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КР по ЛаиФКП по теме

„Дифференциальные ФКП“

Вариант-10

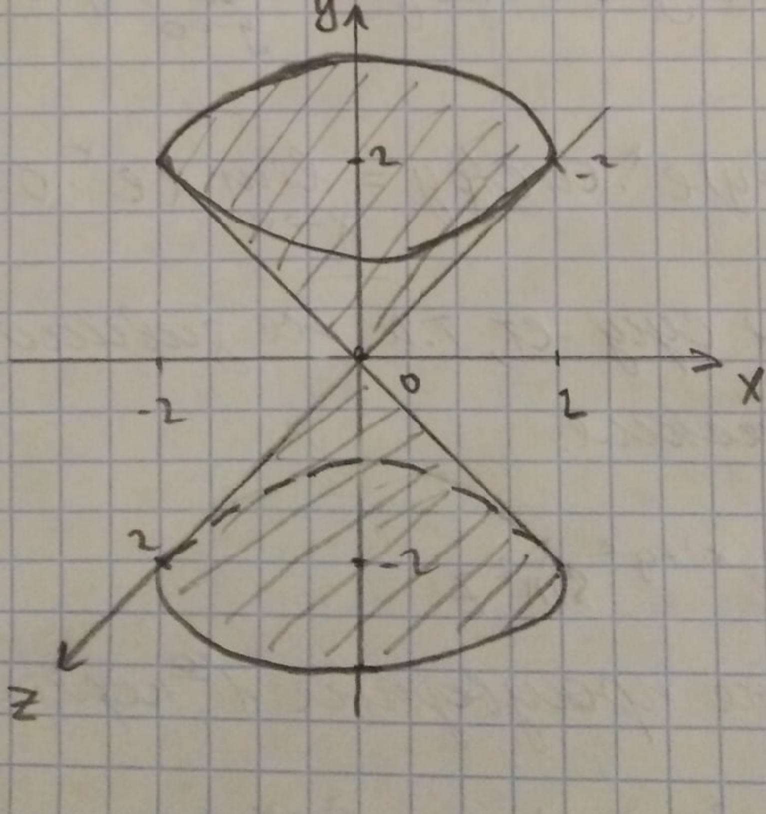
1. $\frac{x^2+y^2-2}{x^2-z^2-1}=C$

$(x_0;y_0;z_0)=(2;2;0)$

Подставим $x_0;y_0;z_0$ в C.

$C=\frac{2^2+2^2-2}{2^2-0^2-1}=C \Rightarrow C=2$

$\frac{x^2+y^2-2}{x^2-z^2-1}=2 \Rightarrow x^2-y^2-2z^2=0$ - коническая пов-ть



(2) $z = tg(3x^2-2y)$

$\frac{\partial z}{\partial x} = (tg(3x^2-2y))'_x = \frac{6x}{\cos^2(3x^2-2y)}$ $\frac{\partial z}{\partial x}\Big|_M = \frac{-6}{\cos^2(3-3)} = -6$

$\frac{\partial z}{\partial y} = (tg(3x^2-2y))'_y = \frac{-2}{\cos^2(3x^2-2y)}$ $\frac{\partial z}{\partial y}\Big|_M = \frac{-2}{1} = -2$

$grad\ z = (-6;-2)$ $|\ell| = \sqrt{(-3)^2+(-1)^2} = \sqrt{10}$ $\ell_0 = \left(\frac{-3}{\sqrt{10}}; \frac{-1}{\sqrt{10}}\right)$

$\frac{\partial z}{\partial \ell} = \frac{\partial z}{\partial x}\Big|_M \cdot \cos\alpha + \frac{\partial z}{\partial y}\Big|_M \cdot \cos\beta = (-6)\cdot\left(\frac{-3}{\sqrt{10}}\right) + (-2)\cdot\left(\frac{-1}{\sqrt{10}}\right) =$

$= \frac{18}{\sqrt{10}} + \frac{2}{\sqrt{10}} = \sqrt{40} = 2\sqrt{10}$

(3) $\lim\limits_{\substack{x\to 0\\ y\to 0}} (x+y)e^x \cdot arctg y = \lim\limits_{y\to 0} y\cdot e^0 \cdot arctg y = 0$

$\lim\limits_{\substack{x\to 0\\ y\to 0}} (x+y)e^x \cdot arctg y = \lim\limits_{x\to 0} x e^x \cdot 0 = 0$

Предел сущ-ет, т.к. не зависит от спос. приближения.

