ЛАиФНП. Вариант 14

a) $3x^2-5y^2+6xy-8\sqrt{10}y+4=0$

в) $4x^2+7y^2+4xy-12\sqrt5 x+6\sqrt5 y+51=0$

c) $5x^2-y^2+z^2+4xy+6xz+2\sqrt{14}x+4\sqrt{14}y-6\sqrt{14}z-28=0$

d) $x^2+2z^2-4xy-4xz+4x+4z-14=0$

a $\quad 3x^2-5y^2+6xy-8\sqrt{10}y+4=0$

1. Запишем матрицу К.Ф. $\quad A=\begin{pmatrix}3&3\\3&-5\end{pmatrix}$

2. $\det(A-\lambda E)$

$$\begin{vmatrix}3-\lambda & 3\\ 3 & -5-\lambda\end{vmatrix}=0 \qquad (3-\lambda)(-5-\lambda)-9=0$$

$$-15-3\lambda+5\lambda+\lambda^2-9=0$$

$$\lambda^2+2\lambda-24=0$$

$$\lambda_1=4 \qquad \lambda_2=-6$$

3. Найдем собств. векторы.

$\lambda=4$

$$\begin{pmatrix}-1&3\\3&1\end{pmatrix}\sim(-1\ 3). \quad (-1\ 3)\begin{pmatrix}x\\y\end{pmatrix}=0$$

$$-x+3y=0 \qquad x-\text{баз. п.}$$

$$X_{\lambda=4}=C_1\begin{pmatrix}3\\1\end{pmatrix} \qquad y-\text{своб. п.}$$

$$y=C_1 \quad x=3C_1$$

$$e_1=\frac{1}{\sqrt{10}}\begin{pmatrix}3\\1\end{pmatrix}$$

$\lambda=-6$

$$\begin{pmatrix}9&3\\3&1\end{pmatrix}\sim(3\ 1) \qquad (3\ 1)\begin{pmatrix}x\\y\end{pmatrix}=0$$

$$X_{\lambda=-6}=C\begin{pmatrix}1\\-3\end{pmatrix} \qquad 3x+y=0 \quad x-\text{своб. пер.}$$

$$x=C_2\Rightarrow y=-3C_2 \quad y-\text{баз. пер.}$$

$$\tilde{e}_2=\frac{1}{\sqrt{10}}\begin{pmatrix}1\\-3\end{pmatrix}$$

$$X=UX'$$

$$U=\begin{pmatrix}\frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}}\\ \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}}\end{pmatrix}$$

$$\begin{cases} x = \frac{3}{\sqrt{10}}x' + \frac{1}{\sqrt{10}}y' \\ y = \frac{1}{\sqrt{10}}x' - \frac{3}{\sqrt{10}}y' \end{cases}$$

$$4x'^2 - 6y'^2 - 8\sqrt{10}\left(\frac{1}{\sqrt{10}}x' - \frac{3}{\sqrt{10}}y'\right) + 4 = 0$$

