Вариант - 10.

9) $3x^2+6y^2-4xy+8\sqrt{5}x+4\sqrt{5}y+36=0$

Матрица кв. формы

$$A=\begin{pmatrix}3 & -2\\ -2 & 6\end{pmatrix}$$

Собств. вектор и собств. знач:

$$|A-\lambda E|=\begin{vmatrix}3-\lambda & -\lambda\\ -\lambda & 6-\lambda\end{vmatrix}=18-9\lambda+\lambda^2-4=$$

$$=\lambda^2-9\lambda+14$$

$\lambda_1=2 \qquad u_1=\begin{pmatrix}2\\1\end{pmatrix} \qquad e_1=\frac{1}{\sqrt{5}}\begin{pmatrix}2\\1\end{pmatrix}$

$\lambda_2=7 \qquad u_2=\begin{pmatrix}1\\-2\end{pmatrix} \qquad e_2=\frac{1}{\sqrt{5}}\begin{pmatrix}1\\-2\end{pmatrix}$

Матрица перехода от иск. базиса к базису e_1, e_2

$$T = \frac{1}{\sqrt{5}}\begin{pmatrix}2 & 1\\ 1 & -2\end{pmatrix}$$

Линейная форма в матричном виде:

$$8\sqrt{5}\,x + 4\sqrt{5}\,y = (8\sqrt{5} \quad 4\sqrt{5})\begin{pmatrix}x\\ y\end{pmatrix} =$$

$$= \sqrt{5}\,(8 \quad 4)\cdot\frac{1}{\sqrt{5}}\begin{pmatrix}2 & 1\\ 1 & -2\end{pmatrix}\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = (20 \quad 0)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 20\bar{x} \cdot 0 \cdot \bar{y}$$

кв. форма в базисе e_1, e_2:

$$\lambda_1 \bar{x}^2 + \lambda_2 \bar{y}^2 = 2\bar{x}^2 + 7\bar{y}^2$$

$$2\bar{x}^2 + 7\bar{y}^2 + 20\bar{x} + 36 = 0$$

$$2(\bar{x}+5)^2 + 7\bar{y}^2 = 14$$

$$\frac{(\bar{x}+5)^2}{(\sqrt{7})^2} + \frac{\bar{y}^2}{(\sqrt{2})^2} = 1 \text{ — эллипс}$$

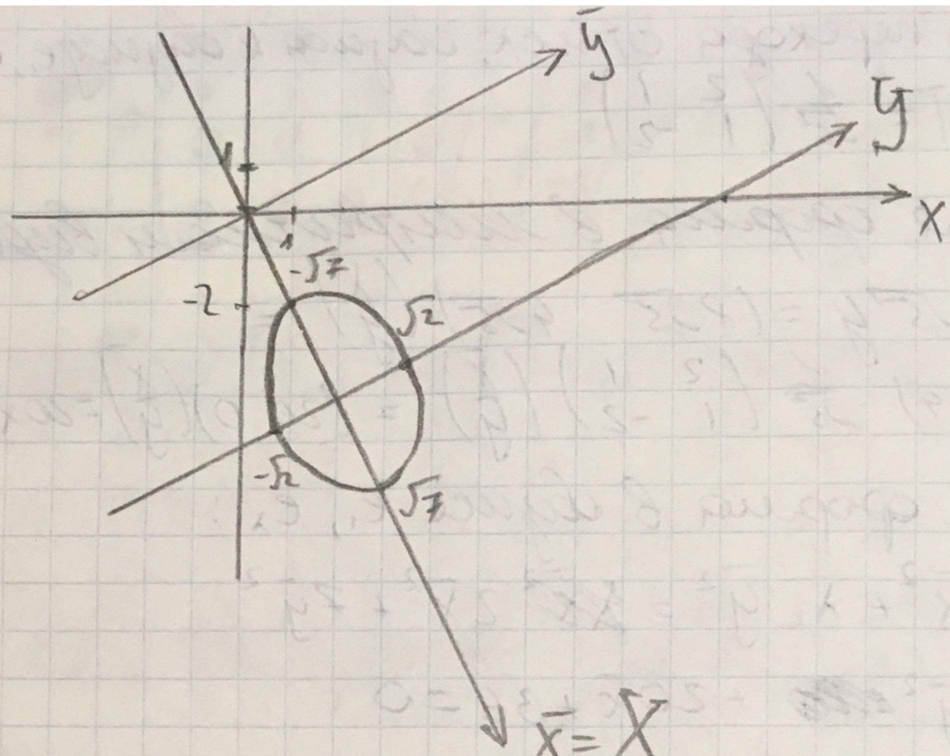
$$\frac{X^2}{(\sqrt{7})^2} + \frac{Y^2}{(\sqrt{2})^2} = 1$$

