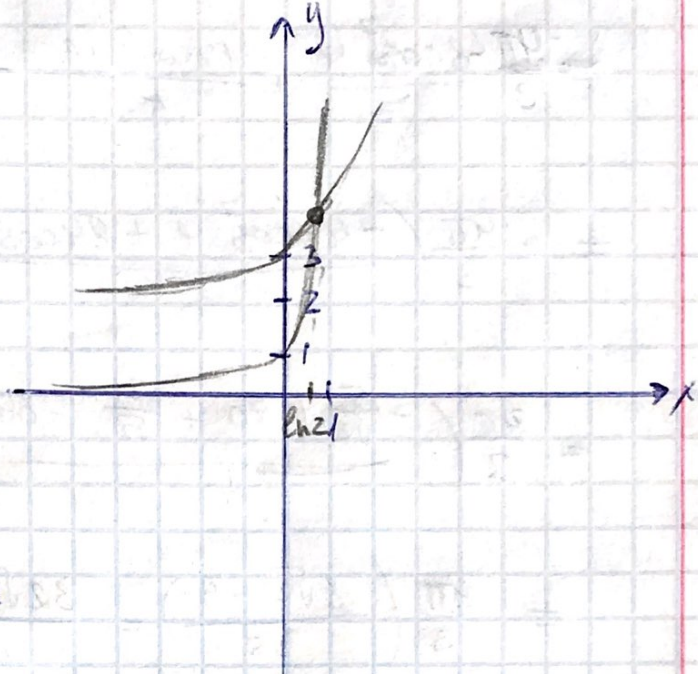
Вар. 1.

№1. $y=e^{2x}$

$y=e^{x}+2$

$x=0$



$e^{x}+2=e^{2x}$

$t^2-t-2=0$

$t_1=-1$ - не явл. реш

$t_2=2$

$e^{x}=2$

$x=\ln 2$

$$S=\int_0^{\ln 2}(e^{x}+2-e^{2x})\,dx=$$

$$=\int_0^{\ln 2}e^{x}dx+2\int_0^{\ln 2}dx-\int_0^{\ln 2}e^{2x}dx=e^{x}\Big|_0^{\ln 2}+2x\Big|_0^{\ln 2}-\frac{e^{2x}}{2}\Big|_0^{\ln 2}=$$

$$=2-1+2\ln 2-\frac{1}{2}(4-1)=1+2\ln 2-\frac{3}{2}=2\ln 2-\frac{1}{2}$$

№2. $\rho=\cos 2\varphi$

$$V=\frac{2\pi}{3}\int_0^{\pi/4}\cos^3(2\varphi)\sin\varphi\,d\varphi=\frac{2\pi}{3}\int_0^{\pi/4}(2\cos^2\varphi-1)^3\sin\varphi\,d\varphi$$

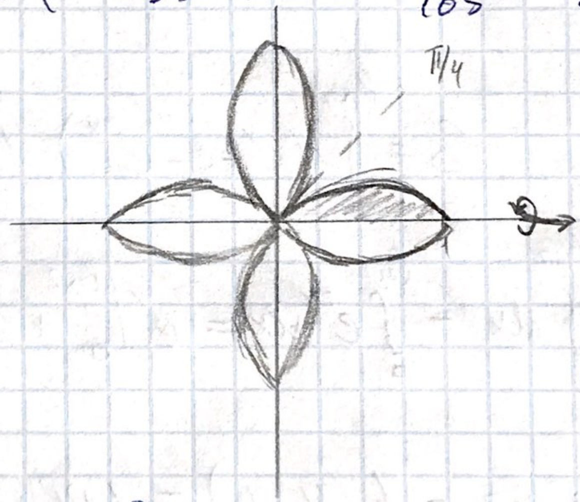
$$=\begin{bmatrix} u=\cos\varphi \\ -du=\sin\varphi\,d\varphi\end{bmatrix}=\frac{2\pi}{3}\int_0^{\pi/4}-(2u^2-1)^3du=\frac{-2\pi}{3}\int_0^{\pi/4}(8u^2-12u^4+6u^2-1)du$$

$$= \frac{2\pi}{3}\left(-\frac{8\cos^7\varphi}{7} + \frac{12\cos^5\varphi}{5} - 2\cos^3\varphi + \cos\varphi\right)\Bigg|_0^{\pi/4}$$

$$= \frac{2\pi}{3}\left(\frac{-40\cos^7\varphi + 84\cos^5\varphi - 70\cos^3\varphi + 70\cos\varphi}{35}\right)\Bigg|_0^{\pi/4} =$$

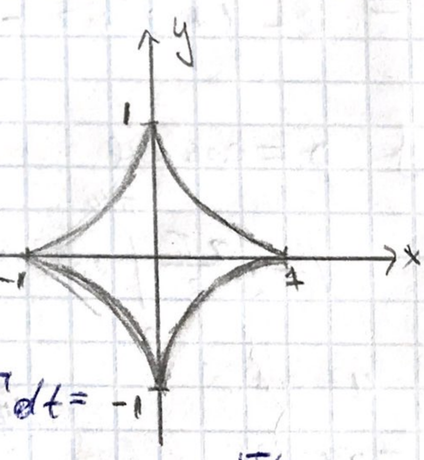
$$= \frac{2\pi}{3}\left(\frac{-\frac{5}{\sqrt{2}} + 40 + \frac{21}{\sqrt{2}} - 84 - \frac{35}{\sqrt{2}} + 70 + \frac{35}{\sqrt{2}} - 35}{35}\right) =$$

$$= \frac{2\pi}{3}\left(\frac{8\sqrt{2} - 9}{35}\right) = \frac{16\sqrt{2}\pi}{105} - \frac{6\pi}{35}$$



№3

$$\begin{cases} x = \cos^3 t \\ y = \sin^3 t \end{cases}$$



$$l = 4\int_0^{\pi/2} \sqrt{9\cos^4 t\sin^2 t + 9\sin^4 t\cos^2 t}\,dt =$$

$$= 4\int_0^{\pi/2} \sqrt{9\sin^2 t\cos^2 t\,\underbrace{(\cos^2 t + \sin^2 t)}_{1}}\,dt =$$

$$= 3\cdot 4\int_0^{\pi/2} \sin t\cos t\,dt = 3\cdot 4\cdot \frac{\sin^2 t}{2}\Bigg|_0^{\pi/2} = 6(1-0) = 6$$

№4 $\int\limits_1^{+\infty} \frac{\ln x}{x} dx = \lim\limits_{a\to+\infty} \int\limits_1^a \frac{\ln x}{x} dx =$ Особые точки: $x = +\infty$

$= \lim\limits_{a\to+\infty} \left(\frac{\ln^2 x}{2} \Big|_1^a \right) = \frac{1}{2} \lim\limits_{a\to+\infty} (\ln^2 a - \ln 1) =$

$= \frac{1}{2}(\infty - 0) = +\infty$ - расходится

№5 $\int\limits_0^{\pi/2} \frac{1-\cos x}{x^3} dx = \left| \begin{matrix} 1-\cos x \sim \frac{x^2}{2} \\ \text{при } x\to 0 \end{matrix} \right| = \int\limits_0^{\pi/2} \frac{x^2}{2x^3} dx =$

Особые точки: $x=0$

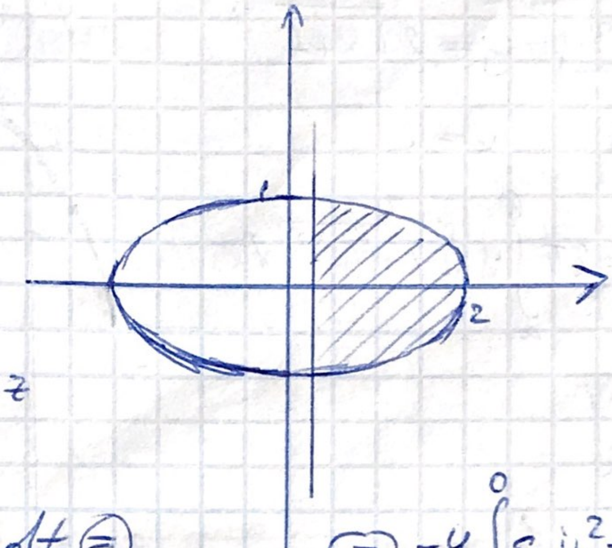
$= \int\limits_0^{\pi/2} \frac{dx}{2x} = \frac{1}{2} \ln x \Big|_0^{\pi/2} = \frac{1}{2}(\ln\frac{\pi}{2} - \ln 0) =$

$= \frac{1}{2}(\ln\frac{\pi}{2} + \infty) = +\infty$ - расходится

Вар. 2

№1 $\begin{cases} x = 2\cos t \\ y = \sin t \end{cases}$

$x = \frac{1}{4}$



$\frac{1}{4} = 2\cos t$

$t = \pm \arccos\frac{1}{8} + 2\pi k, k \in \mathbb{Z}$

$$S = -2\int_{\arccos\frac{1}{8}}^{0} \sin t(-2\sin t)\,dt = -4\int_{\arccos\frac{1}{8}}^{0} \sin^2 t\,dt =$$

$$= -4\int_{\arccos\frac{1}{8}}^{0} \frac{1-\cos 2t}{2}\,dt = -2\left(\int_{\arccos\frac{1}{8}} dt - \int_{\arccos\frac{1}{8}} \cos 2t\,dt\right) =$$

$$= -2\left(t\Big|_{\arccos\frac{1}{8}}^{0} - \frac{\sin 2t}{2}\Big|_{\arccos\frac{1}{8}}^{0}\right) = -2\left(-\arccos\frac{1}{8} + \frac{\sin(2\arccos\frac{1}{8})}{2}\right) =$$

$$= 2\arccos\frac{1}{8} - \sin\left(2\arccos\frac{1}{8}\right)$$

№2

$\rho = \sin 2\varphi$

$$V = \frac{2\pi}{3}\int_0^{\pi/2} \sin^3 2\varphi\,d\varphi = \frac{2\pi}{3}\int_0^{\pi/2} (1-\cos^2 2\varphi)(-d\cos 2\varphi) =$$

