а) $\underbrace{24xy-16x^2-9y^2}_{I}+\underbrace{70x+10y}_{II}\underbrace{-125}_{III}=0$

$$\begin{pmatrix}-16 & 12\\ 12 & -9\end{pmatrix}$$

$$\begin{vmatrix}-16-\lambda & 12\\ 12 & -9-\lambda\end{vmatrix}=(16+\lambda)(9+\lambda)-144=\lambda^2+144+16\lambda+9\lambda-144=0$$

$\lambda(25+\lambda)=0 \Rightarrow -25x'^2$ – I канон. вид кв. ф.

$\lambda=0 \quad \lambda=-25$

$$\lambda=0 \quad \begin{pmatrix}-16 & 12\\ 12 & -9\end{pmatrix}\underset{II/3}{\overset{I/4}{\sim}}\begin{pmatrix}-4 & 3\\ 4 & -3\end{pmatrix}\overset{II+I}{\sim}\begin{pmatrix}-4 & 3\\ 0 & 0\end{pmatrix}$$

x – базисн.
y – свободн.

$-4x+3y=0$

$x=\frac{3}{4}y$

$y=C$, тогда $x=\frac{3}{4}C$

$$X^{\lambda=0}=\begin{pmatrix}x\\ y\end{pmatrix}=C\begin{pmatrix}\frac{3}{4}\\ 1\end{pmatrix}$$

$C=4 \Rightarrow E_1=\begin{pmatrix}3\\ 4\end{pmatrix}$

Нормируем

$\bar{e_1} = \frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\lambda = -25$

$\begin{pmatrix} 9 & 12 \\ 12 & 16 \end{pmatrix} \xrightarrow[II/4]{I/3} \begin{pmatrix} 3 & 4 \\ 3 & 4 \end{pmatrix} \xrightarrow{II-I} \begin{pmatrix} 3 & 4 \\ 0 & 0 \end{pmatrix}$

x - базисн.
y - свободная

$x = -\frac{4}{3}y$

Пусть $y = c$, $x = -\frac{4}{3}c$. $X^{\lambda=-25} = \begin{pmatrix} x \\ y \end{pmatrix} = c\begin{pmatrix} -\frac{4}{3} \\ 1 \end{pmatrix}$

Пусть $c = -3 \Rightarrow E_2 = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

Нормируем: $\bar{e_2} = \frac{1}{5}\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$V = \frac{1}{5}\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$

$X = VX' \Rightarrow \begin{cases} x = \frac{1}{5}(3x' + 4y') \\ y = \frac{1}{5}(4x' - 3y') \end{cases}$

