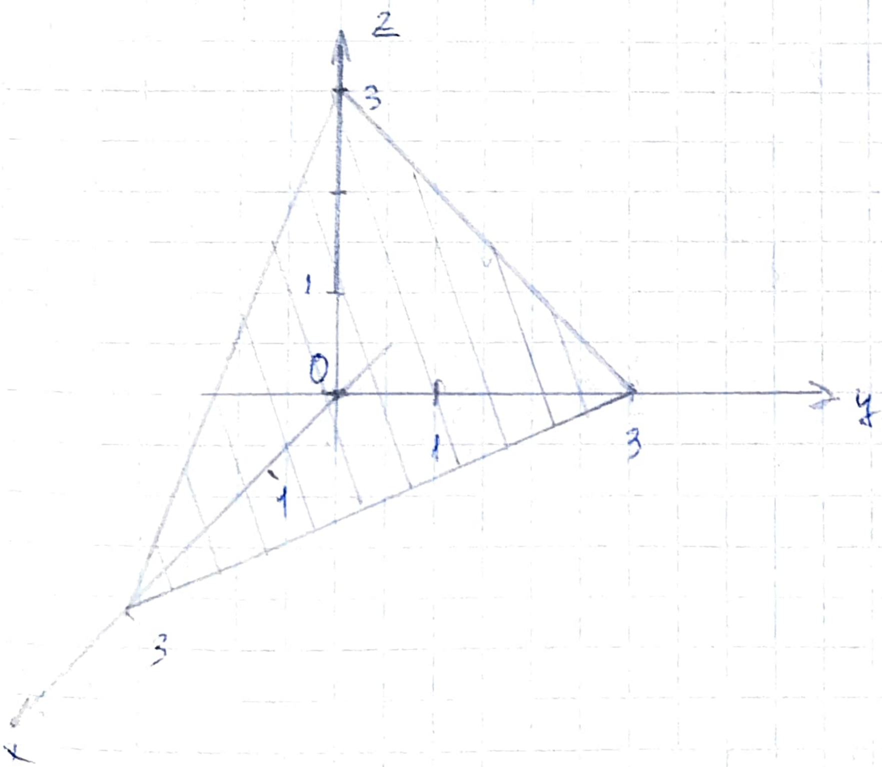
Подставим $x_0; y_0; z_0$ в C

$$C = \frac{0+3+0-1}{0+3+0+1} = \frac{2}{4} = \frac{1}{2}$$

$$C = \frac{x+y+z-1}{x+y+z+1} = \frac{1}{2}$$

$$2x+2y+2z-2 = x+y+z+1$$

$x+y+z = 3$ – плоскость



№2

$z = \sin(x^3 - 2y^2)$; $M = (2; -2)$; $\vec{l} = 4\vec{i} + 3\vec{j}$

$\operatorname{grad} z$? ; $\frac{\partial z}{\partial l}$ - ?

Решение

$$\frac{\partial z}{\partial x} = \cos(x^3 - 2y^2) \cdot 3x^2 \; ; \; z'_x(M) = 12$$

$$\frac{\partial z}{\partial y} = -4\cos(x^3 - 2y^2) \cdot y \; ; \; z'_y(M) = 8$$

$$\operatorname{grad} z = \left.\frac{\partial z}{\partial x}\right|_M \vec{i} + \left.\frac{\partial z}{\partial y}\right|_M \vec{j} = 12\vec{i} + 8\vec{j} \Rightarrow$$

$$\operatorname{grad} z = (12; 8)$$

$$\cos\alpha = \frac{l_x}{|l|} = \frac{4}{\cancel{2}5} \; ; \; \cos\beta = \frac{l_y}{|l|} = \frac{3}{\cancel{2}5}$$

$$\frac{\partial z}{\partial l} = \frac{\partial z}{\partial x} \cdot \cos\alpha + \frac{\partial z}{\partial y} \cos\beta = \frac{12 \cdot 4}{\cancel{2}5} + \frac{8 \cdot 3}{\cancel{2}5} =$$

$$= \frac{48 + 24}{\cancel{2}5} = \frac{72}{5}$$

Ответ: $\operatorname{grad} z = (12; 8)$; $\frac{\partial z}{\partial l} = \frac{72}{5}$

№3. ~~lim~~ $\exists \lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{xy}{x^2+y^2}$ - ?

Решение

Пусть $y = kx$, тогда

$$\lim_{\substack{x\to 0\\ \cancel{y\to 0}}} \frac{x\cdot kx}{x^2+k^2x^2} = \lim_{x\to 0} \frac{x^2\cdot k}{x^2(1+k^2)} = \lim_{x\to 0}\left(\frac{k}{1+k^2}\right) -$$

Результат зависит только от $k \Rightarrow \lim \nexists$.

№4

$$z = \varphi(y+ax) + \psi(y-ax)$$

$$a^2\frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x^2} = 0$$, для любых дважды диф-мых

φ, ψ?

Решение:

$$\frac{\partial z}{\partial x} = \varphi'(y+ax)a + \psi'(y-ax)(-a)$$

$$\frac{\partial z}{\partial y} = \varphi'(y+ax) + \cancel{\varphi}\,\psi'(y-ax)$$

$$\frac{\partial^2 z}{\partial x^2} = a^2\varphi''(y+ax) + \psi^2(y-ax)(-a)^2$$

$$\frac{\partial^2 z^*}{\partial y^2} = \varphi''(y+ax) + \psi''(y-ax)$$

Подставим в условие:

$$0 = a^2 \cdot (\varphi''(y+ax) + \psi''(y-ax)) - a^2\varphi(y+ax) \mp \psi''(y-ax)a$$

$$= 0 \Rightarrow 0 = 0$$

ч.т.д

N5.

$$\operatorname{arctg}(z-x) = yz - 2 \; ; \; (1, 2, 1)$$

$$M_0 = (1, 2) \; ;$$

$$dz\Big|_{M_0} - ? \; ; \; z(1,2; 1,9) - ?$$

Решение:

$$F(x, y, z) = \operatorname{arctg}(z-x) - yz + 2$$

$$z'_x = -\frac{F'_x}{F'_z} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} = -\frac{\partial z}{\partial x} = -\frac{-\frac{1}{1+(z-x)^2}}{\frac{1}{1+(z-x)^2} - y} =$$

$$= \frac{1}{1 - y - y(z-x)^2}$$

$$z'_y = -\frac{F'_y}{F'_z} = -\frac{-z}{\frac{1}{1+(z-x)^2}-y} = \frac{z}{\frac{1}{1+(z-x)^2}-y}$$

$$z'_x(1;2;1) = \frac{1}{1-2-2(1-1)^2} = -1$$

$$z'_y(1;2;1) = \frac{z}{\frac{1}{1+(z-x)^2}-y} = \frac{1}{\frac{1}{1+(1-1)^2}-2} = -1$$

Тогда $dz = z'_x \Delta x + z'_y \Delta y = -1\Delta x - 1\Delta y =$
$= -\Delta x - \Delta y$

$$\begin{aligned} x &= x_0 + \Delta x \\ y &= y_0 + \Delta y \end{aligned} \Rightarrow \begin{aligned} \Delta x &= 0{,}2 \\ \Delta y &= -0{,}1 \end{aligned}$$

$$z(1{,}2;1{,}9) \approx z(M_0) + dz = 1 - 0{,}2 + 0{,}1 = 0{,}9$$

Ответ: $dz = -\Delta x - \Delta y$

$z(1{,}2;\ 1{,}9) \approx 0{,}9$