Преобразование перехода от исходной системы координат к канонической:

$$\begin{pmatrix}x\\ y\end{pmatrix} = T\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = T\begin{pmatrix}X-5\\ Y\end{pmatrix} = \frac{1}{\sqrt{5}}\begin{pmatrix}2 & 1\\ 1 & -2\end{pmatrix}\begin{pmatrix}X-5\\ Y\end{pmatrix}$$

или:

$$\begin{cases} x = \frac{1}{\sqrt{5}}(2X + Y) - \frac{10}{\sqrt{5}} \\ y = \frac{1}{\sqrt{5}}(X - 2Y) - \frac{5}{\sqrt{5}} \end{cases}$$



$\bar{x}, \bar{y}$ - поворот X, Y – парал. перенос.

b) $6xy - 3x^2 + 5y^2 + 8\sqrt{10}\,y + 80 = 0$

$$A = \begin{pmatrix} -3 & 3 \\ 3 & 5 \end{pmatrix}$$

$$|A - \lambda E| = \begin{vmatrix} -3-\lambda & 3 \\ 3 & 5-\lambda \end{vmatrix} =$$

$$= 15 - 2\lambda + \lambda^2 - 9 = \lambda^2 - 2\lambda - 24$$

$\lambda_1 = -4 \quad u_1 = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \quad e_1 = \frac{1}{\sqrt{10}}\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$\lambda_2 = 6 \quad u_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad e_2 = \frac{1}{\sqrt{10}}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

Матрица перехода от исх. базиса к базису e_1, e_2

$$T = \frac{1}{\sqrt{10}}\begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix}$$

$$\lambda_1 \bar{x}^2 + \lambda_2 \bar{y}^2 = -4\bar{x}^2 + 6\bar{y}^2$$

Линейная форма:

$$8\sqrt{10}\,y=(0\ \ 8\sqrt{10})\begin{pmatrix}x\\y\end{pmatrix}=\sqrt{10}\,(0\ \ 8)\frac{1}{\sqrt{10}}\begin{pmatrix}3&1\\-1&3\end{pmatrix}\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=$$

$$=(-8\ \ 24)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$-4\bar{x}^2+6\bar{y}^2-8\bar{x}+24\bar{y}+80=0$$

$$-2\bar{x}+3\bar{y}^2-4\bar{x}+12\bar{y}+40=0$$

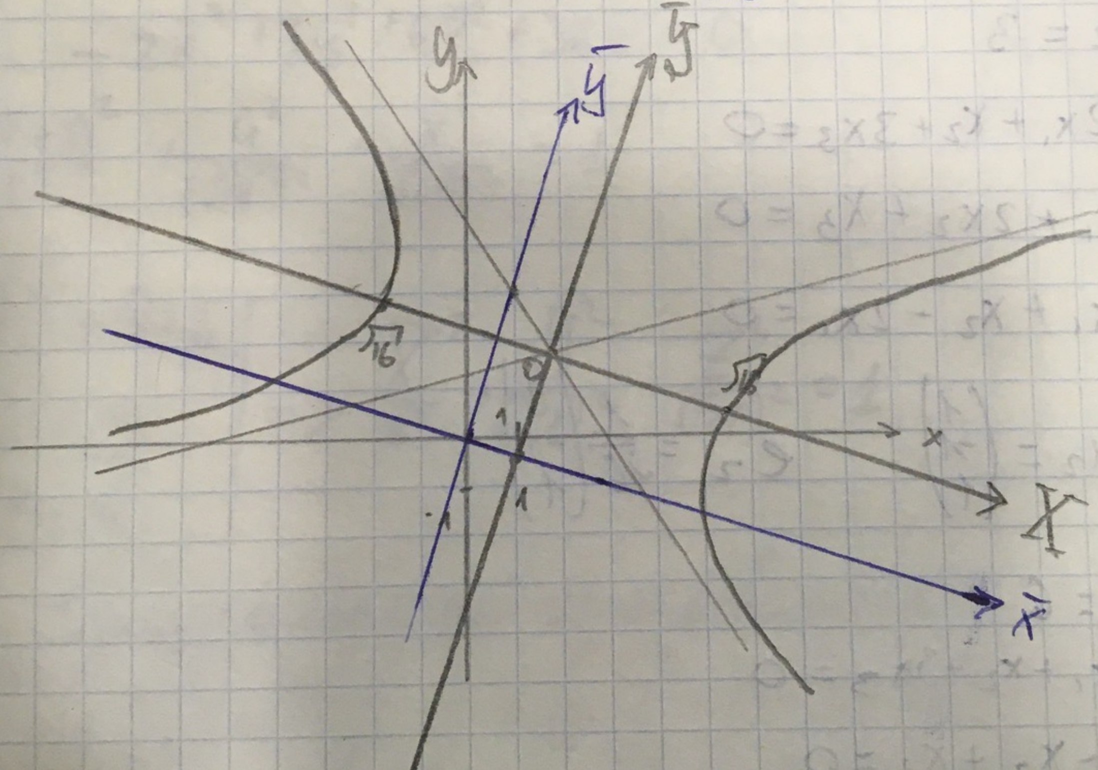
$$-2(\bar{x}^2+2\bar{x}+1)+3(\bar{y}^2+4\bar{y}+4)+30=0$$