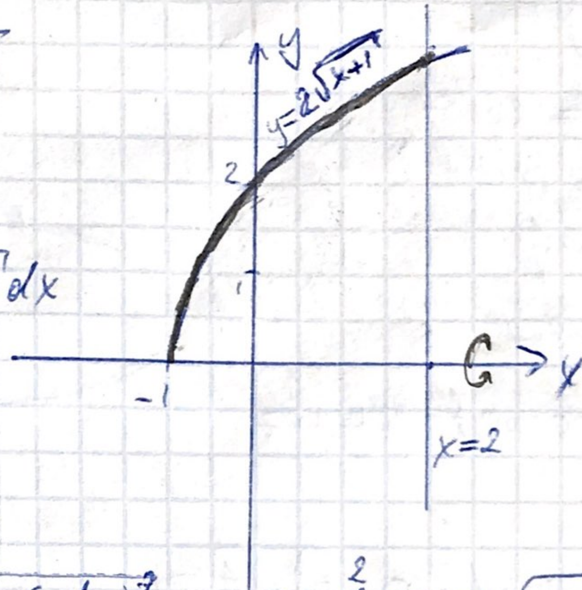
$$= \frac{2\pi}{3}\left(\int_0^{\pi/2} \cos^2 2\varphi\,d\cos 2\varphi - \int_0^{\pi/2} d\cos 2\varphi\right) = \frac{2\pi}{3}\left(\frac{\cos^3 2\varphi}{6}\Big|_0^{\pi/2} - \frac{\cos 2\varphi}{2}\Big|_0^{\pi/2}\right) =$$

$$= \frac{2\pi}{3}\left(\frac{1}{3} + \frac{1}{3}\right) = \frac{2\pi}{3}\cdot\frac{2}{3} = \frac{4\pi}{9}$$

№3 $y = 2\sqrt{x+1}$

$x = 2$

$$S_x = 2\pi\int_a^b y\sqrt{1+(y')^2}\,dx$$



$a = -1$

$b = 2$

$$S_x = 2\pi\int_{-1}^{2} 2\sqrt{x+1}\cdot\sqrt{1+\left(\frac{1}{\sqrt{x+1}}\right)^2}\,dx = 4\pi\int_{-1}^{2}\sqrt{x+1}\cdot\underbrace{\sqrt{1+\frac{1}{x+1}}}_{=\sqrt{\frac{x+2}{x+1}}}\,dx =$$

$$= 4\pi\int_{-1}^{2}\sqrt{\cancel{(x+1)}\frac{x+2}{\cancel{(x+1)}}}\,dx = 4\pi\int_{-1}^{2}\sqrt{x+2}\,dx =$$

$$= 4\pi\cdot\frac{2\sqrt{(x+2)^3}}{3}\Bigg|_{-1}^{2} = 4\pi\cdot\frac{2}{3}\cdot(8-1) = 4\pi\cdot\frac{14}{3} = \boxed{\frac{56\pi}{3}}$$

№4 $$\int_1^{+\infty}\frac{\sin\frac{1}{x}}{\sqrt{x+1}}\,dx = \left[\sin\left(\frac{1}{x}\right)\sim\frac{1}{x} \text{ при } x\to+\infty\right] =$$

Особые точки: $x = +\infty$

$$= \int_1^{+\infty}\frac{dx}{x\sqrt{x+1}} =$$

$f(x) = x\sqrt{x+1}$. Найти $g(x): g(x)\sim f(x)$. Пусть $g(x) = x^k$

$$\lim_{x\to+\infty}\frac{f(x)}{g(x)}=\lim_{x\to+\infty}\frac{x^{3/2}\sqrt{1+\frac{1}{x}}}{x^k}=[k=3/2]=$$

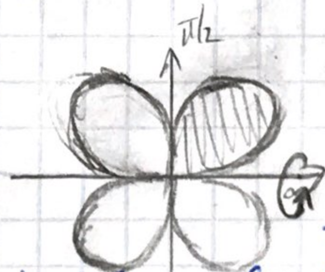
$$=\lim_{x\to+\infty}\sqrt{1+1/x}=1\Rightarrow f(x)\sim g(x)=x^{3/2}$$

$$\ominus\int_1^{+\infty}\frac{dx}{x^{3/2}}=-\frac{2}{\sqrt{x}}\Big|_1^{+\infty}=2 \text{ - сходится}$$

№5 $\int_0^1\frac{\ln(1+\sqrt[5]{x^3})}{e^x-1}dx=\left[\begin{array}{l}\ln(1+\sqrt[5]{x^3})\sim x^{3/5}, x\to 0\\ e^x-1\sim x, x\to 0\end{array}\right]\ominus$ Особые точки: $x=0$

$$\ominus\int_0^1\frac{x^{3/5}}{x}dx=\int_0^1\frac{dx}{x^{2/5}}=\frac{5}{3}\left(\sqrt[5]{x^3}\right)\Big|_0^1=$$

$$=\frac{5}{3}(1-0)=\frac{5}{3} \text{ - сходится}$$



№2 $\rho=\sin 2\varphi$

$$V=\frac{2\pi}{3}\int_0^{\pi/2}\sin^3 2\varphi\,\sin\varphi\,d\varphi=\frac{2\pi}{3}\int_0^{\pi/2}8\cos^3\varphi\sin^4\varphi\,d\varphi=$$

$$=\frac{16\pi}{3}\int_0^{\pi/2}\cos\varphi\sin^4\varphi(1-\sin^2\varphi)\,d\varphi=\frac{16\pi}{3}\int u^4(1-u^2)\,du=$$

$$=\frac{16\pi}{3}\left(\int_0^{\pi/2}u^4du-\int_0^{\pi/2}u^6du\right)=\frac{16\pi}{3}\left(\frac{\sin^5\varphi}{5}-\frac{\sin^7\varphi}{7}\right)\Big|_0^{\pi/2}=$$

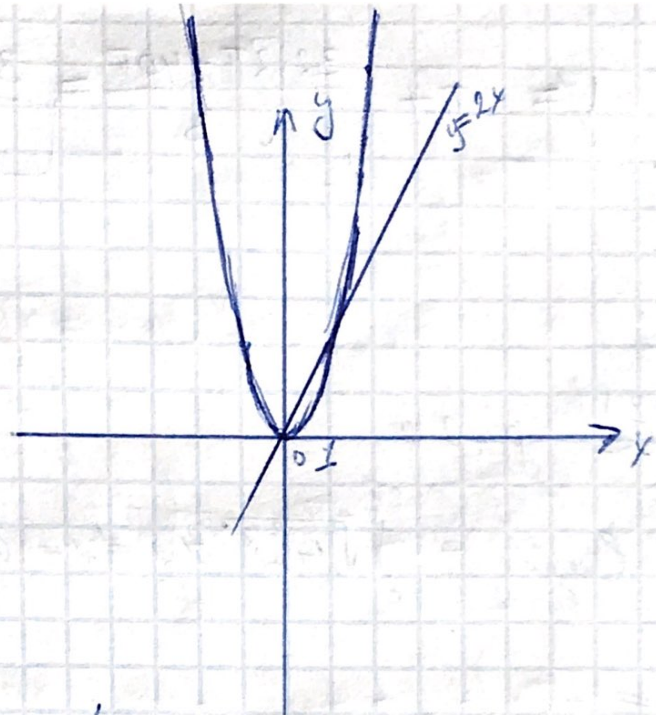
$$=16\pi\left(\frac{\sin^5\varphi}{15}-\frac{\sin^7\varphi}{21}\right)\Big|_0^{\pi/2}=16\pi\left(\frac{1}{15}-\frac{1}{21}\right)=\boxed{\frac{32\pi}{105}}$$

Вар. 3

№1

$y = 2x^2$

$y = 2x$



$2x^2 = 2x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$

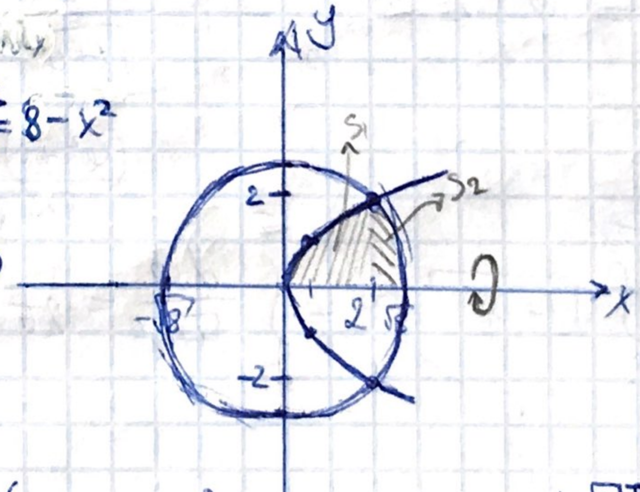
$x = 1$

$$S = \int_0^1 (2x - 2x^2)\,dx = 2\int_0^1 (x - x^2)\,dx = 2\left(\int_0^1 x\,dx - \int_0^1 x^2\,dx\right) =$$

$$= \left(x^2 - \frac{2x^3}{3}\right)\Big|_0^1 = \frac{1}{3}$$

№2 $y^2 = 2x$

$x^2 + y^2 = 8 \Rightarrow y^2 = 8 - x^2$



$V = V_1 + V_2$

$$V_1 = \pi\int_0^2 2x\,dx = \pi \cdot x^2\Big|_0^2 = 4\pi$$

$$V_2 = \pi\int_2^{2\sqrt{2}} (8 - x^2)\,dx = \left(8\pi x - \frac{\pi x^3}{3}\right)\Big|_2^{2\sqrt{2}} = 16\sqrt{2}\pi - \frac{16\sqrt{2}\pi}{3} -$$

$$-16\pi + \frac{8\pi}{3} = \frac{32\sqrt{2}\pi - 40\pi}{3}$$

$$V = 4\pi + \frac{32\sqrt{2}\pi - 40\pi}{3} = \boxed{\frac{32\sqrt{2}\pi - 28\pi}{3}}$$

№3 $\rho = a\varphi$

$$l_{\text{первого витка}} = \int_0^{2\pi} \sqrt{a^2 + a^2\varphi^2}\, d\varphi = a \underbrace{\int_0^{2\pi} \sqrt{1+\varphi^2}\, d\varphi}_{I} =$$

$$=$$

$$I = \int \sqrt{1+\varphi^2}\, d\varphi = \left[\begin{array}{ll} u = \sqrt{1+\varphi^2} & ;\ dv = d\varphi \\ du = \frac{\varphi\, d\varphi}{\sqrt{1+\varphi^2}} & ;\ v = \varphi \end{array}\right] =$$