$-25x'^2 + \frac{70}{5}(3x' + 4y') + \frac{10}{5}(4x' - 3y') - 125 = 0$

$-25x'^2 + 42x' + 56y' + 8x' - 6y' - 125 = 0$

$-25x'^2 + 50x' + 50y' - 125 = 0 \quad | -25$

$x'^2 - 2x' - 2y' + 5 = 0$

$x'^2 - 2x' + 1 - 2y' + 4 = 0$

$(x'-1)^2 - 2y' + 4 = 0$

$2y' = (x'-1)^2 + 4$

$2(\underbrace{y'-2}_{y''}) = (\underbrace{x'+1}_{x''})^2$

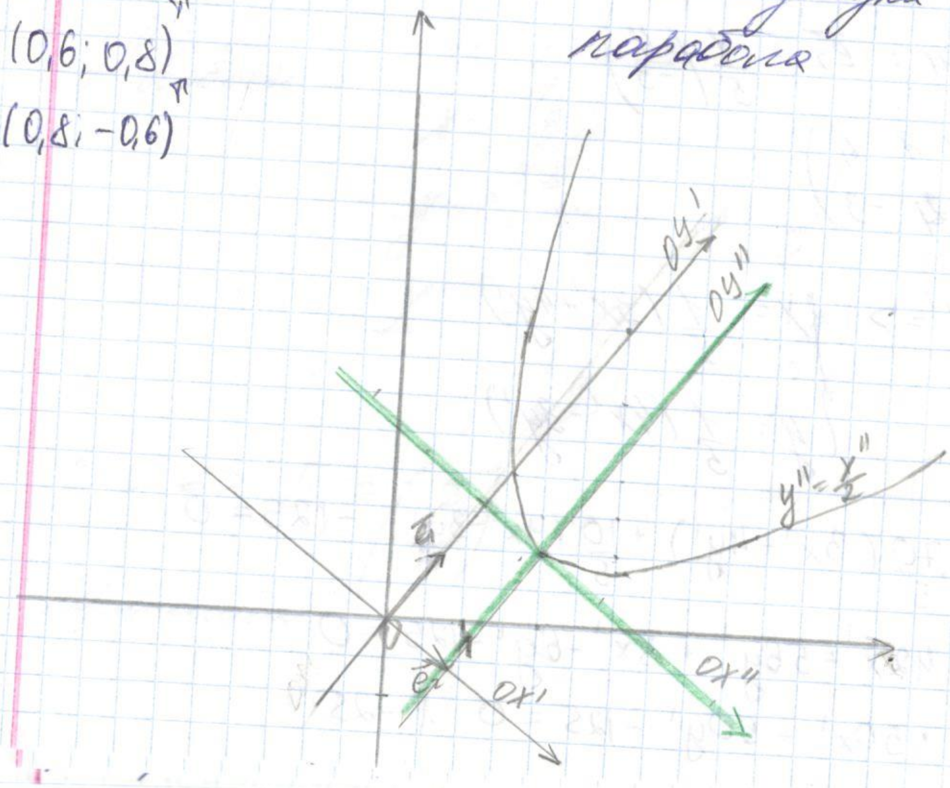
$\begin{cases} x'' = x'+1 \\ y'' = y'-2 \end{cases} \Rightarrow \quad 2y'' = x''^2$

$y'' = \dfrac{x''^2}{2}$ - кривая второго порядка

парабола

$\vec{e}_1 = (0,6;\ 0,8)^T$

$\vec{e}_2 = (0,8;\ -0,6)^T$



$$\underbrace{5x^2+8y^2-4xy}_{I}+\underbrace{16\sqrt5 x+8\sqrt5 y}_{II}\underbrace{+64}_{III}=0$$

$\begin{pmatrix}5 & -2\\ -2 & 8\end{pmatrix}$ - м. кв. ф.

$$\begin{vmatrix}5-\lambda & -2\\ -2 & 8-\lambda\end{vmatrix}=40-8\lambda-5\lambda+\lambda^2-4=0$$

$$\lambda^2-13\lambda+36=0$$

$$\lambda_1=9 \quad \lambda_2=4$$

$$9x'^2+4y'^2 = I$$

$\lambda=9$

$$\begin{pmatrix}-4 & -2\\ -2 & -1\end{pmatrix} \underset{I/2}{\overset{II+I/2}{\sim}} \begin{pmatrix}2 & 1\\ 0 & 0\end{pmatrix}$$

x баз.
y - своб.
$y=-2x$

$y=-2 \quad x=1$

$X=\begin{pmatrix}1\\-2\end{pmatrix} \quad \vec{e}=\frac{1}{\sqrt5}\begin{pmatrix}1\\-2\end{pmatrix}$

$\lambda=4$

$$\begin{pmatrix}1 & -2\\ -2 & 4\end{pmatrix} \overset{II+2I}{\sim} \begin{pmatrix}1 & -2\\ 0 & 0\end{pmatrix}$$

x баз.
y - своб.
$x=2y$

$y=1 \quad x=2$

$X = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\bar{e}_2 = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$V = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

$X = V \cdot X' \Rightarrow \begin{cases} x = \frac{1}{\sqrt{5}}(x' + 2y') \\ y = \frac{1}{\sqrt{5}}(-2x' + y') \end{cases}$