(4) $u = e^{x^2+y^2}\sin^2 z$

Частные производные 1-го порядка

$\frac{\partial u}{\partial x} = (e^{x^2+y^2}\sin^2 z)'_x = 2xe^{x^2+y^2}\sin^2 z$

$\frac{\partial u}{\partial y} = (e^{x^2+y^2}\sin^2 z)'_y = 2ye^{x^2+y^2}\sin^2 z$

$$\frac{\partial u}{\partial z} = \left(e^{x^2+y^2}\sin^2 z\right)'_z = e^{x^2+y^2}\, 2\sin z \cdot \cos z = e^{x^2+y^2}\cdot \sin 2z$$

Частные производные 2^го порядка

$$\frac{\partial^2 u}{\partial x^2} = \left(2xe^{x^2+y^2}\sin^2 z\right)'_x = 2\sin^2 z\left(e^{x^2+y^2} + 2x^2 e^{x^2+y^2}\right)$$

$$\frac{\partial^2 u}{\partial y^2} = \left(2ye^{x^2+y^2}\sin^2 z\right)'_y = 2\sin^2 z\, e^{x^2+y^2}\left(1+2y^2\right)$$

$$\frac{\partial^2 u}{\partial z^2} = \left(e^{x^2+y^2}\sin 2z\right)'_z = 2e^{x^2+y^2}\cos 2z$$

$$\frac{\partial^2 u}{\partial x \partial y} = \left(2xe^{x^2+y^2}\sin^2 z\right)'_y = 2\cdot 2xye^{x^2+y^2}\sin^2 z$$

$$\frac{\partial^2 u}{\partial x \partial z} = \left(2xe^{x^2+y^2}\sin^2 z\right)'_z = 4xe^{x^2+y^2}\sin z \cdot \cos z = 2xe^{x^2+y^2}\cdot \sin 2z$$

$$\frac{\partial^2 u}{\partial y \partial x} = \left(2ye^{x^2+y^2}\sin^2 z\right)'_x = 4xye^{x^2+y^2}\sin^2 z$$

$$\frac{\partial^2 u}{\partial y \partial z} = \left(2ye^{x^2+y^2}\sin^2 z\right)'_z = 2ye^{x^2+y^2}\sin 2z$$

$$\frac{\partial^2 u}{\partial z \partial x} = \left(e^{x^2+y^2}\sin 2z\right)'_x = 2xe^{x^2+y^2}\sin 2z$$

$$\frac{\partial^2 u}{\partial z \partial y} = \left(e^{x^2+y^2}\sin 2z\right)'_y = 2ye^{x^2+y^2}\sin 2z$$

5. $z^4 - xz - y = 0 \quad F(x,y,z) = z^4 - xz - y$

$$z'_x = \frac{F'_x}{F'_z} = \frac{-z}{4z^3 - x} \qquad z'_y = \frac{F'_y}{F'_z} = \frac{-1}{4z^3 - x}$$

Найдём производные при $x_0 = 1, y_0 = 2, z = -1$

$$z'_x\Big|_{(1,2,-1)} = \frac{1}{-4-1} = -\frac{1}{5} \qquad z'_y\Big|_{(1,2,-1)} = \frac{-1}{4-1} = \frac{1}{5}$$

$$dz = z'_x \Delta x + z'_y \Delta y = -\frac{1}{5}\Delta x + \frac{1}{5}\Delta y$$

$z(1,2;\ 1,9)$

$x = x_0 + \Delta x \Rightarrow x_0 = 1;\ \Delta x = 0,2$

$y = y_0 + \Delta y \Rightarrow y_0 = 2;\ \Delta y = -0,1$

$$z\Big|_{x=1,2;\ y=1,9} \approx z\Big|_{x=1;\ y=2} + dz = -1 - \frac{1}{5}(0,2) + \frac{1}{5}(-0,1) = -1,06$$