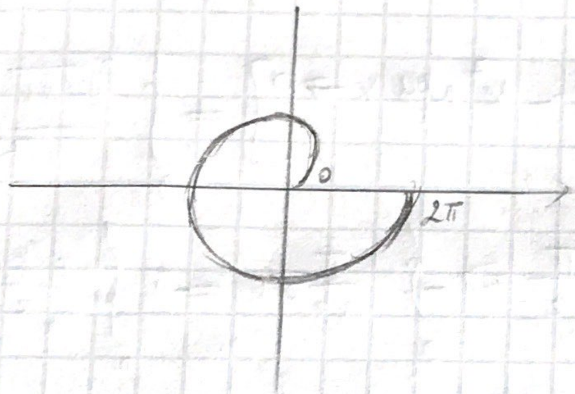
$$= \left(\varphi\sqrt{1+\varphi^2}\right)\Big|_0^{2\pi} - \int_0^{2\pi} \underbrace{\frac{\varphi^2\, d\varphi}{\sqrt{1+\varphi^2}}}_{= \frac{(\varphi^2+1)-1}{\sqrt{1+\varphi^2}}} = 2\pi\sqrt{1+4\pi^2} -$$

$$- \underbrace{\int_0^{2\pi} \sqrt{1+\varphi^2}\, d\varphi}_{= l} + \int_0^{2\pi} \frac{d\varphi}{\sqrt{1+\varphi^2}} = 2\pi\sqrt{1+4\pi^2} - l + \ln\left(\varphi + \sqrt{1+\varphi^2}\right)\Big|_0^{2\pi} =$$

$$= 2\pi\sqrt{1+4\pi^2} - l + \ln\left(2\pi + \sqrt{1+4\pi^2}\right)$$

$$l = \pi\sqrt{1+4\pi^2} + \frac{1}{2}\ln\left(2\pi + \sqrt{1+4\pi^2}\right)$$

(5) $\boxed{a\left(\pi\sqrt{1+4\pi^2}+\frac{1}{2}\ln\left(2\pi+\sqrt{1+4\pi^2}\right)\right)}$



$\boxed{№4}$

$I=\int\limits_1^{+\infty}\frac{\sqrt{\operatorname{arctg}x}}{4+x^2}dx$ $\frac{\sqrt{\operatorname{arctg}x}}{4+x^2}<\frac{\pi}{4+x^2}$ Особые точки $x=+\infty$

$$\int\limits_1^{+\infty}\frac{dx}{4+x^2}=\lim_{a\to+\infty}\int\limits_1^{a}\frac{dx}{4+x^2}=\lim_{a\to+\infty}\left(\frac{\operatorname{arctg}\left(\frac{x}{2}\right)}{2}\right)\Bigg|_1^a=$$

$$=\lim_{a\to+\infty}\left(\frac{\operatorname{arctg}\left(\frac{a}{2}\right)}{2}-\frac{\operatorname{arctg}\left(\frac{1}{2}\right)}{2}\right)=\frac{\pi}{4}-\frac{\operatorname{arctg}\left(\frac{1}{2}\right)}{2}$$

$\Rightarrow\int\limits_1^{+\infty}\frac{\pi}{4+x^2}$ - сходится $\Rightarrow$ I - сходится

№5

$$I=\int_0^1 \frac{\sqrt[3]{tg\,x}}{\ln(1+x^2)}\,dx =$$

Особые точки $x=0$

$$= \left[\ln(1+x^2) \sim x^2 \text{ при } x \to 0 \right] = \int_0^1 \frac{\sqrt[3]{tg\,x}}{x^2}\,dx \Rightarrow$$

$$\Rightarrow \frac{\sqrt[3]{tg\,x}}{x^2} > \frac{-\pi}{x^2} \Rightarrow -\int_0^1 \frac{\pi}{x^2}\,dx = -\pi \int_0^1 \frac{dx}{x^2} = -\pi \cdot \left(\frac{-1}{x}\right)\Bigg|_0^1 =$$

$$= -\pi \cdot (-1 + \infty) = -\infty$$ — расходится $\Rightarrow I$ — расходится

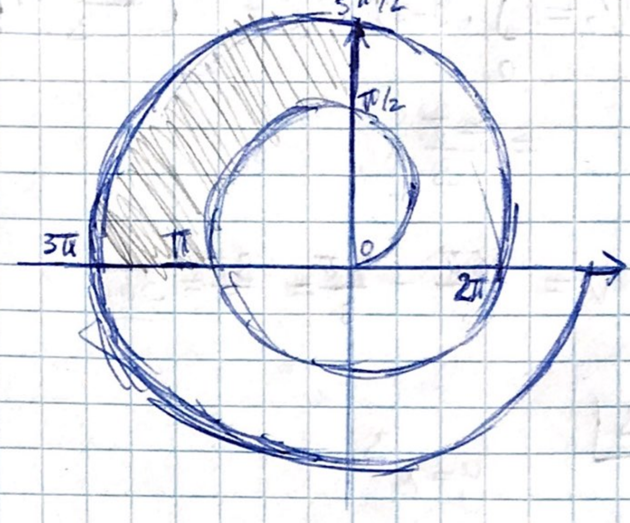
Вар. №4.

№1

$\rho = \varphi$

$\varphi = \pi/2$

$\varphi = \pi$



$S = S_1 - S_2$

$S_1 = \frac{1}{2}\int\limits_{5\pi/2}^{3\pi} \varphi^2 d\varphi = \frac{1}{2}\cdot\frac{\varphi^3}{3}\Big|_{\frac{5\pi}{2}}^{3\pi} = \frac{1}{2}\cdot\frac{27\pi^3}{3} - \frac{1}{2}\cdot\frac{125\pi^3}{3\cdot 8} =$

$= \frac{27\pi^3}{6} - \frac{125\pi^3}{48} = \frac{(216-125)\pi^3}{48} = \frac{91\pi^3}{48}$

$S_2 = \frac{1}{2}\int\limits_{\pi/2}^{\pi} \varphi^2 d\varphi = \frac{1}{2}\frac{\varphi^3}{3}\Big|_{\pi/2}^{\pi} = \frac{1}{2}\cdot\frac{\pi^3}{3} - \frac{1}{2}\cdot\frac{\pi^3}{24} =$

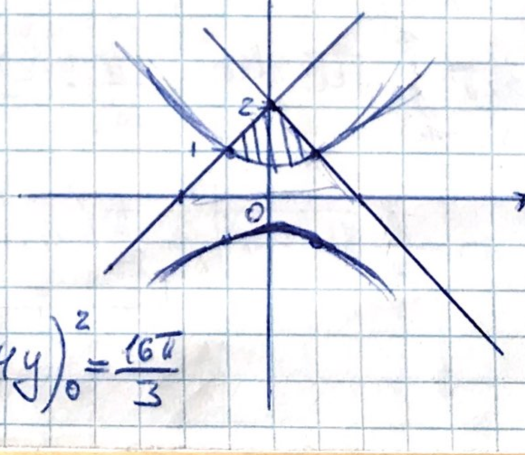
$= \frac{\pi^3}{6} - \frac{\pi^3}{48} = \frac{7\pi^3}{48}$

$S = \frac{91\pi^3}{48} - \frac{7\pi^3}{48} = \frac{84\pi^3}{48} = \boxed{\frac{21\pi^3}{12}}$

№2

$x^2 = 5y^2 - 4$

$x^2 = (y-2)^2$



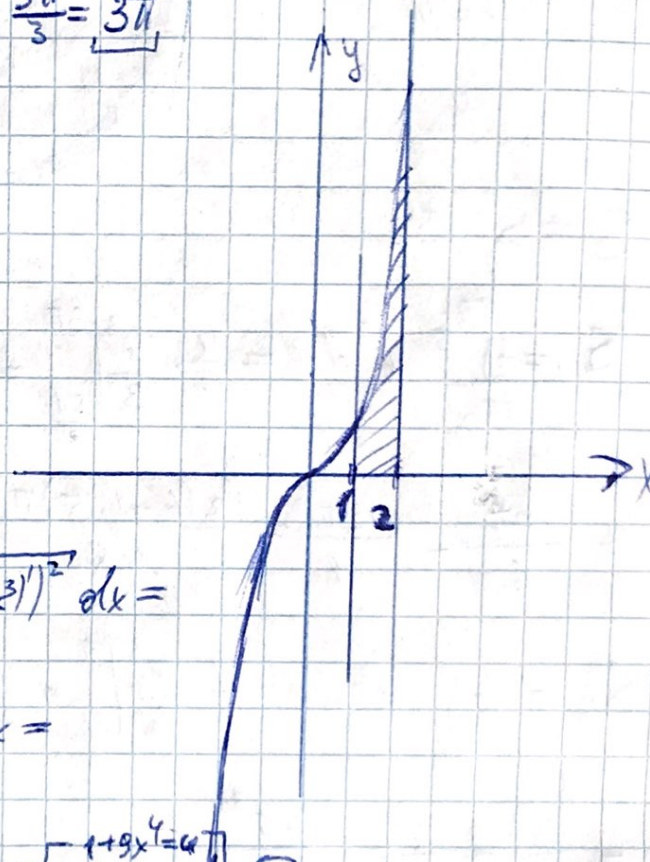
$V = V_1 - V_2$

$V_1 = \pi\int\limits_0^2 (5y^2 - 4)dy = \pi\left(\frac{5y^3}{3} - 4y\right)\Big|_0^2 = \frac{16\pi}{3}$

$$V_2=\pi\int_0^1 (y-2)^2\,dy=\pi\left(\frac{(y-2)^3}{3}\right)\Bigg|_0^1=\frac{\pi}{3}(-1+8)=$$

$$=\frac{7\pi}{3}$$

$$V=\frac{16\pi}{3}-\frac{7\pi}{3}=\frac{9\pi}{3}=\boxed{3\pi}$$



№3

$y=x^3$

$x=1$

$x=2$

$$S_{ox}=2\pi\int_1^2 |x^3|\sqrt{1+((x^3)')^2}\,dx=$$

$$=2\pi\int_1^2 x^3\sqrt{1+(3x^2)^2}\,dx=$$

$$=2\pi\int_1^2 x^3\sqrt{1+9x^4}\,dx=\left[\begin{array}{l}1+9x^4=u\\ x^3dx=\frac{du}{36}\end{array}\right]=$$

$$\cancel{=\frac{2\pi}{4}\int_1^2\sqrt{1+9x^4}\,dx^4=\frac{\pi}{2}\int_1^2\sqrt{1+9x^4}\,dx^4}$$

$$=\frac{2\pi}{36}\int\sqrt{u}\,du=\frac{\pi}{18}\cdot\frac{2u^{3/2}}{3}+C=\frac{\pi}{18}\cdot\frac{2}{3}\cdot\sqrt{(1+9x^4)^3}\Rightarrow$$

$$\Rightarrow\frac{\pi}{27}\sqrt{(1+9x^4)^3}\Big|_1^2=\boxed{\frac{\pi}{27}\left(145\sqrt{145}-10\sqrt{10}\right)}$$