$$4x'^2 - 6y'^2 - 8x' + 24y' + 4 = 0$$

$$4(x'-1)^2 - 4 - 6(y'-2)^2 + 24 + 4 = 0$$

$$\frac{(x'-1)^2}{6} - \frac{(y'-2)^2}{4} = -1$$

$x'' = (x'-1) \qquad a = \sqrt{6} \approx 2,5$

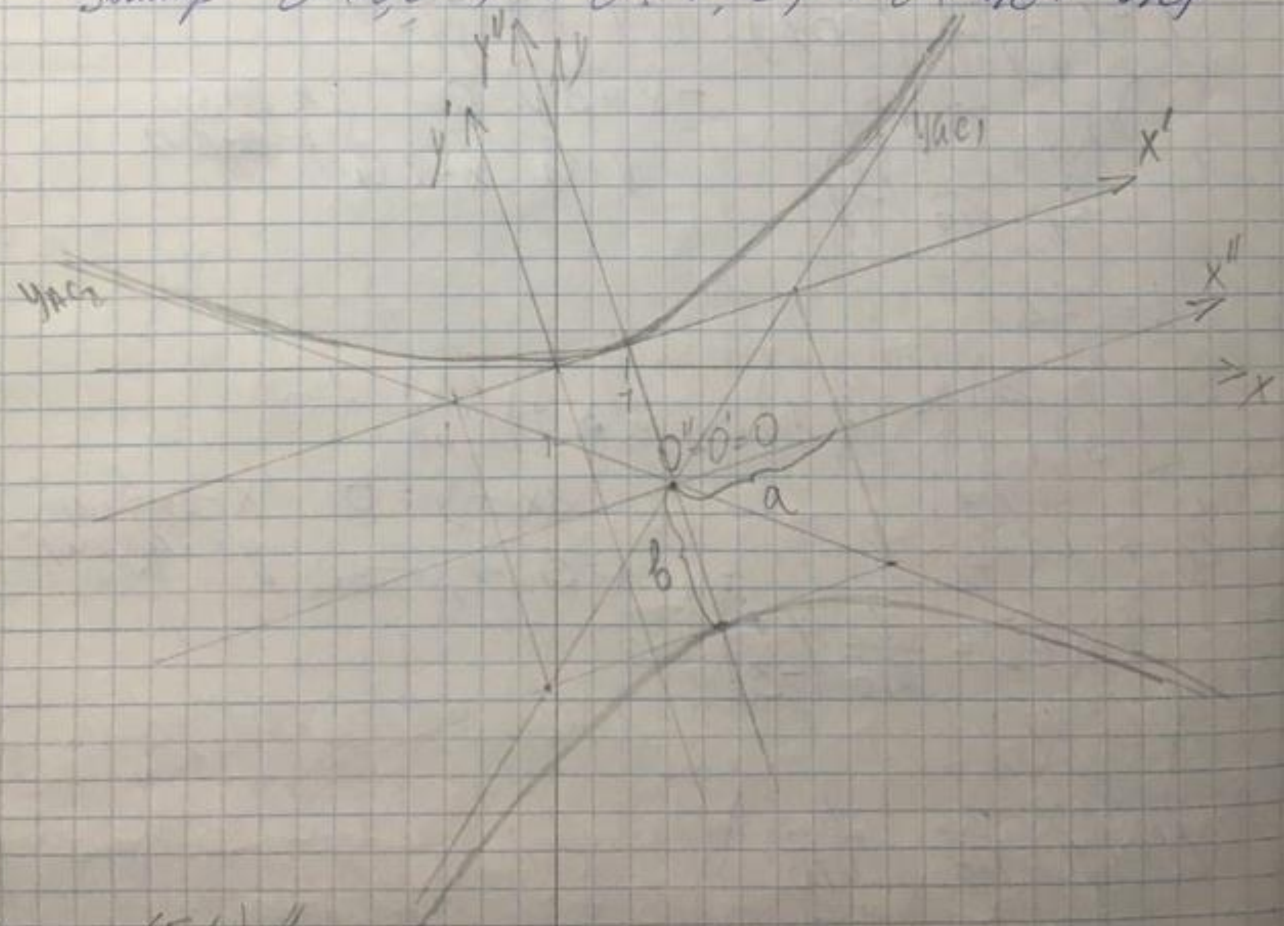
$y'' = (y'-2) \qquad b = \sqrt{4} = 2.$

Вид: сопряженная гипербола.

$$x = \frac{1}{\sqrt{10}}(3x'+y') = \frac{1}{\sqrt{10}}(3(x''+1) + y''+2) = \frac{1}{\sqrt{10}}(3x''+y''+5)$$

$$y = \frac{1}{\sqrt{10}}(x''+1+3(y''-2)) = \frac{1}{\sqrt{10}}(x''-3y''-5)$$

Центр $O''(0,0) \to O'(1,2) \to O\left(\frac{5}{\sqrt{10}}; -\frac{5}{\sqrt{10}}\right)$



$y_{ac_1} = (\sqrt{6}/2)x''$

$y_{ac_2} = -(\sqrt{6}/2)x''$

(6) $4x^2+7y^2+4xy-12\sqrt{5}x+6\sqrt{5}y+51=0$

Матрица КФ: $A=\begin{pmatrix}4&2\\2&7\end{pmatrix}$

$\det(A-\lambda E)$

$\begin{vmatrix}4-\lambda&2\\2&7-\lambda\end{vmatrix}=0 \quad (4-\lambda)(7-\lambda)-4=0$

$\lambda^2-11\lambda+24=0$

Найдем собств. векторы $\lambda_1=8 \quad \lambda_2=3$

$\lambda_1=8 \quad \begin{pmatrix}-4&2\\2&-1\end{pmatrix}\sim\begin{pmatrix}2&-1\end{pmatrix} \quad \begin{pmatrix}2&-1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=0$

$2x-y=0$ x - св. пер. y - баз пер.

