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1. $z = \ln\sqrt{x^2 - y^2}$

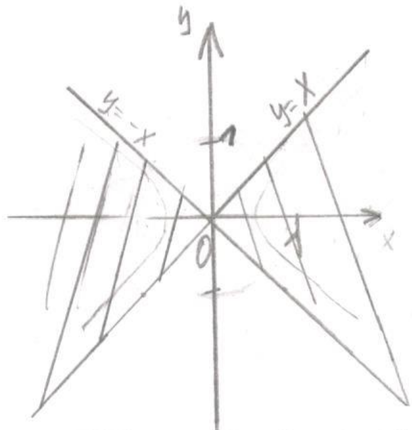
$x^2 - y^2 \geq 0$

$D(z) = \{ (x,y) \in \mathbb{R}^2 : x^2 - y^2 > 0 \}$

$x^2 - y^2 = 0$

$y^2 = x^2$

$\left[\begin{array}{l} y = x \\ y = -x \end{array}\right.$



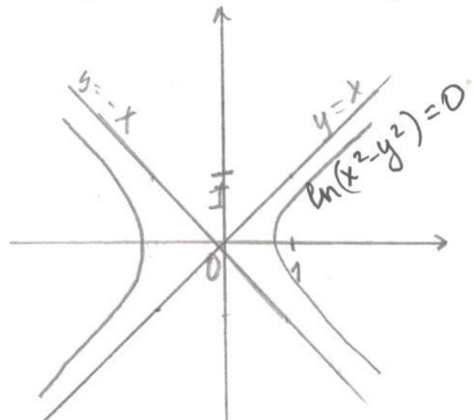
$c = \ln\sqrt{x^2 - y^2}$, где c – const

Подставим $(-1;0)$

$c = \ln\sqrt{(-1)^2 - 0^2} = \ln 1 = 0$

Таким образом, $0 = \ln\sqrt{x^2 - y^2}$

$\ln\sqrt{x^2 - y^2} = 0$ - линия уровня



2. $u = (x^2 + y^2 + z^2)^{\frac{3}{2}}$ $M(1,-2,2)$

MN, $N(3,0,3)$

$\frac{\partial u}{\partial x} = \frac{3}{2}(x^2 + y^2 + z^2)^{\frac{1}{2}} \cdot 2x = 3x\sqrt{x^2 + y^2 + z^2}$ $\quad \frac{\partial u}{\partial x} = 3\sqrt{1+4+4} = 9$

$\frac{\partial u}{\partial y} = 3y\sqrt{x^2 + y^2 + z^2}$ $\quad \frac{\partial u}{\partial y} = -6\sqrt{9} = -18$

$\frac{\partial u}{\partial z} = 3z\sqrt{x^2 + y^2 + z^2}$ $\quad \frac{\partial u}{\partial z} = 6 \cdot 3 = 18$

$\text{grad}\, u = \{9; -18; 18\}$

$\overline{MN} = (2; 2; 1)$; $|MN| = 3$

$\cos\alpha = \frac{\overline{MN}_x}{|\overline{MN}|} = \frac{2}{3}$ $\cos\beta = \frac{\overline{MN}_y}{|\overline{MN}|} = \frac{2}{3}$ $\cos\gamma = \frac{\overline{MN}_z}{|\overline{MN}|} = \frac{1}{3}$

$$\frac{\partial u}{\partial MN} = u'_x \cos\alpha + u'_y \cos\beta + u'_z \cos\gamma$$

$$\frac{\partial u}{\partial MN} = \frac{9 \cdot 2}{3} + \frac{(-18)\cdot 2}{3} + 18 \cdot \frac{1}{3} = 6 - 12 + 6 = 0$$

$$\frac{\partial u}{\partial MN} = 0$$

3. $$\lim_{\substack{x\to 0\\ y\to 0}} e^{x-\frac{1}{y^2}} = \left| y = kx \right| = \lim_{x\to 0} e^{x-\frac{1}{(kx)^2}} = e^{0-\frac{1}{0}} = e^{-\infty} = 0$$

Предел существует

4. $z = f(u(x,y), v(x,y))$

$f = tg(u - v)$

$u = arctg\, xy$; $v = arctg \frac{x}{y}$;

$z = tg\left(arctg\, xy - arctg \frac{x}{y}\right)$

$$z'_x = \frac{1}{\cos^2(arctg\, \cancel{\frac{x}{y}}\, xy - arctg\frac{x}{y})} \cdot \left(\frac{y}{1 + x^2y^2} - \frac{1}{\left(1 + \frac{x^2}{y^2}\right)} \cdot \frac{1}{y} \right)$$

$$z'_y = \frac{1}{\cos^2(arctg\, xy - arctg\frac{x}{y})} \left(\frac{x}{1 + x^2y^2} - \frac{x}{\left(1 + \frac{x^2}{y^2}\right)} \left(\frac{-1}{y^2}\right) \right)$$

5. $xy - z = e^z$, $M(1,1,0)$. $M_0(1,1)$ Приближ. $(0,9; 1,2)$

$xy - z - e^z = 0$

$$\cancel{\begin{cases} F'_x = y - z - e^z \\ F'_y = x - z - e^z \\ F'_z = -1 - e^z \end{cases}}$$

$$\cancel{z'_x = -\frac{F'_x}{F'_z} = -\frac{y - z - e^z}{-1 - e^z} = \frac{y - z - e^z}{1 + e^z}.\ z'_x = \frac{1 - 0 - e^0}{1 + e^0} = 0}$$

$$\cancel{z'_y = -\frac{F'_y}{F'_z} = -\frac{x - z - e^z}{-1 - e^z} = \frac{x - z - e^z}{1 + e^z}.\ z'_y = \frac{1 - 0 - e^0}{1 + e^0} = 0}$$

$$\begin{cases} F'_x = y \\ F'_y = x \\ F'_z = -1 - e^z \end{cases}$$

$$z'_x = -\frac{y}{-1 - e^z} = \frac{y}{1 + e^z}.\ z'_x = \frac{1}{1 + e^0} = \frac{1}{2}$$

$$z'_y = -\frac{x}{-1 - e^z} = \frac{x}{1 + e^z}.\ z'_y = \frac{1}{1 + e^0} = \frac{1}{2}$$

$$dz = \frac{1}{2}\Delta x + \frac{1}{2}\Delta y$$

$$x = x_0 + \Delta x \qquad 0{,}9 = 1 + \Delta x$$

$$y = y_0 + \Delta y \qquad 1{,}2 = 1 + \Delta y$$

$$\Delta x = -0{,}1$$

$$\Delta y = 0{,}2$$

Приближ. значение функции

$z = 0$; $dz = dx + dy$;

$$dz = -0{,}1 + 0{,}2 = 0{,}1 \approx \Delta z$$

$$z(0{,}9; 1{,}2) \approx 0 + \Delta z = 0{,}1$$