№4 $I=\int\limits_1^{+\infty}\frac{\sin x}{x\sqrt{x}+1}dx$ Особые точки: $x=+\infty$

$$\frac{\sin x}{x\sqrt{x}+1}<\frac{1}{x\sqrt{x}+1}=f(x)$$

Найти $g(x)$: $f(x)\sim g(x)$. Пусть $g(x)=\left(\frac{1}{x}\right)^k$

$$\lim_{x\to+\infty}\frac{x^k}{x\sqrt{x}+1}=\lim_{x\to+\infty}\frac{x^k}{x^{3/2}\left(1+\frac{1}{x^{3/2}}\right)}=\left[k=3/2\right]=$$

$$=\lim_{x\to+\infty}\frac{1}{1+\frac{1}{x^{3/2}}}=1$$

$$\int\limits_1^{+\infty}\frac{dx}{x^{3/2}}=-\frac{2}{\sqrt{x}}\Big|_1^{+\infty}=2 \text{ - сходится} \Rightarrow I \text{ - сходится}$$

№5 $I=\int\limits_0^1\frac{2^x-1}{\sin^2 x}dx=\left[\begin{matrix}2^x-1\underset{x\to0}{\sim}x\ln 2,\\ \sin^2 x\underset{x\to0}{\sim}x^2\end{matrix}\right]=$ Особые точки: $x=0$

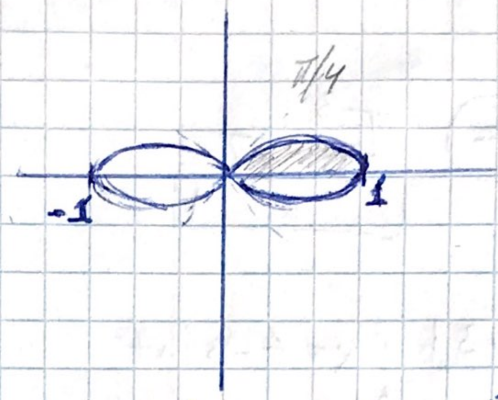
$$=\int\limits_0^1\frac{x\ln 2\,dx}{x^2}=\int\limits_0^1\frac{\ln 2\,dx}{x}=\ln 2\int\limits_0^1\frac{dx}{x}=\ln 2\cdot\ln|x|\Big|_0^1=$$

$=\ln 2(+\infty)=+\infty$ - расходится $\Rightarrow I$ - расходится

Вар. 5

№3

№1 $r^2 = \cos 2\varphi$



~~$r = \sqrt{\cos 2\varphi}$~~

$$S = 4\int_0^{\pi/4} \frac{\cos 2\varphi}{2} d\varphi = 2\int_0^{\pi/4} \cos 2\varphi \, d\varphi = 2\,\frac{\sin 2\varphi}{2}\Big|_0^{\pi/4} =$$

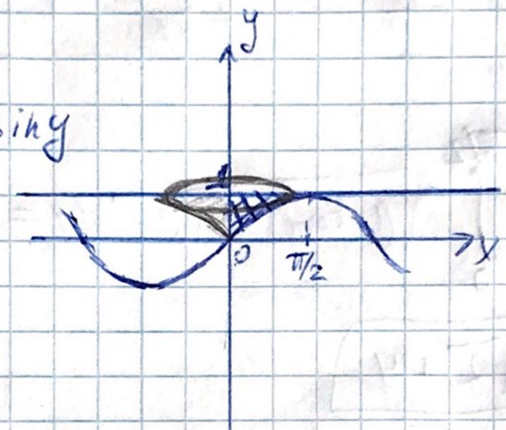
$$= \sin 2\varphi \Big|_0^{\pi/4} = 1$$

№2

$y = \sin x \Rightarrow x = \arcsin y$

$y = 1$

$x = 0$



~~$\sin x = 1$~~

~~$x = \frac{\pi}{2} + 2\pi n, n \in \mathbb{Z}$~~

$$V_{oy} = \pi \int_0^1 \arcsin^2 y \, dy = \left[\begin{matrix} u = \arcsin^2 y & dv = dy \\ du = \frac{2\arcsin y}{\sqrt{1-y^2}} dy & v = y \end{matrix}\right] =$$

$$= \pi\Big(y\arcsin^2 y - \underbrace{\int_0^1 \frac{2y\arcsin y}{\sqrt{1-y^2}} dy}_{I}\Big) = \Big(\pi y \arcsin^2 y + 2\pi\sqrt{1-y^2}\arcsin y - 2\pi y\Big)\Big|_0^1 =$$

$$I = \int_0^1 \frac{2y\arcsin y}{\sqrt{1-y^2}} dy = \left[\begin{matrix} u = \arcsin y & dv = \frac{y\,dy}{\sqrt{1-y^2}} \\ du = \frac{dy}{\sqrt{1-y^2}} & v = -\sqrt{1-y^2} \end{matrix}\right] =$$

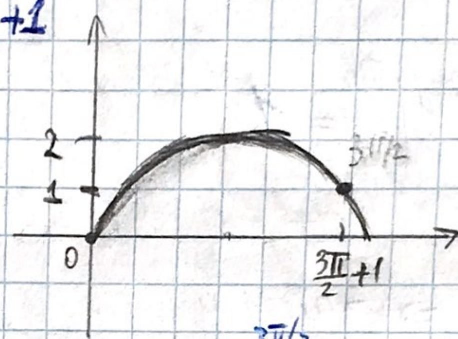
$$= 2\left(-\sqrt{1-y^2}\arcsin y + \int dy\right) = 2y - 2\sqrt{1-y^2}\arcsin y$$

$= \frac{\pi^3}{4} - 2\pi$

N3 $\begin{cases} x = t - \sin t \\ y = 1 - \cos t \end{cases}$ $\quad A(0;0) \quad B\left(\frac{3\pi}{2}+1; 1\right) \quad x' = 1 - \cos t \quad y' = \sin t$

$t - \sin t = \frac{3\pi}{2} + 1$

t	0	$\pi/2$	π	$3\pi/2$
x	0	$\pi/2 - 1$	π	$3\pi/2 + 1$
y	0	1	2	1



$$l = \int_0^{3\pi/2} \sqrt{(1-\cos t)^2 + \sin^2 t}\, dt = \underbrace{\int_0^{3\pi/2} \sqrt{2 - 2\cos t}\, dt}_{I} =$$

$$= \boxed{2\sqrt{2} + 4}$$

$$I = \int_0^{3\pi/2} \sqrt{2 - 2\cos t}\, dt = \cancel{\left[\begin{matrix} u = tg\frac{x}{2} \\ du = \frac{2}{2\cos\left(\frac{x}{2}\right)} dx \end{matrix}\right]} \int_0^{3\pi/2} \sqrt{2(1-\cos t)}\, dt =$$

$$= \int_0^{3\pi/2} \sqrt{4 \cdot \sin^2\left(\frac{t}{2}\right)}\, dt = \left[\begin{matrix} u = t/2 \\ dt = 2du \end{matrix}\right] =$$

$$= 2\int_0^{3\pi/2} \left|\sin\frac{t}{2}\right| dt = \qquad = 2\int_0^{3\pi/2} 2|\sin u|\, du =$$

$$= 4\int \sin u\, du = 4(-\cos u)\Big|_0^{\frac{3\pi}{2}} = -4\cos\frac{t}{2}\Big|_0^{3\pi/2} = \boxed{2\sqrt{2} + 4}$$

№4

$$I=\int_1^{+\infty}\frac{\operatorname{arctg}x}{\sqrt{x^3+5}}\,dx$$ Особые точки: $x=+\infty$

Найти $g(x)$: $g(x)\sim f(x)$. Пусть $g(x)=\frac{1}{x^k}$

$$\frac{\operatorname{arctg}x}{\sqrt{x^3+5}}<\frac{\pi/2}{\sqrt{x^3+5}}\qquad \lim_{x\to+\infty}\frac{x^k}{\sqrt{x^3+5}}=$$

$$=\lim_{x\to+\infty}\frac{x^k}{x^{3/2}\sqrt{1+5/x^{3/2}}}=\left[k=3/2\right]=\lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{5}{x^{3/2}}}}=$$

$$=1\qquad g(x)=\frac{1}{x^{3/2}}$$

$$\int_1^{+\infty}\frac{dx}{x^{3/2}}=-\frac{2}{\sqrt{x}}\Big|_1^{+\infty}=2$$ — сходится $\Rightarrow$ I — сходится

№5

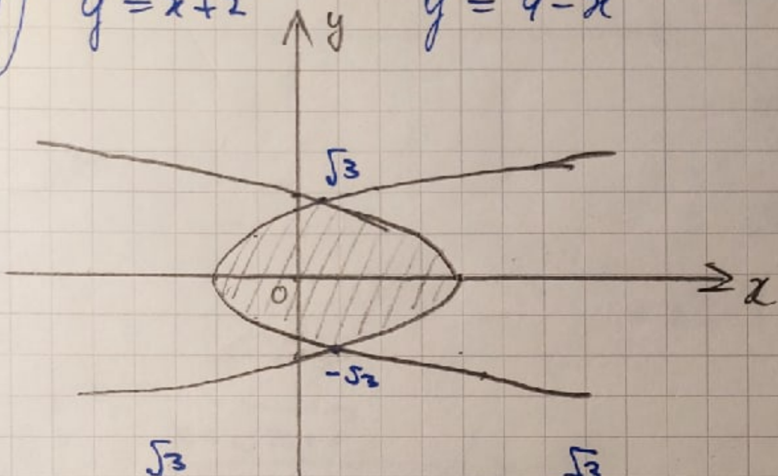
$$I=\int_0^{\pi/2}\frac{dx}{x\sqrt[3]{\sin x}}=\left[\sin x\sim x,\ x\to 0\right]=$$ Особые точки: $x=0$

$$=\int_0^{\pi/2}\frac{dx}{x\sqrt[3]{x}}=\int_0^{\pi/2}\frac{dx}{x^{4/3}}=-\frac{3}{\sqrt[3]{x}}\Big|_0^{\pi/2}$$ $\Rightarrow$ расходится $\Rightarrow$

$\Rightarrow$ I — расходится

Вариант. 6.