$x=C_1$

$X=C_1\begin{pmatrix}1\\2\end{pmatrix} \quad 2C_1=y$

$\tilde{e}_1=\frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\end{pmatrix}$

$\lambda_2=3 \quad \begin{pmatrix}1&2\\2&4\end{pmatrix}\sim\begin{pmatrix}1&2\end{pmatrix} \quad \begin{pmatrix}1&2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=0$

$x+2y=0$ y - своб. п. x - баз п.

$X=C_2\begin{pmatrix}-2\\1\end{pmatrix} \quad y=C_2$

$x=-2C_2$

$\tilde{e}_2=\frac{1}{\sqrt{5}}\begin{pmatrix}-2\\1\end{pmatrix}$

$U=\begin{pmatrix}-2/\sqrt{5}&+1/\sqrt{5}\\1/\sqrt{5}&+2/\sqrt{5}\end{pmatrix} \quad X=UX'$

$\begin{cases}x=\frac{1}{\sqrt{5}}(-2x'+y')\\y=\frac{1}{\sqrt{5}}(x'+2y')\end{cases}$

$3x'^2+8y'^2-12\sqrt{5}\left(\frac{-2}{\sqrt{5}}x'+\frac{1}{\sqrt{5}}y'\right)+6\sqrt{5}\left(\frac{1}{\sqrt{5}}x'+\frac{2y'}{\sqrt{5}}\right)+51=$