$I + II + III = 0$

$9x'^2 + 4y'^2 + \frac{16\sqrt{5}}{\sqrt{5}}(x' + 2y') + \frac{8\sqrt{5}}{\sqrt{5}}(-2x' + y') + 64 = 0$

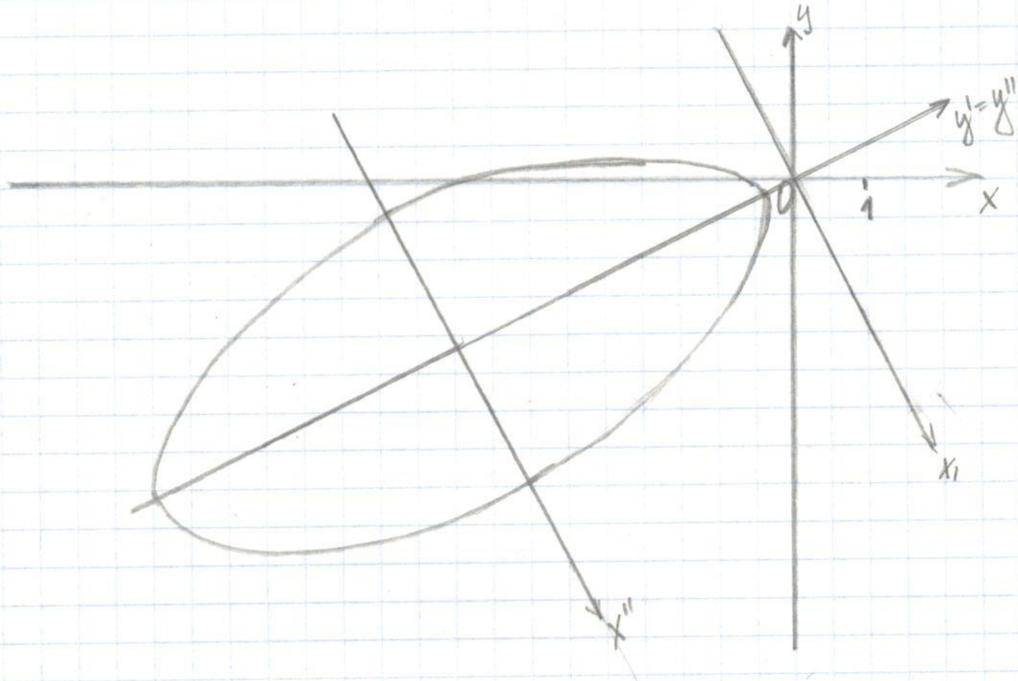
$9x'^2 + 4y'^2 + 16x' + 32y' - 16x' + 8y' + 64 = 0$

$9x'^2 + 4y'^2 + 40y' + 64 = 0$

$9x'^2 + 4(y'^2 + 5)^2 = 36 \quad | : 36$

$\frac{x'^2}{4} + \frac{(y' + 5)^2}{9} = 1$

$\begin{cases} x'' = x' \\ y'' = y' + 5 \end{cases} \Rightarrow \frac{x''}{4} + \frac{y''}{9} = 1$



y
y'=y''
0
1
x
x₁
x''

~~[illegible]~~

$$\underbrace{2x^2+2y^2+3z^2+4xy+2xz+2yz}_{I}\underbrace{-20\sqrt{2}x+20\sqrt{2}y}_{II}+\underbrace{40}_{III}=0$$

$$\begin{pmatrix} 2 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix} - \text{м. кв. ф}$$

$$\begin{vmatrix} 2-\lambda & 2 & 1 \\ 2 & 2-\lambda & 1 \\ 1 & 1 & 3-\lambda \end{vmatrix} = (4+\lambda^2-4\lambda)(3-\lambda)+2+2-(2-\lambda)-(2-\lambda)$$

$$-4(3-\lambda)=0$$

$$12-4\lambda+3\lambda^2-\lambda^3-12\lambda+4\lambda^2+4-2+\lambda-2+\lambda-12+4\lambda=0$$

$$-\lambda^3+7\lambda^2-10\lambda=0$$

$$-\lambda(\lambda^2-7\lambda+10)=0$$

$\lambda=0 \quad \lambda=2 \quad \lambda=5 \Rightarrow 2y'^2+5z'^2 \rightarrow I$