1) $y^2 = x + 2 \qquad y^2 = 4 - x$



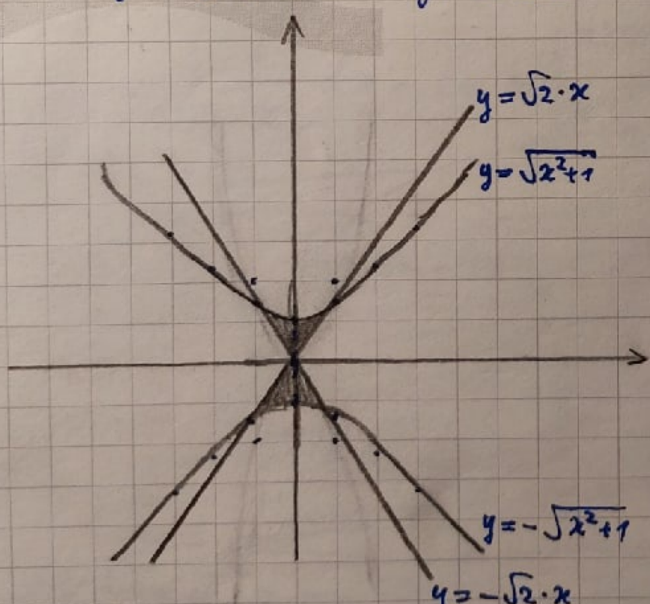
$$x = y^2 - 2$$
$$x = 4 - y^2$$
$$y^2 - 2 = 4 - y^2$$
$$2y^2 = 6$$
$$y = \pm\sqrt{3}$$

$$S = \int_{-\sqrt{3}}^{\sqrt{3}} (4 - y^2 - y^2 + 2)\,dy = \int_{-\sqrt{3}}^{\sqrt{3}} (6 - 2y^2)\,dy = 6y\Big|_{-\sqrt{3}}^{\sqrt{3}} - \frac{2}{3}y^3\Big|_{-\sqrt{3}}^{\sqrt{3}} =$$

$$= 6\sqrt{3} + 6\sqrt{3} - \frac{2}{3}(3\sqrt{3} + 3\sqrt{3}) = 12\sqrt{3} - 4\sqrt{3} = \boxed{8\sqrt{3}}$$

2) Вращение вокруг Ox.

$y^2 = x^2 + 1 \qquad y^2 = 2x^2$



$$x^2 + 1 = 2x^2 \Rightarrow x = 1,\ x = -1$$

$$V = \pi \cdot \int_{-1}^{1} \left((\sqrt{x^2+1})^2 - (\sqrt{2}\cdot x)^2\right) dx =$$

$$= \pi \cdot \int_{-1}^{1} (x^2 + 1 - 2x^2)\,dx = \pi \cdot \int_{-1}^{1} (1 - x^2)\,dx =$$

$$= \pi\left(x - \frac{x^3}{3}\right)\Big|_{-1}^{1} = \pi\left(1 - \frac{1}{3} + 1 - \frac{1}{3}\right) =$$

$$= \boxed{\frac{4}{3}\pi}$$

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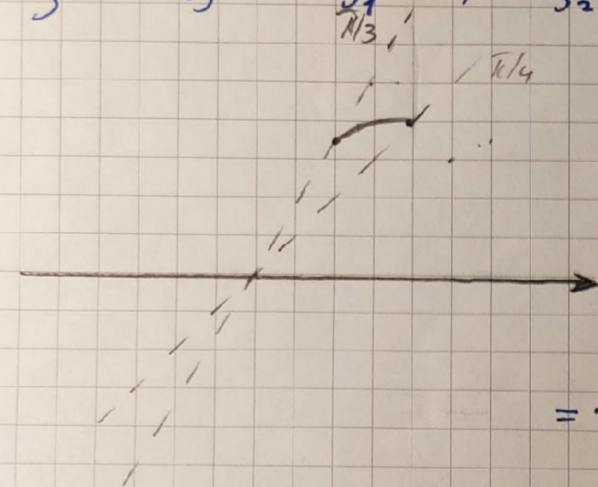
3) $y = \cos\varphi \qquad \varphi_1 = \pi/4 \qquad \varphi_2 = \pi/3$



$$S = 2\pi \cdot \int_{\pi/4}^{\pi/3} \cos\varphi \cdot dl =$$

$$= 2\pi \cdot \int_{\pi/4}^{\pi/3} \cos\varphi \sqrt{\cos^2\varphi + \sin^2\varphi}\, d\varphi =$$

$$= 2\pi \cdot \int_{\pi/4}^{\pi/3} \cos\varphi \cdot d\varphi = 2\pi \cdot \sin\varphi \Big|_{\pi/4}^{\pi/3} =$$

$$= 2\pi \cdot \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \right) = \boxed{(\sqrt{3} - \sqrt{2})\pi}$$

4) $$\int_1^{+\infty} \frac{x + \sqrt{x+1}}{x^2 + 2 \cdot \sqrt[6]{x^4+1}}\, dx \; = $$

$$\frac{1}{x^k} - ?$$

$$\lim_{x \to +\infty} \frac{x^k (x + \sqrt{x+1})}{x^2 + 2 \cdot \sqrt[6]{x^4+1}} = \lim_{x \to +\infty} \frac{x^k \left(\frac{1}{x} + \sqrt{\frac{1}{x^3} + \frac{1}{x^4}} \right)}{1 + 2 \cdot \sqrt[6]{\frac{1}{x^8} + \frac{1}{x^{12}}}} = \cancel{\lim_{x \to +\infty} \frac{x^{k-1}}{1+2}} =$$

$= 1$ при $k = 1$

$$= \sim \int_1^{+\infty} \frac{1}{x}\, dx = \ln x \Big|_1^{+\infty} = \ln(+\infty) - \ln 1 = +\infty$$ — расходится

5) $I = \int\limits_0^1 \frac{\sqrt{x}}{\ln(1+x)}dx = \left[\begin{matrix} \ln(1+x) \sim x \\ \text{при } x \to 0 \end{matrix} \right] \sim \int\limits_0^1 \frac{\sqrt{x}}{x}dx = \int\limits_0^1 \frac{dx}{\sqrt{x}} = 2\sqrt{x}\Big|_0^1 = 2 - 0 =$

Особые точки: $x = 0$

$= 2 \Rightarrow$ сходится $\Rightarrow$

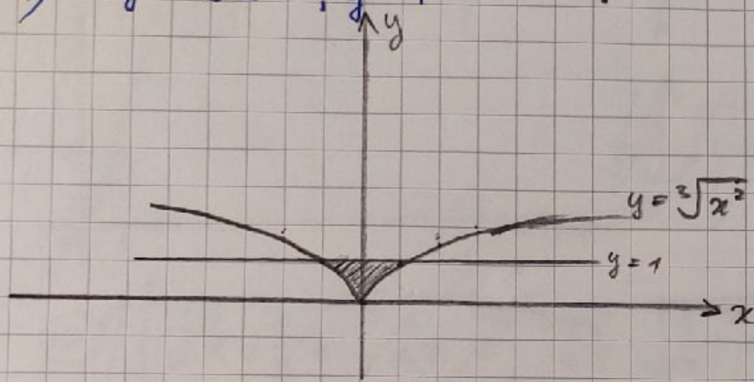
$\Rightarrow I$ - сходится

Вариант 7.

1) $y=\sqrt[3]{x^2}$, $y=1$ $\quad S - ?$

$\sqrt[3]{x^2}=1$

$x=\pm 1$



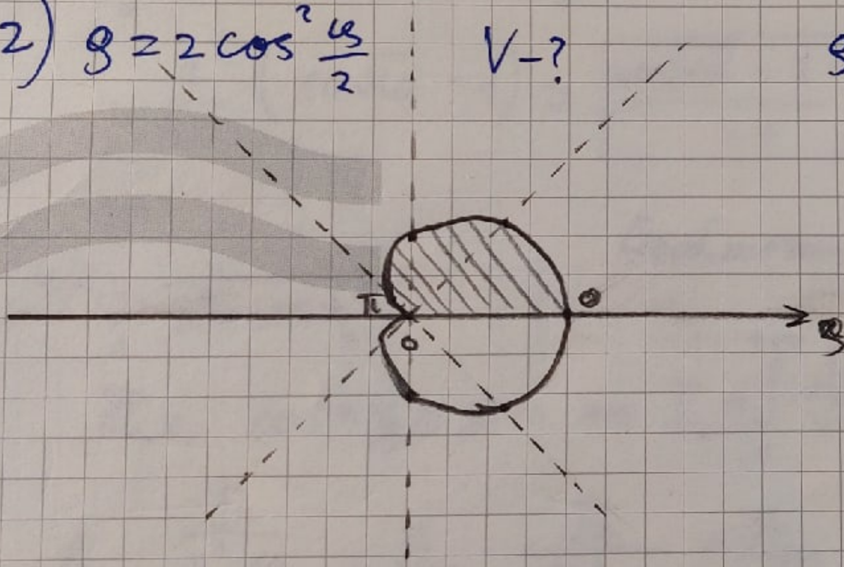
$$S=\int_{-1}^{1}\left(1-\sqrt[3]{x^2}\right)dx=\int_{-1}^{1}dx-\int_{-1}^{1}x^{\frac{2}{3}}dx=x\Big|_{-1}^{1}-\frac{3}{5}\cdot x^{\frac{5}{3}}\Big|_{-1}^{1}=$$