$3x'^2+8y'^2+24x'\cancel{-12y'}+6x'\cancel{+12y'}+51=3x'^2+8y'^2+30x'+51$

$3(x'+5)^2-75+8y'^2+51=0$

$\frac{(x'+5)^2}{8}+\frac{y'^2}{3}=1 \quad x''=x'+5 \quad x'=x''-5$

$y''=y'$

$$\frac{(x'')^2}{8} + \frac{(y'')^2}{3} = 1$$

Тип линии: эллипс

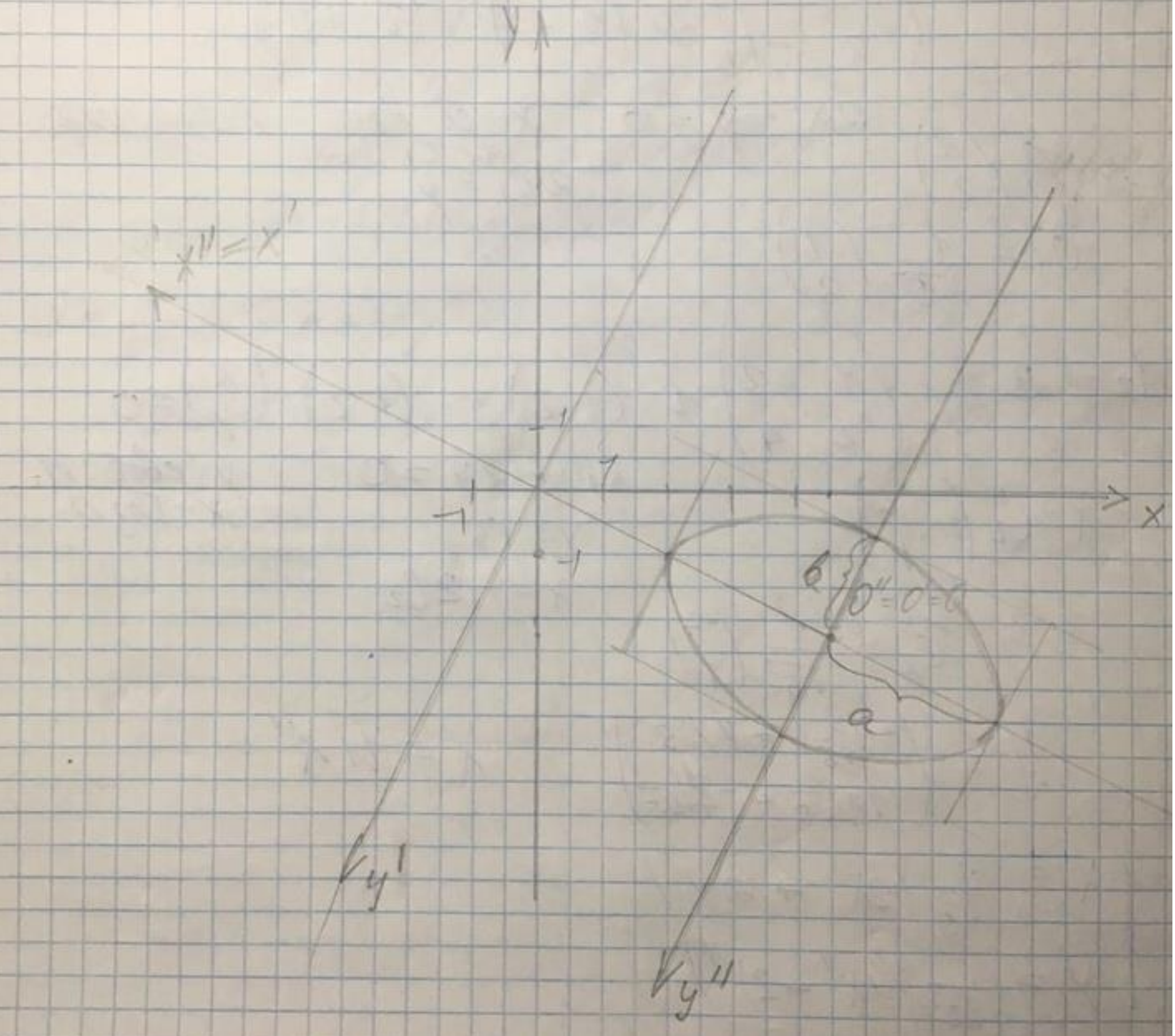
$a = \sqrt{8} \approx 2{,}8$

$b = \sqrt{3} \approx 1{,}73$

$O''(0\,0)$ $O'(-5;0)$ $O\left(\frac{10}{\sqrt{5}}; -\frac{5}{\sqrt{5}}\right)$

$$x = \frac{2x'}{\sqrt{5}} + \frac{y'}{\sqrt{5}} = \frac{2x''}{\sqrt{5}} + \frac{10}{\sqrt{5}} + \frac{y''}{\sqrt{5}}$$

$$y = \frac{-x'}{\sqrt{5}} + \frac{2y'}{\sqrt{5}} = \frac{-x''}{\sqrt{5}} - \frac{5}{\sqrt{5}} + \frac{2y''}{\sqrt{5}}$$



(c) $5x^2-y^2+z^2+4xy+6xz+2\sqrt{14}x+4\sqrt{14}y-6\sqrt{14}z-28=0$

$$A=\begin{pmatrix}5&2&3\\2&-1&0\\3&0&1\end{pmatrix}$$

$\det(A-\lambda E)=0$

$$\begin{vmatrix}5-\lambda&2&3\\2&-1-\lambda&0\\3&0&1-\lambda\end{vmatrix}=0 \Rightarrow -\lambda^3+5\lambda^2+14\lambda=0$$

$\lambda_1=0 \quad \lambda_2=7 \quad \lambda_3=-2$

1) $\lambda=0$

$$\begin{pmatrix}5&2&3\\2&-1&0\\3&0&1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=0 \quad \begin{pmatrix}5&2&3\\2&-1&0\\3&0&1\end{pmatrix}\sim\begin{pmatrix}5&2&3\\0&3&2\end{pmatrix}$$

x_1, x_2 - баз.
x_3 - своб.