$$2(\bar{x}+1)^2-3(\bar{y}+2)^2=30$$

$$\frac{(\bar{x}+1)^2}{(\sqrt{15})^2}-\frac{(\bar{y}+2)^2}{(\sqrt{10})^2}=1 \text{ — гипербола}$$

$$\frac{X^2}{(\sqrt{15})^2}-\frac{Y^2}{(\sqrt{10})^2}=1$$

$$\begin{pmatrix}x\\y\end{pmatrix}=T\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=T\begin{pmatrix}X-1\\Y-2\end{pmatrix} \text{ или } \begin{cases}x=\frac{1}{\sqrt{10}}(3X+Y)-\frac{5}{\sqrt{10}}\\ y=\frac{1}{\sqrt{10}}(-X+3Y)-\frac{5}{\sqrt{10}}\end{cases}$$



c) $x^2+5y^2+z^2+2xy+6xz+2yz+6\sqrt{6}x+12\sqrt{6}y+6\sqrt{6}z=\ldots$

$$A=\begin{pmatrix}1&1&3\\1&5&1\\3&1&1\end{pmatrix} \qquad |A-\lambda E|=\begin{vmatrix}1-\lambda&1&3\\1&5-\lambda&1\\3&1&1-\lambda\end{vmatrix} =$$

$$= (1-\lambda)(\lambda^2-6\lambda+4)+2+\lambda+9\lambda-4\lambda=$$

$$=-\lambda^3+7\lambda^2-36=-(\lambda+2)(\lambda-6)(\lambda-3)$$

$\lambda=-2$

$$\begin{cases}3x_1+x_2+3x_3=0\\x_1+7x_2+x_3=0\\3x_1+x_2+3x_3=0\end{cases}$$

$$u_1=\begin{pmatrix}1\\0\\-1\end{pmatrix} \qquad e_1=\frac{1}{\sqrt{6}}\begin{pmatrix}1\\0\\-1\end{pmatrix}$$

$\lambda_2=3$

$$\begin{cases}-2x_1+x_2+3x_3=0\\x_1+2x_2+x_3=0\\3x_1+x_2-2x_1=0\end{cases}$$

$$u_2=\begin{pmatrix}1\\-1\\1\end{pmatrix} \qquad e_2=\frac{1}{\sqrt{6}}\begin{pmatrix}1\\-1\\1\end{pmatrix}$$

$\lambda_3=6$

$$\begin{cases}-5x_1+x_2+3x_3=0\\x_1-x_2+x_3=0\end{cases}$$

$6\bar{e}_2=0$

$\lfloor 3x_1+x_2-5x_3=0$

$u_3=\begin{pmatrix}1\\2\\1\end{pmatrix} \qquad e_3=\frac{1}{\sqrt{6}}\begin{pmatrix}1\\2\\1\end{pmatrix}$

В базисе e_1, e_2, e_3 кв. форма:

$$\lambda_1\bar{x}^2+\lambda_2\bar{y}^2+\lambda_3\bar{z}^2=-2\bar{x}^2+3\bar{y}^2+6\bar{z}^2$$

$$T=\frac{1}{\sqrt{6}}\begin{pmatrix}1&1&1\\0&-1&2\\-1&1&1\end{pmatrix}$$

Линейная форма

$$6\sqrt{6}x+12\sqrt{6}y+6\sqrt{6}z=\sqrt{6}(6,12,6)\begin{pmatrix}x\\y\\z\end{pmatrix}=$$

$$=\sqrt{6}(6\ 12\ 6)\cdot\frac{1}{\sqrt{6}}\begin{pmatrix}1&1&1\\0&-1&2\\-1&1&1\end{pmatrix}\begin{pmatrix}\bar{x}\\\bar{y}\\\bar{z}\end{pmatrix}=\cancel{(0\ 0\ 36}$$

$$=(0\ 0\ 36)\begin{pmatrix}\bar{x}\\\bar{y}\\\bar{z}\end{pmatrix}=36\bar{z}$$

$$-2\bar{x}^2+3\bar{y}^2+6\bar{z}^2+36\bar{z}=0$$

$$\frac{(\bar{z}+3)^2}{3^2}+\frac{\bar{y}^2}{(\sqrt{18})^2}-\frac{\bar{x}^2}{(\sqrt{27})^2}=1$$