$$=(1+1)-\left(\frac{3}{5}+\frac{3}{5}\right)=2-\frac{6}{5}=\frac{4}{5}$$

2) $\rho=2\cos^2\frac{\varphi}{2}$ $\quad V - ?$

$$\rho=2\cdot\cos^2\frac{\varphi}{2}=2\cdot\frac{1+\cos\varphi}{2}=$$

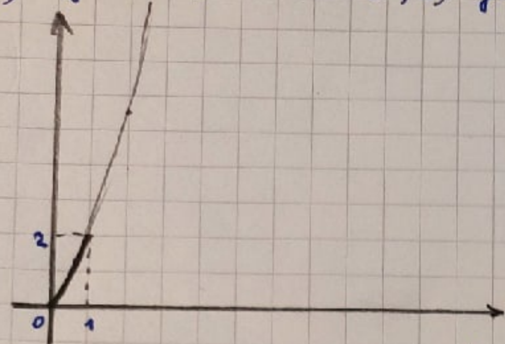
$$=1+\cos\varphi$$



$$V=2\pi\int_0^{\pi}\frac{\rho^3(\varphi)}{3}\cdot\sin\varphi\,d\varphi=-\frac{2\pi}{3}\cdot\int_0^{\pi}(1+\cos\varphi)^3d(\cos\varphi+1)=-\frac{2\pi}{3}\cdot\frac{(1+\cos\varphi)^4}{4}\Big|_0^{\pi}$$

$$=-\frac{2\pi}{3}(0-4)=\boxed{\frac{8\pi}{3}}$$

3) $y = 2x\sqrt{x}$ от $(0,0)$ до $(1,2)$



$$l = \int_0^1 dl = \int_0^1 \sqrt{1+(y'(x))^2}\,dx =$$

$$= \int_0^1 \sqrt{1+9x}\,dx = \frac{1}{9}\int_0^1 \sqrt{1+9x}\,d(9x+1) = \frac{1}{9}\cdot\frac{2}{3}(9x+1)\sqrt{9x+1}\Big|_0^1 =$$

$$= \frac{2}{27}\left(10\sqrt{10}-1\right) = \boxed{\frac{20\sqrt{10}-2}{27}}$$

4) $I_1 = \int_1^{+\infty} \frac{\sqrt{x}}{x^3+\cos x}\,dx$ Особ. точки: $x = +\infty$

Т.к. $\cos x \geq 0 > -10$, то $I_1 = \int_1^{+\infty} \frac{\sqrt{x}}{x^3+\cos x}\,dx < \int_1^{+\infty} \frac{\sqrt{x}}{x^3-10}\,dx = I_2$

$$\lim_{x\to+\infty} \frac{\frac{\sqrt{x}}{x^3-10}}{x^k} = \lim_{x\to+\infty} \frac{\sqrt{x}}{x^k(x^3-10)} = \lim_{x\to+\infty} \frac{x^{-2,5}}{x^k(1-10\cdot x^{-2,5})} = 1 \text{ при } k=-2,5$$

Тогда $I_2 \sim \int_1^{+\infty} \frac{dx}{x^{3,5}} = -\frac{2}{3}\cdot x^{-\frac{5}{2}}\Big|_1^{+\infty} = -\frac{2}{3x^2\sqrt{x}}\Big|_1^{+\infty} = -\left(0-\frac{2}{3}\right) = \frac{2}{3}$

– сходится => I_1 – сходится

Особые точки: $x = 0$

$$5)\ I = \int_0^1 \frac{\sqrt{x}}{\sin^2 x}\,dx \sim \int_0^1 \frac{1}{x\sqrt{x}}\,dx = \int_0^1 x^{-\frac{3}{2}}\,dx = -2\cdot x^{-\frac{1}{2}}dx\Big|_0^1 = \frac{-2}{\sqrt{x}}\Big|_0^1 = \left(\frac{-2}{1} - \frac{-2}{0}\right) =$$

$= \infty$ - расходится

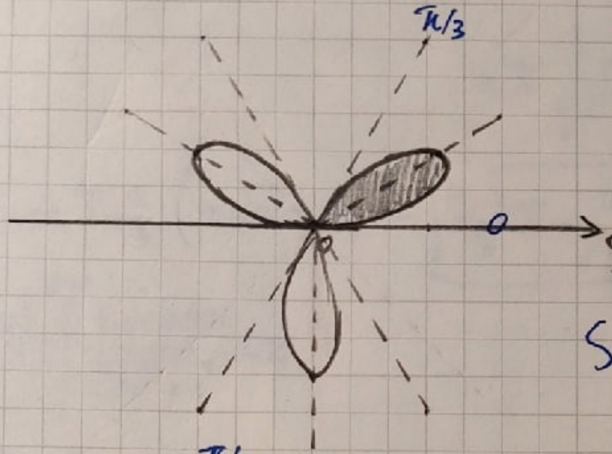
$$\lim_{x\to 0}\frac{\sqrt{x}}{\sin^2 x} = \begin{bmatrix}\sin x \sim x \\ \text{при } x \to 0\end{bmatrix} = \lim_{x\to 0}\frac{\sqrt{x}}{x^2} = \lim_{x\to 0}\frac{1}{x\sqrt{x}}$$

$\Rightarrow$

$\Rightarrow$ исходный интеграл - расходится

Вариант 8.

1) $\rho = \sin 3\varphi$ $\quad S$ одного лепестка – ?



$\sin(3\varphi) \geq 0$

$2\pi n \leq 3\varphi \leq \pi + 2\pi n$

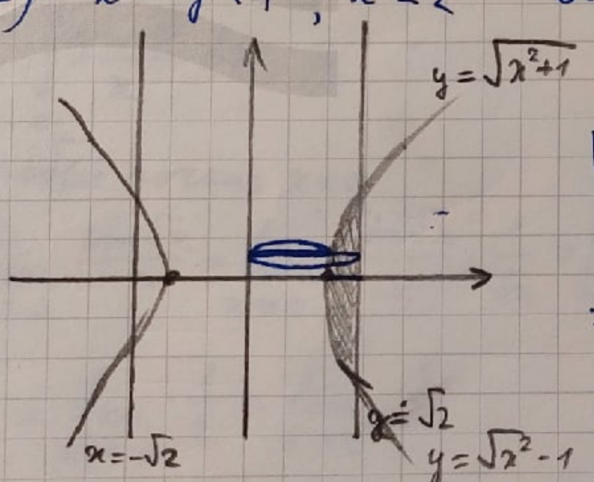
$\frac{2\pi}{3} n \leq \varphi \leq \frac{\pi}{3} + \frac{2\pi}{3} n$

$$S = \int_0^{\pi/3} \frac{\rho^2(\varphi)}{2} d\varphi = \frac{1}{3} \int_0^{\pi/3} \frac{\sin^2(3\varphi)}{2} d(3\varphi) =$$

$$= \frac{1}{6} \int_0^{\pi/3} \sin^2(3\varphi) d(3\varphi) = \frac{1}{12} \int_0^{\pi/3} \frac{1 - \cos 6\varphi}{2} d(6\varphi) =$$

$$= \frac{1}{12} \left(3\varphi - \frac{\sin 6\varphi}{2} \right) \Bigg|_0^{\frac{\pi}{3}} = \frac{1}{12} (\pi - 0) = \boxed{\frac{\pi}{12}}$$

2) $x^2 = y^2 + 1$, $x^2 = 2$ вокруг Oy



$y^2 + 1 = 2$

$y^2 = 1 \implies y = \pm 1$

$$V = 2\pi \int_0^1 \left((\sqrt{2})^2 - (\sqrt{y^2+1})^2 \right) dy =$$

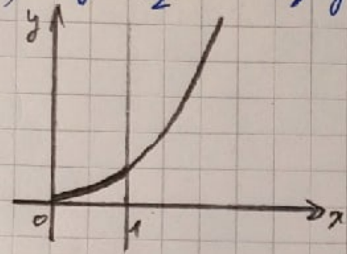
$$= 2\pi \cdot \int_0^1 (2 - y^2 - 1) dy = 2\pi \left(y - \frac{y^3}{3} \right) \Bigg|_0^1 =$$

$$= 2\pi \left(1 - \frac{1}{3} \right) = \boxed{\frac{4\pi}{3}}$$

$x^2 = y^2 + 1$

$x = \pm\sqrt{y^2+1}$

3) $y = \frac{x^2}{2}$ вокруг Oy. $x=0,\ x=1$



$$S = 2\pi \cdot \int_0^1 |x| \cdot \sqrt{1+\left(\left(\frac{x^2}{2}\right)'\right)^2}\,dx = 2\pi \cdot \int_0^1 x \cdot \sqrt{1+x^2}\,dx =$$

$$= \pi \int_0^1 \sqrt{1+x^2}\,d(x^2+1) = \pi \cdot \frac{2}{3}(x^2+1)\sqrt{x^2+1}\Big|_0^1 =$$

$$= \pi \cdot \frac{2}{3}\left(2\sqrt{2}-1\right) = \boxed{\frac{4\sqrt{2}}{3}\pi - \frac{2}{3}\pi}$$

4) $\int_1^{+\infty} \frac{4+\cos x}{\sqrt{x^3+1}}\,dx$

Особые точки: $x = +\infty$

т.к. $\cos x \leq 1 < 10$, то $\int_1^{+\infty} \frac{4+\cos x}{\sqrt{x^3+1}}\,dx < \int_1^{+\infty} \frac{14}{\sqrt{x^3+1}}\,dx = 14 \cdot \int_1^{+\infty} \frac{dx}{\sqrt{x^3+1}} \sim$

$$\sim \int_1^{+\infty} \frac{dx}{x^{\frac{3}{2}}} = -2 \cdot \frac{1}{\sqrt{x}}\Big|_1^{+\infty} = (-2\cdot 0 + 2\cdot 1) = 2 \text{ – сходится}$$

=> исходный интеграл – сходится

5) $\int_0^1 \frac{\sin\sqrt{x}}{x}\,dx$

Особые точки: $x = 0$

$$\lim_{x\to 0} \frac{\sin\sqrt{x}}{x} = \lim_{x\to 0} \frac{\sin\sqrt{x}}{\sqrt{x}} \cdot \frac{1}{\sqrt{x}} = \begin{bmatrix} t = \sqrt{x} \\ t \to 0 \text{ при } x \to 0 \end{bmatrix} = \lim_{t\to 0} \frac{\sin t}{t} \cdot \frac{1}{t} =$$

$$= \lim_{t\to 0} \frac{1}{t} = \lim_{x\to 0} \frac{1}{\sqrt{x}}$$

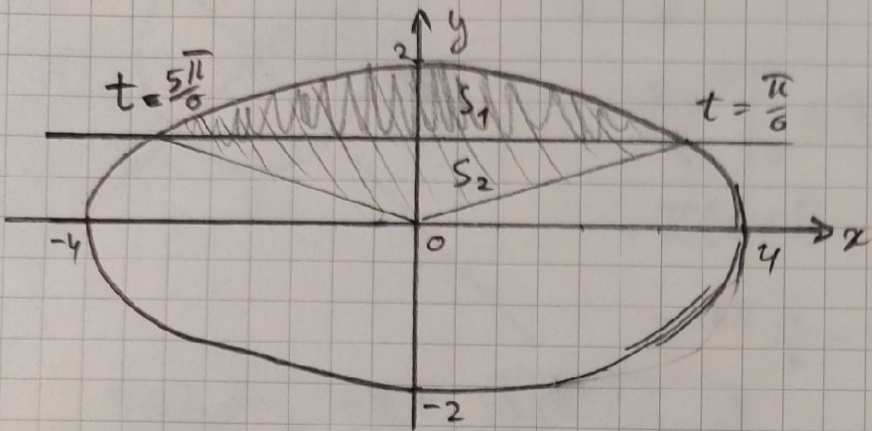
$$\lim_{x\to 0} \frac{\frac{1}{\sqrt{x}}}{\frac{1}{x^k}} = 1 \text{ при } x = \frac{1}{2} \Rightarrow \int_0^1 \frac{\sin\sqrt{x}}{x}\,dx \sim \int_0^1 \frac{dx}{\sqrt{x}} = 2\sqrt{x}\Big|_0^1 = 2 \text{ – сходится}$$

=> исходный интеграл – сходится

Вариант 9.