$5x_1+2x_2+3x_3=0 \quad x_3=C_1$

$3x_2+2C_1=0 \quad x_2=-\frac{2C_1}{3} \quad x_1=-\frac{1}{3}C_1$

$$X=C_1\begin{pmatrix}-\frac{1}{3}\\-\frac{2}{3}\\1\end{pmatrix} \quad C=+3$$

$$\tilde{e}_1=\frac{1}{\sqrt{14}}\begin{pmatrix}-1\\-2\\+3\end{pmatrix}$$

2) $\lambda=7$

$$\begin{pmatrix}-2&2&3\\2&-8&0\\3&0&-6\end{pmatrix}\sim\begin{pmatrix}-2&2&3\\0&-6&3\end{pmatrix}$$

x_3 - своб.
x_1, x_2 - баз.

$-x_1+2x_2+3x_3=0$

$-2x_2+C_2=0$

$x_3=C_2$

$x_2=\frac{C_2}{2}$

$x_1=2C_2$

$$X=C_2\begin{pmatrix}2\\\frac{1}{2}\\1\end{pmatrix} \quad C_2=2$$

$$\tilde{e}_2=\frac{1}{\sqrt{21}}\begin{pmatrix}4\\1\\2\end{pmatrix}$$

3) $\lambda=-2$

$$\begin{pmatrix}7&2&3\\2&1&0\\3&0&3\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=0 \quad \begin{pmatrix}7&2&3\\2&1&0\\3&0&3\end{pmatrix}\sim\begin{pmatrix}7&2&3\\0&1&-2\end{pmatrix}$$

x_3 - своб
x_1, x_2 - баз

$x_3=C_3$

$7x_1+2x_2+3x_3=0$

$x_2-2C_3=0 \quad x_2=2C_3$

$x_1=-C_3$

$X = c_3\begin{pmatrix}-1\\2\\1\end{pmatrix} \quad c_3=1 \quad \vec{e}_3 = \frac{1}{\sqrt6}\begin{pmatrix}-1\\2\\1\end{pmatrix}$

$X = UX'$

$$U = \begin{pmatrix} -\frac{1}{\sqrt6} & -1/\sqrt{14} & 4/\sqrt{21} \\ 2/\sqrt6 & -2/\sqrt{14} & 1/\sqrt{21} \\ 1/\sqrt6 & 3/\sqrt{14} & 2/\sqrt{21} \end{pmatrix}$$

$x = (-1/\sqrt6)x' + (-1/\sqrt{14}\,y') + 4/\sqrt{21}\,z'$

$y = (2/\sqrt6)x' - 2/\sqrt{14}\,y' + 1/\sqrt{21}\,z'$

$z = 1/\sqrt6\,x' + 3/\sqrt{14}\,y' + 2/\sqrt{21}\,z'$

$-2x'^2 + 0y' + 7z'^2 + 2\sqrt{14}\left(-1/\sqrt6\,x' - \frac{1}{\sqrt{14}}y' + 4/\sqrt{21}\,z'\right) +$
$+ 4\sqrt{14}\left(\frac{2}{\sqrt6}x' - 2/\sqrt{14}\,y' + 1/\sqrt{21}\,z'\right) - 6\sqrt{14}\left(1/\sqrt6\,x' + 3/\sqrt{14}\,y' +\right.$
$\left.+ 2/\sqrt{21}\,z'\right) - 28 = -2x'^2 + 7z'^2 + 28y' - 28 = 0$