1) $\lambda=0$ $\begin{pmatrix} 2 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix} \overset{I-II}{\underset{III\sim I}{\sim}} \begin{pmatrix} 1 & 1 & 3 \\ 2 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix} \overset{II-2I}{\sim} \begin{pmatrix} 1 & 1 & 3 \\ 0 & 0 & -5 \\ 0 & 0 & 0 \end{pmatrix}$

~~[illegible]~~ $z=0$

$x=-y$. При $x=1$

$X^{\lambda=0}=(1;-1;0)^T$. $\vec{e}_1=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

$\lambda = 2$

$$\begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} \overset{II-2I}{\sim} \begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & -1 \\ 0 & 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$z = -2,\ y = 1,\ x = 1.\quad X = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}\quad \vec{e}_1 = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

$\lambda = 5$

$$\begin{pmatrix} -3 & 2 & 1 \\ 2 & -3 & 1 \\ 1 & 1 & -2 \end{pmatrix} \underset{II-2III}{\overset{I+3III}{\sim}} \begin{pmatrix} 0 & 5 & -5 \\ 0 & -5 & 5 \\ 1 & 1 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -2 \\ 0 & 5 & -5 \\ 0 & 0 & 0 \end{pmatrix}$$

$x = 1,\ y = 1,\ z = 1$

$X = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\quad \vec{e}_2 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$V = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} \qquad X = VX'$$

$$\begin{cases} x = \frac{1}{\sqrt{6}}(\sqrt{3}\,x' + y' + \sqrt{2}\,z') \\ y = \frac{1}{\sqrt{6}}(-\sqrt{3}\,x' + y' + \sqrt{2}\,z') \\ z = \frac{1}{\sqrt{6}}(-2y' + \sqrt{2}\,z') \end{cases}$$

$$2y'^2 + 5z'^2 - \frac{20\sqrt{2}}{\sqrt{2}\cdot\sqrt{3}}(\sqrt{3}x' + y' + \sqrt{2}z') +$$

$$+\frac{20\sqrt{2}}{\sqrt{2}\cdot\sqrt{3}}(-\sqrt{3}x' + y' + \sqrt{2}z' + 40 = 0$$

$$2y'^2 + 5z'^2 - 20x' - \frac{20y'\sqrt{3}}{3} - \frac{20\sqrt{6}z'}{3} - 20x' + \frac{20\sqrt{3}y'}{3} +$$

$$+\frac{20z'\sqrt{6}}{3} + 40 = 0$$

$$2y'^2 + 5z'^2 - 40x' + 40 = 0$$

$$2y'^2 + 5z'^2 = 40(x' - 1)$$

$$\begin{cases} y'' = y' \\ z'' = z' \\ x'' = x' - 1 \end{cases} \Rightarrow \quad 2y''^2 + 5z''^2 = 40x'' \quad |:40$$

$$\frac{y''^2}{20} + \frac{5z''^2}{8} = x''$$ – ~~кано~~

каноническое уравнение параболоида

d) $\underbrace{x^2+2z^2-4xy-4xz}_{I}-12x+12z+18=0$

$\begin{pmatrix} 1 & -2 & -2 \\ -2 & 0 & 0 \\ -2 & 0 & 2 \end{pmatrix}$ – м. кв. формы

$\begin{vmatrix} 1-\lambda & -2 & -2 \\ -2 & -\lambda & 0 \\ -2 & 0 & 2-\lambda \end{vmatrix} = (\lambda^2-\lambda)(2-\lambda)+4\lambda-2\cdot2(2-\lambda)=$