1) $x = 4\cdot\cos t$ $\qquad y = 1;$ содержит $(0;2)$

$y = 2\cdot\sin t$



$S = S_1 + S_2$

$S_1 - ?$

$$S = \int\limits_{5\pi/6}^{\pi/6} y(t)\cdot x'(t)\,dt = \int\limits_{5\pi/6}^{\pi/6} 2\cdot\sin t\cdot(-4\sin t)\,dt = 8\int\limits_{\pi/6}^{5\pi/6} \sin^2 dt = 8\cdot\int\limits_{\pi/6}^{5\pi/6} \frac{1-\cos 2t}{2}\,dt =$$

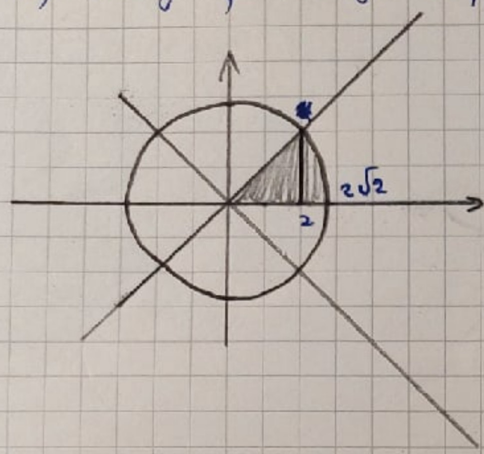
$$= 2\cdot\int\limits_{\pi/6}^{5\pi/6}(1-\cos 2t)\,d(2t) = 2\cdot(2t - \sin 2t)\Big|_{\pi/6}^{5\pi/6} = 2\cdot\left(\frac{10\pi}{6} + \frac{\sqrt{3}}{2} - \frac{2\pi}{6} + \frac{\sqrt{3}}{2}\right) =$$

$$= 2\sqrt{3} + \frac{8\pi}{3}$$

$$S_2 = 4\cos\frac{\pi}{6}\cdot 2\sin\frac{\pi}{6} = 8\cdot\frac{\sqrt{3}}{4} = 2\sqrt{3}$$

$$S_1 = S - S_2 = 2\sqrt{3} + \frac{8\pi}{3} - 2\sqrt{3} = \boxed{\frac{8\pi}{3}}$$

2) $x^2=y^2$, $x^2+y^2=8$, содержит $(1,0)$; вокруг Ox.



$$V=\int_0^2 \pi\cdot x^2\,dx+\int_2^{2\sqrt2}\pi\cdot(8-x^2)\,dx=$$

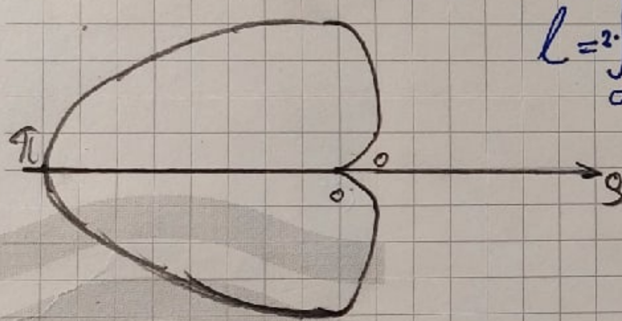
$$=\frac{\pi x^3}{3}\Big|_0^2+\pi\left(8x-\frac{x^3}{3}\right)\Big|_2^{2\sqrt2}=\frac{\pi\cdot 8}{3}+$$

$$+\pi\left(8\cdot2\sqrt2-\frac{16\sqrt2}{3}-16+\frac{8}{3}\right)=\frac{8\pi}{3}+16\pi\sqrt2-$$

$$-\frac{16\pi\sqrt2}{3}-16\pi+\frac{8}{3}\pi=\frac{16\pi}{3}+\frac{32\pi\sqrt2}{3}-16\pi=$$

$$=\frac{32\pi\sqrt2}{3}-\frac{32\pi}{3}=\boxed{\frac{32\pi}{3}(\sqrt2-1)}$$

3) $\rho=4(1-\cos\varphi)$



$$l=2\cdot\int_0^{\pi}\sqrt{\rho^2+\rho'^2}\,d\varphi=2\cdot\int_0^{\pi}\sqrt{16(\cos^2\varphi-2\cos\varphi+1)+16\sin^2\varphi}\,d\varphi$$

$$=8\cdot\int_0^{\pi}\sqrt{2-2\cos\varphi}\,d\varphi=32\cdot\int_0^{\pi}\cos\frac{\varphi}{2}\,d\frac{\varphi}{2}=$$

$$=32\cdot\sin\frac{\varphi}{2}\Big|_0^{\pi}=32\cdot\left(\sin\frac{\pi}{2}-\sin 0\right)=$$

$$=\boxed{32}$$

4) $I=\int_1^{+\infty}\frac{\operatorname{arctg}x^3}{x^2+2x}\,dx<\int_1^{+\infty}\frac{100}{x^2+2x}\,dx=\int_1^{+\infty}\frac{100}{(x+1)^2-1}\,d(x+1)=50\cdot\ln\left|\frac{x}{x+2}\right|\Big|_1^{+\infty}=$

$=50\cdot\left(\overset{=0}{\ln\frac{+\infty}{+\infty+2}}-\ln\frac{1}{3}\right)=-50\cdot\ln\frac13$ − сходится => I − сходится

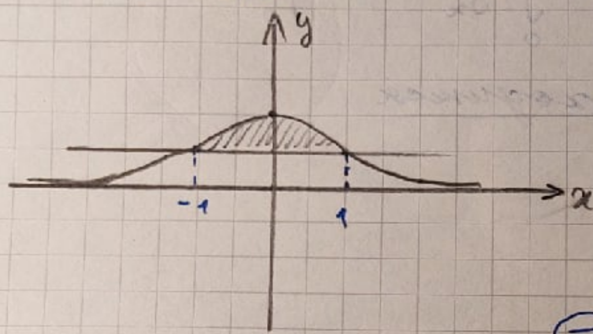
5) $I=\int_0^1\frac{x\sqrt x}{\ln(1+x^2)}\,dx\Rightarrow\int_0^1\frac{x\sqrt x}{\ln(1+x^2)}\,dx\sim\left[\begin{matrix}\ln(1+x^2)\sim x^2\\ \text{при } x^2\to0\end{matrix}\right]\sim\int_0^1\frac{x\sqrt x}{x^2}\,dx=\int_0^1\frac{dx}{\sqrt x}=$

$=2\sqrt x\Big|_0^1=2-0=2$ − сходится => I − сходится

Вариант 16.

1) $y=\frac{1}{1+x^2}$, $y=\frac{1}{2}$

$$\frac{1}{1+x^2}=\frac{1}{2} \Rightarrow x=\pm 1$$

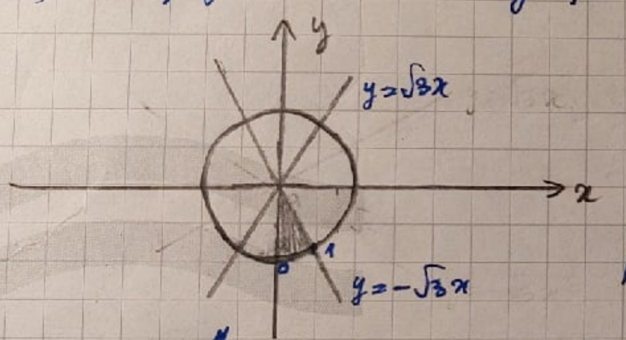


$$S=\int_{-1}^{1}\left(\frac{1}{1+x^2}-\frac{1}{2}\right)dx =$$

$$=\left(\operatorname{arctg} x-\frac{x}{2}\right)\Big|_{-1}^{1}=\frac{\pi}{4}-\frac{1}{2}+\frac{\pi}{4}-\frac{1}{2}=$$

$$=\boxed{\frac{\pi}{2}-1}$$

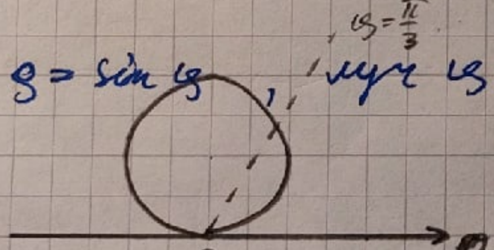
2) Вокруг Ox: $3x^2=y^2$, $x^2+y^2=4$, содержит $(0;-1)$



$$V=2\cdot\pi\cdot\int_{0}^{1}(4-x^2-3x^2)dx=2\pi\int_{0}^{1}(4-4x^2)dx=8\pi\cdot\left(x-\frac{x^3}{3}\right)\Big|_{0}^{1}=8\pi\left(1-\frac{1}{3}\right)=$$

$$=8\pi-\frac{8\pi}{3}=\frac{16\pi}{3}$$

3) $\rho=\sin\varphi$, луч $\varphi=\frac{\pi}{3}$, меньшая дуга окр-ти.