$x'' = z'$

$y'' = x'$

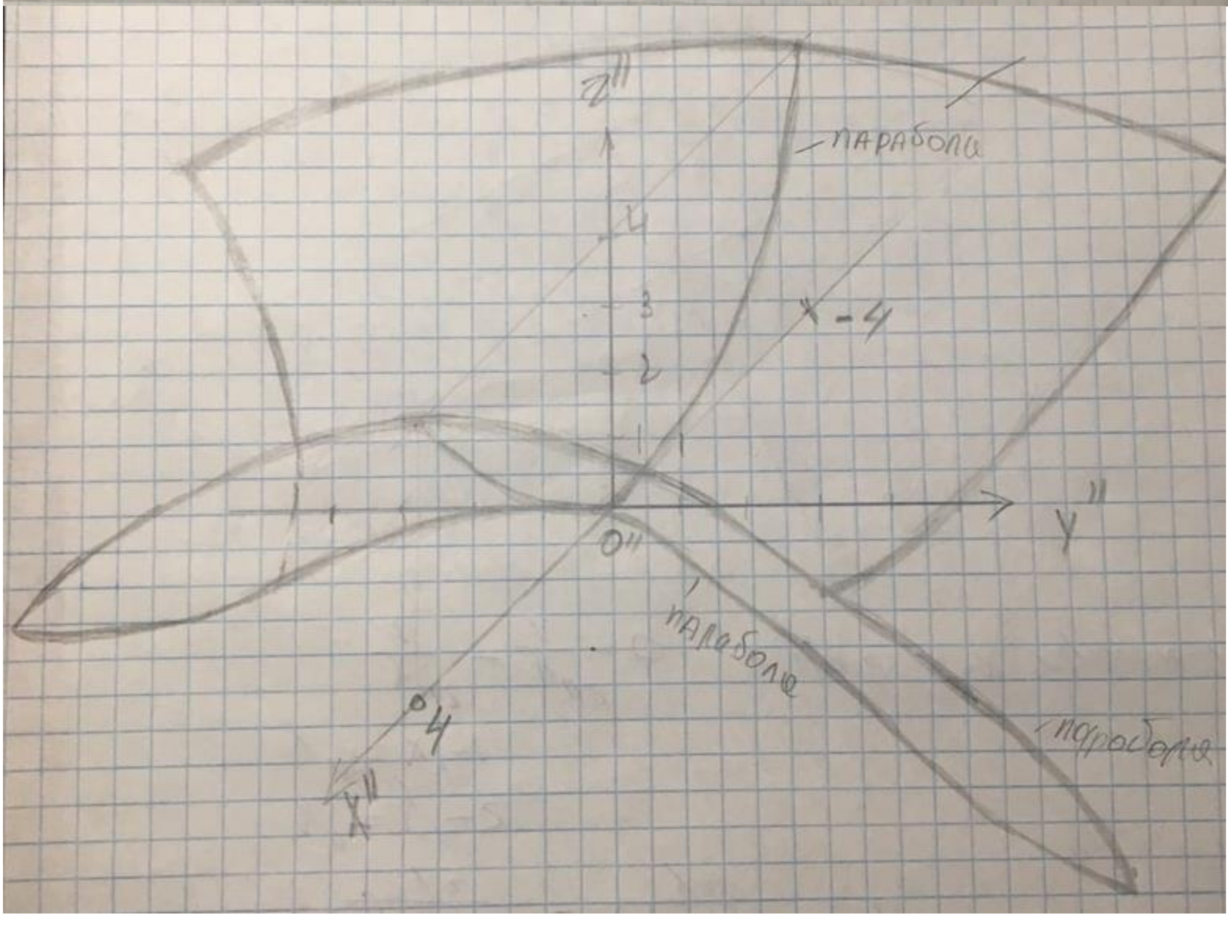
$z'' = y' + 1$

$-\frac{x'^2}{14} + \frac{z'^2}{4} = y' + 1$

$\frac{x''^2}{2} - \frac{y''^2}{7} = 2z''$

Тип: гипербол. параболоид.

$O''(000) \Rightarrow O'(0\,-1\,0) \Rightarrow O\left(\frac{1}{\sqrt{14}}; \frac{2}{\sqrt{14}}; -\frac{3}{\sqrt{14}}\right)$



(d) $x^2 + \quad 2z^2 - 4xy - 4xz + 4x + 4z - 14 = 0$

$$A = \begin{pmatrix} 1 & -2 & -2 \\ -2 & 0 & 0 \\ -2 & 0 & 2 \end{pmatrix}$$

I

$\det(A - E\lambda)$ $$\begin{vmatrix} 1-\lambda & -2 & -2 \\ -2 & 0-\lambda & 0 \\ -2 & 0 & 2-\lambda \end{vmatrix} = 0$$

$\lambda^3 - 3\lambda^2 - 6\lambda + 8 = 0$

$\Downarrow$

$\lambda_1 = 1 \quad \lambda_2 = 4 \quad \lambda_3 = -2$

$\lambda_1 = 1$ $$\begin{pmatrix} 0 & -2 & -2 \\ -2 & -1 & 0 \\ -2 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & -2 & -2 \\ -2 & -1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

x_1, x_2 - баз. п. $\quad x_3$ - своб п.