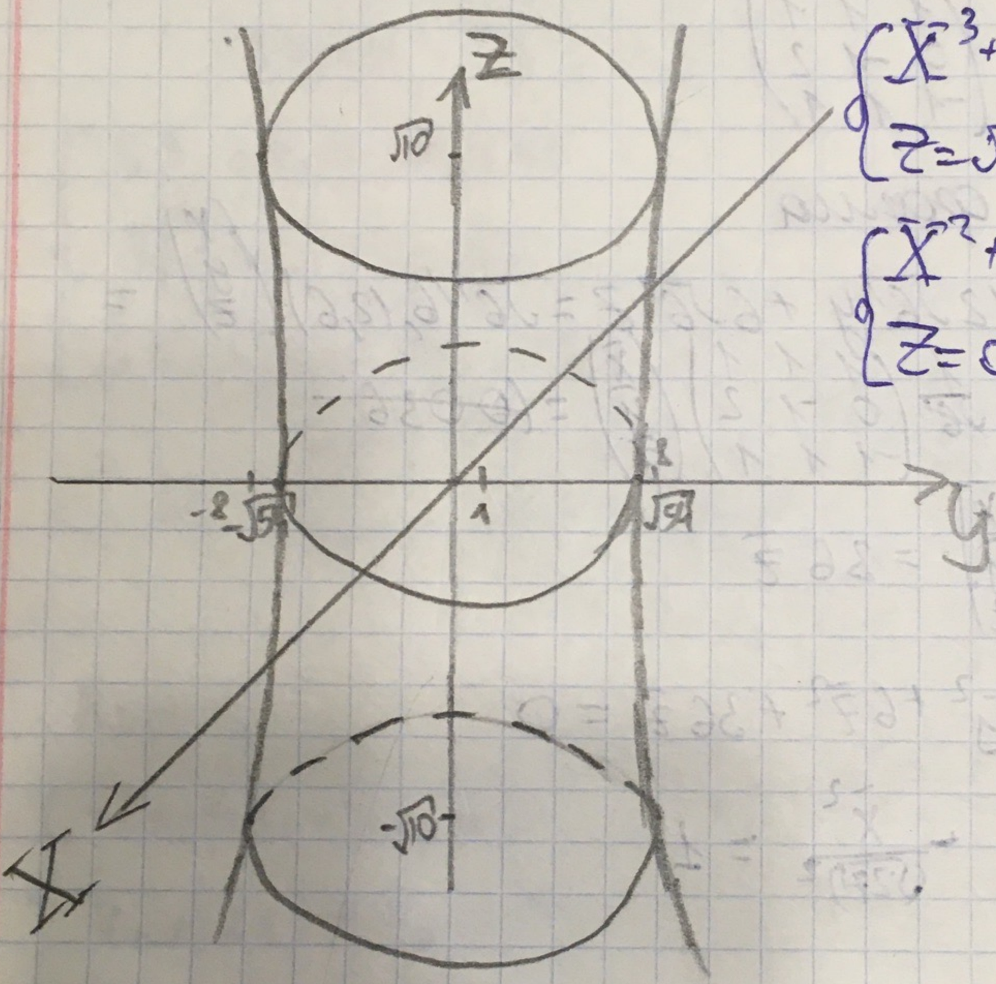
$$\left.\begin{aligned}X&=\bar{z}+3\\Y&=\bar{y}\\Z&=\bar{x}\end{aligned}\right| \Rightarrow \frac{X^2}{3^2}+\frac{Y^2}{(\sqrt{18})^2}-\frac{Z^2}{(\sqrt{27})^2}=1$$

Однополостный гиперболоид

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=T\begin{pmatrix}\bar{x}\\\bar{y}\\\bar{z}\end{pmatrix}=T\begin{pmatrix}Z\\Y\\X-3\end{pmatrix}=\frac{1}{\sqrt{6}}\begin{pmatrix}1&1&1\\0&-1&2\\-1&1&1\end{pmatrix}\begin{pmatrix}Z\\Y\\X-3\end{pmatrix}$$