$=2\lambda^2-\lambda^3-2\lambda+\lambda^2+4\lambda-8+4\lambda=0$

$-\lambda^3+3\lambda^2+6\lambda-8=0$

$\lambda_1=-2,\ \lambda_2=1,\ \lambda_3=4 \Rightarrow -2x'^2+y'^2+4z'^2 \to I$

$\lambda_1=-2$

$\begin{pmatrix} 3 & -2 & -2 \\ -2 & 2 & 0 \\ -2 & 0 & 4 \end{pmatrix} \overset{I+II}{\sim} \begin{pmatrix} 1 & 0 & -2 \\ -2 & 2 & 0 \\ -2 & 0 & 4 \end{pmatrix} \overset{III+2II}{\sim} \begin{pmatrix} 1 & 0 & -2 \\ -2 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sim$

$\overset{II+2I}{\sim} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & -4 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad z=1,\ y=2,\ x=2 \quad \bar{X}=\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad \bar{e}_1=\frac{1}{3}\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

$\lambda=1$

$\begin{pmatrix} 0 & -2 & -2 \\ -2 & -1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \overset{III-II}{\sim} \begin{pmatrix} 0 & -2 & -2 \\ -2 & -1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \underset{\text{поменяем}}{\overset{I+2III}{\sim}} \begin{pmatrix} -2 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

$z = +2$
$y = -2$
$x = 1$

$X = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ $\vec{e_2} = \frac{1}{3}\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

$\lambda = 4$

$$\begin{pmatrix} -3 & -2 & -2 \\ -2 & -4 & 0 \\ -2 & 0 & -2 \end{pmatrix} \overset{\text{III}-\text{II}}{\sim} \begin{pmatrix} -3 & -2 & -2 \\ -2 & -4 & 0 \\ 1 & 2 & 0 \end{pmatrix} \overset{\text{II}+2\text{III}}{\underset{\text{I}\cdot(-1)}{\sim}} \begin{pmatrix} 3 & 2 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{pmatrix} \overset{\text{I}-\text{III}\cdot 3}{\sim}$$

$$\begin{pmatrix} 0 & -4 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 \\ 0 & -4 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$ $x = 2y$, $4y = 2z$ $x = 2,\ y = -1,\ z = -2$

$X = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ $\vec{e_3} = \frac{1}{3}\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$

$U = \frac{1}{3}\begin{pmatrix} 2 & 1 & 2 \\ 2 & -2 & -1 \\ 1 & 2 & -2 \end{pmatrix}$ $X = UX'$

$$\begin{cases} x = \frac{1}{3}(2x' + y' + 2z') \\ y = \frac{1}{3}(2x' - 2y' - z') \\ z = \frac{1}{3}(x' + 2y' - 2z') \end{cases}$$

$$-2x'^2 + y'^2 + 4z'^2 - \frac{12}{3}(2x' + y' + 2z') + 4(x' + 2y' - 2z') + 18 = 0$$

$$-2x'^2 + y'^2 + 4z'^2 - \underline{8x'} - 4y' - 8z' + \underline{4x'} + 8y' - 8z' + 18 = 0$$

$$-2x'^2 + y'^2 + 4z'^2 - 4x' + 4y' - 16z' + 18 = 0$$

$$-2(x'^2 + 2x' + 1) + 2 + y'^2 + 4y' + 4 - 4 + 4(z'^2 - 4z' + 4) - 16 + 18 = 0$$

$$-2(x'+1)^2 + (y'+2)^2 + 4(z'-2)^2 = 0$$

$$\frac{(y'+2)^2}{4} + \frac{(z'-2)^2}{1} - \frac{(x'+1)^2}{2} = 0$$

$$\begin{cases} y'' = y' + 2 \\ z'' = z' - 2 \\ x'' = x' + 1 \end{cases} \Rightarrow \frac{y''^2}{4} + \frac{z''^2}{1} - \frac{x''^2}{2} = 0 \text{ -}$$

каноническое ур-ие конуса 2-го порядка