$2x_1 + x_2 = 0$

$x_2 + x_3 = 0 \quad x_3 = C_1$

$x_2 = -C_1$

$x_1 = +\frac{1}{2} C_1$

$$X = C_1 \begin{pmatrix} \frac{1}{2} \\ -1 \\ 1 \end{pmatrix} \quad C_1 = 2$$

$$\tilde{e}_1 = \frac{1}{3}\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$\lambda_2 = 4$ $$\begin{pmatrix} -3 & -2 & -2 \\ -2 & -4 & 0 \\ -2 & 0 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} -3 & -2 & -2 \\ -2 & -4 & 0 \\ -2 & 0 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \end{pmatrix}$$

x_3 - своб. $\quad x_1, x_2$ - баз.

$x_1 + x_3 = 0$

$2x_2 - x_3 = 0 \quad x_3 = C_2$

$x_2 = \frac{C_2}{2}$

$x_1 = -C_2$

$$X = C_2 \begin{pmatrix} -1 \\ \frac{1}{2} \\ 1 \end{pmatrix} \quad C_2 = -2$$

$$\tilde{e}_2 = \frac{1}{3}\begin{pmatrix} +2 \\ -1 \\ -2 \end{pmatrix}$$

$\lambda_3 = -2$ $$\begin{pmatrix} 3 & -2 & -2 \\ -2 & 2 & 0 \\ -2 & 0 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} 3 & -2 & -2 \\ -2 & 2 & 0 \\ -2 & 0 & 4 \end{pmatrix} \sim \begin{pmatrix} 3 & -2 & -2 \\ 0 & 1 & -2 \end{pmatrix}$$

$x_1 - 2x_3 = 0$ x_3 - своб. п.

$x_2 + 2x_3 = 0$ x_1, x_2 - базис. п.

$x_3 = c_3$ $x_2 = +2c_3$

$x_1 = 2c_3$

$$X = c_3 \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad c_3 = 1$$

$$\hat{e}_3 = \frac{1}{3}\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

$$U = \begin{pmatrix} \frac{1}{3} & 2/3 & 2/3 \\ -2/3 & -1/3 & 2/3 \\ 2/3 & -2/3 & 1/3 \end{pmatrix} \qquad X = UX'$$

$$x = \frac{1}{3}x' + \frac{2}{3}y' + 2/3\,z'$$

$$y = -\frac{2}{3}x' - \frac{1}{3}y' + 2/3\,z'$$

$$z = \frac{2}{3}x' - \frac{2}{3}y' + 1/3\,z'$$

$$= x'^2 + 4y'^2 - 2z'^2$$

$$4x + 4z = \frac{4}{3}x' + \cancel{\frac{8}{3}y'} + 8/3\,z' + 8/3\,x' - \cancel{8/3\,y'} + 4/3\,z' =$$

$$= 4x' + 4z'$$

$$x'^2 + 4y'^2 - 2z'^2 + 4x' + 4z' - 14 = 0$$

$$(x'+2)^2 - 4 + 4y'^2 - 2(z'-1)^2 + 2 - 14 = 0$$

$$(x'+2)^2 + 4y'^2 - 2(z-1)^2 = 16$$

$$\frac{(x'+2)^2}{16} + \frac{y'^2}{4} - \frac{(z-1)^2}{8} = 1$$

$$\begin{cases} x'' = x' + 2 \\ y'' = y' \\ z'' = z' - 1 \end{cases} \qquad \frac{x''^2}{(\sqrt{16})^2} + \frac{y''^2}{(\sqrt{4})^2} - \frac{z''^2}{(\sqrt{8})^2} = 1$$