или

$$\begin{cases} x = \dfrac{\bar{Z}+\bar{Y}+\bar{X}}{\sqrt{6}} - \dfrac{3}{\sqrt{6}} \\ y = \dfrac{-\bar{Y}+2\bar{X}}{\sqrt{6}} - \dfrac{6}{\sqrt{6}} \\ z = \dfrac{-\bar{Z}+\bar{Y}+\bar{X}}{\sqrt{6}} - \dfrac{3}{\sqrt{6}} \end{cases}$$



$$\begin{cases} \bar{X}^3+\bar{Y}^2=\bar{Z}^2+54 \text{ - окр} \quad CR=8 \\ \bar{Z}=\sqrt{10} \end{cases}$$

$$\begin{cases} \bar{X}^2+\bar{Y}^2=\bar{Z}+54 \text{ - окр} \quad CR=\sqrt{54} \\ \bar{Z}=0 \end{cases}$$

d) $x^2+2y^2+4z^2+4xz-4\sqrt{5}x-4y+2\sqrt{5}z-18=0$

$$A=\begin{pmatrix}1&0&2\\0&2&0\\2&0&4\end{pmatrix}$$

$$|A-\lambda E|=\begin{vmatrix}1-\lambda&0&2\\0&2-\lambda&0\\2&0&4-\lambda\end{vmatrix}=(1-\lambda)(8-6\lambda+\lambda^2)+2(2\lambda-4)=$$

$$=-\lambda(\lambda-5)(\lambda-2)$$

$\lambda_1=0$

$$\begin{cases}x_1+2x_3=0\\2x_2=0\\2x_1+4x_3=0\end{cases}$$

$$u_1=\begin{pmatrix}2\\0\\-1\end{pmatrix}\quad e_1=\frac{1}{\sqrt{5}}\begin{pmatrix}2\\0\\-1\end{pmatrix}$$

$\lambda_2=2$

$$\begin{cases}-x_1+2x_3=0\\0=0\\2x_1+2x_3=0\end{cases}$$

$$u_2=\begin{pmatrix}0\\1\\0\end{pmatrix}\quad e_2=\cancel{\frac{1}{\sqrt{5}}}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

$\lambda_3=5$

$$\begin{cases}-4x_1+2x_3=0\\-3x_2=0\end{cases}$$

$2x_1 - x_3 = 0$

$$u_3 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \quad e_3 = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$T = \begin{pmatrix} \frac{2}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{5}} & 0 & \frac{2}{\sqrt{5}} \end{pmatrix}$$

Линейная форма:

$$-4\sqrt{5}x - 4y + 2\sqrt{5}z = (-4\sqrt{5} \;\; -4 \;\; 2\sqrt{5})$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = (-4\sqrt{5} \;\; 4 \;\; 2\sqrt{5}) \cdot \begin{pmatrix} \frac{2}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{5}} & 0 & \frac{2}{\sqrt{5}} \end{pmatrix} =$$

$$= (10 \;\; 4 \;\; 0)\begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix}$$

$$2\bar{y}^2 + 5\bar{z}^2 + 10\bar{x} + 4 \cdot \bar{y} \cdot 18 = 0$$

$$2(\bar{y}^2 + 2\bar{y} + 1) + 5\bar{z}^2 + 10\bar{x} \cancel{+} -20 = 0$$

$$\frac{(\bar{y}+1)^2}{(\sqrt{10})^2} + \frac{\bar{z}^2}{2^2} - \frac{\bar{x}}{2} = 1$$

$$\left.\begin{aligned} X &= \bar{z} \\ Y &= \bar{y}+1 \\ Z &= \cancel{x+2}\,\frac{x+2}{4} \end{aligned}\right| \Rightarrow \frac{X^2}{2^2} + \frac{Y^2}{(\sqrt{10})^2} - 2Z = 0 -$$

– эллиптический параболоид

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = T\begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = T\begin{pmatrix} 4Z - X \\ Y - 1 \\ X \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{2}{\sqrt5} & 0 & \frac{1}{\sqrt5} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt5} & 0 & \frac{2}{\sqrt5} \end{pmatrix} \begin{pmatrix} 4Z - 2 \\ Y - 1 \\ X \end{pmatrix}$$

или

$x = \frac{2}{\sqrt5}(4Z - \lambda) + \frac{1}{\sqrt5} X$

$y = Y - 1$

$z = \frac{-4Z - 2}{\sqrt5} + \frac{2}{\sqrt5} X$

$$\begin{cases} Z = \frac{Y^2}{20} & (\text{в } YOZ) \\ X = 0 \end{cases}$$

$$\begin{cases} Z = \frac{X^2}{8} & (\text{в } XOZ) \\ Y = 0 \end{cases}$$