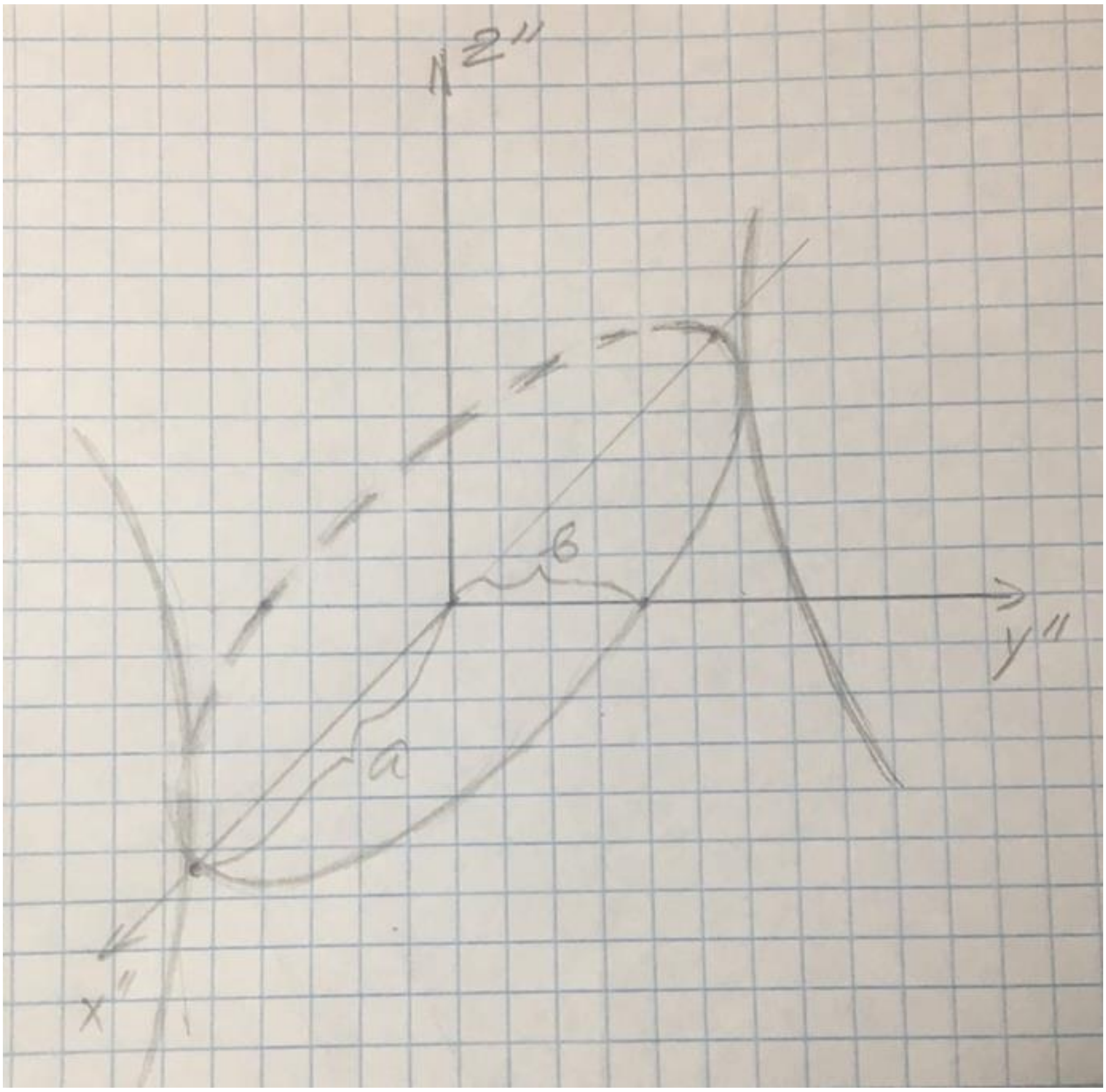
Тип: однополостный гиперболоид.

$$x = \frac{1}{3}(x''-2) + \frac{2}{3}\cdot y'' + \frac{2}{3}(z''+1) = \frac{x''}{3} + \frac{2y''}{3} + \frac{2z''}{3}$$

$$y = -\frac{2}{3}(x''-2) - \frac{1}{3}y'' + 2/3\,(z''+1) = -\frac{2x''}{3} - \frac{y''}{3} + \frac{2z''}{3} - \frac{2}{3}$$

$$z = \frac{2}{3}(x''-2) - \frac{2}{3}y'' + \frac{1}{3}(z''+1) = \frac{2x''}{3} - \frac{2y''}{3} + \frac{z''}{3} - 1$$

$$O(0,0,0) \qquad O'(-2;0;1) \qquad O\left(0; -\frac{2}{3}; -1\right)$$


z''
y''
x''
в
a