$$l=\int_{0}^{\pi/3}2\pi\cdot\sin\varphi\cdot\sqrt{\sin^2\varphi+\cos^2\varphi}\,d\varphi=$$

$$=-2\pi\cdot\cos\varphi\Big|_{0}^{\pi/3}=-2\pi\cdot\frac{1}{2}+2\pi=\boxed{\pi}$$

$$4)\ \int_1^{+\infty} \frac{2-\sin x}{x+1}\,dx \geqslant \int_1^{+\infty} \frac{1}{x+1}\,dx = \ln|x+1|\Big|_1^{+\infty} = +\infty$$ – расходится

(т.к. $\sin x \leqslant 1$)

$\Rightarrow$ исходный интеграл – <u>расходится</u>

$$5)\ \int_0^1 \frac{x\sqrt{x}}{\operatorname{tg}^2 x}\,dx \sim \left[\begin{matrix}\operatorname{tg} x \sim x \\ \text{при } x\to 0\end{matrix}\right] \sim \int_0^1 \frac{x\sqrt{x}}{x^2}\,dx = \int_0^1 \frac{dx}{\sqrt{x}} = 2\sqrt{x}\Big|_0^1 = 2-0=2$$ – сходится

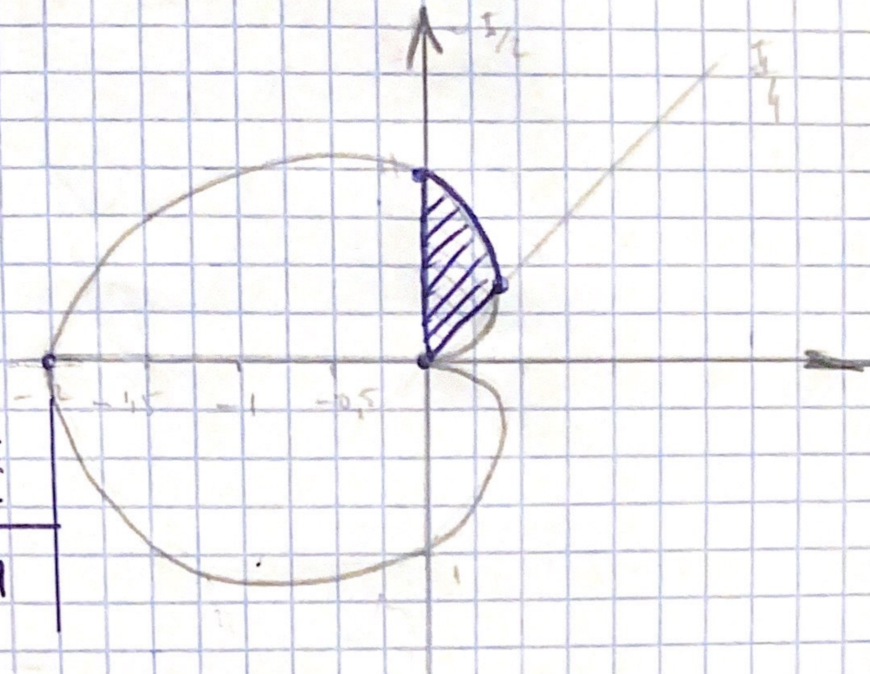
$\Rightarrow$ исходный интеграл – <u>сходится</u>

Билет 11

№1

$$\begin{cases} \rho = 2\sin^2\frac{\varphi}{2} \\ \varphi_1 = \frac{\pi}{4} = a \\ \varphi_2 = \frac{\pi}{2} = b \end{cases}$$

φ	0	π	$\frac{\pi}{2}$
ρ	0	-2	1



1) $\cancel{\rho(a) = 2\sin^2\frac{\varphi_1}{2} = 2\sin^2\left(\frac{\pi}{8}\right) = 2\cdot\frac{1-\cos\left(\frac{\pi}{4}\right)}{2} = 1-\frac{\sqrt{2}}{2}}$

2) $\cancel{\rho(b) = 2\sin^2\frac{\pi/2}{2} = 2\sin^2\left(\frac{\pi}{4}\right) = 2\cdot\left(\frac{\sqrt{2}}{2}\right)^2 = 1}$

$$S = \frac{1}{2}\int_{\pi/4}^{\pi/2}\rho^2(\varphi)\,d\varphi = \frac{4}{2}\int_{\pi/4}^{\pi/2}\sin^4\frac{\varphi}{2}\,d\varphi = 4\int_{\pi/4}^{\pi/2}\sin^4\frac{\varphi}{2}\,d\left(\frac{\varphi}{2}\right) =$$

$$= 4\int_{\pi/4}^{\pi/2}\left(\frac{1-\cos(\varphi)}{2}\right)^2\frac{1}{2}\,d\varphi = \int_{\pi/4}^{\pi/2}\left(1-2\cos(\varphi)+\cos^2(\varphi)\right)d\varphi =$$

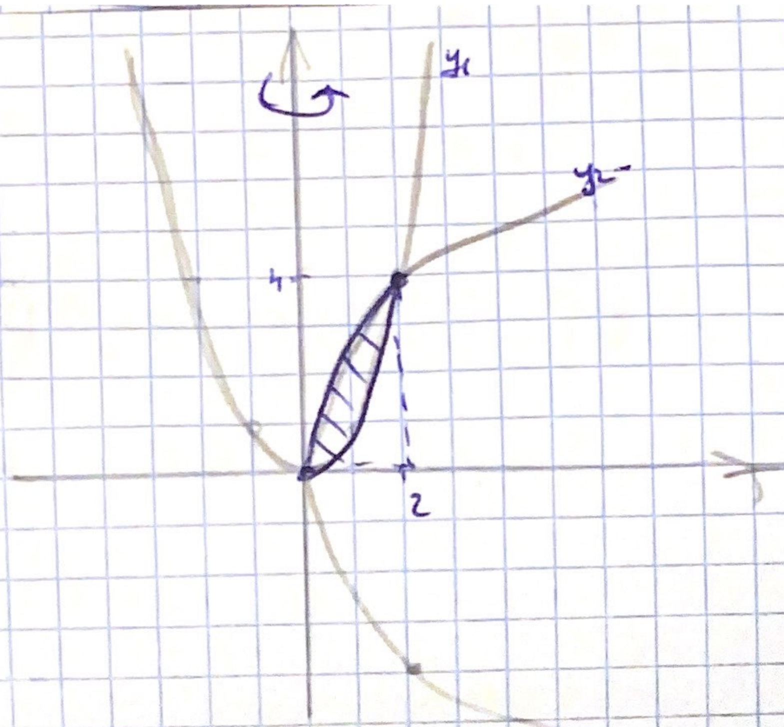
$$= \left(\frac{3}{2}\varphi - 2\sin\varphi + \frac{1}{4}\sin 2\varphi\right)\Big|_{\pi/4}^{\pi/2} = \frac{3\pi}{8} - 2 + \sqrt{2} - \frac{1}{4} =$$

$$= \frac{3\pi}{8} - \frac{9}{4} + \sqrt{2}$$

№2

↻ Oy

$$\begin{cases} y_1 = x^2 \\ y_2^2 = 8x \Rightarrow x = \frac{y^2}{8} \end{cases}$$



$x^4 - 8x = 0$

$x(x^3 - 8) = 0$

$x_1 = 0$; $x_2 = 2$

$V = V_1 - V_2$; 1) $V_2 = 2\pi \int_0^2 \left(\frac{y^2}{8}\right)^2 dy = \frac{\pi}{32} \int_0^2 y^4 dy$ (2)

(2) $\frac{\pi}{32} \cdot \frac{1}{5} y^5 \Big|_0^2 = \frac{\pi}{5}$

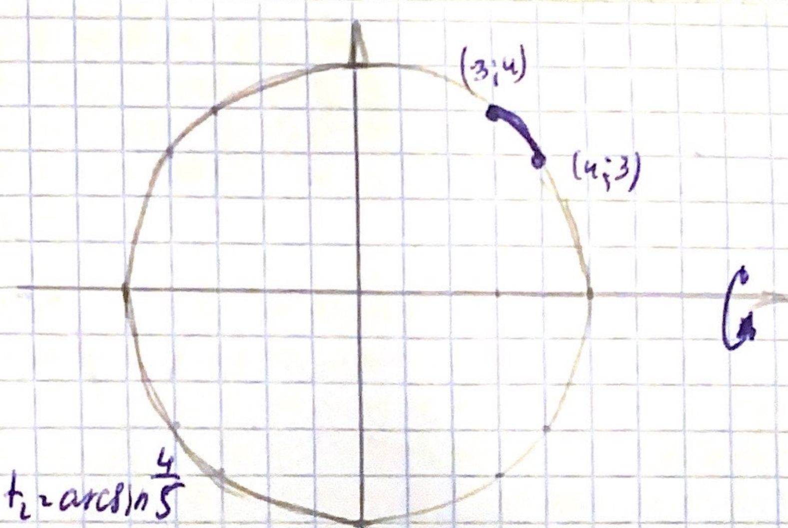
$V_1 = 2\pi \int_0^2 y\, dy = \pi \cdot y^2 \Big|_0^2 = 4\pi$

$V = 4\pi - \frac{\pi}{5} = \frac{19\pi}{5}$

№3.

$$\begin{cases} x = 5\sin t \\ y = 5\cos t \end{cases}$$

t	0	$\pi/2$	π	$3\pi/2$
x	0	5	0	-5
y	5	0	-5	0



$t_1 = \arcsin\frac{3}{5}$; $t_2 = \arcsin\frac{4}{5}$

$$S_{0x} = 2\pi\int_{t_1}^{t_2} y(t)\cdot\sqrt{(x'(t))^2 + (y'(t))^2}\,dt =$$

$$= 2\pi\int_{t_2}^{t_2} 5\cos t \cdot \sqrt{\underbrace{25\cos^2 t + 25\sin^2 t}_{25\underbrace{(\cos^2 t + \sin^2 t)}_{1}}}\,dt =$$

$$= 2\pi\cdot 25\int_{t_1}^{t_2}\cos t\,dt = 50\pi\cdot\sin t\Big|_{t_1}^{t_2} = 50\pi\left(\frac{4}{5} - \frac{3}{5}\right) =$$

$$= 10\pi$$

№4 — ?

$$I = \int_1^{+\infty}\frac{x+1}{x^3+\sin x}\,dx$$

~~[illegible]~~ сходится

$$I = \int_1^{+\infty}\frac{x}{x^3+\sin x}\,dx + \underbrace{\lim_{x\to+\infty}\frac{1}{x^3+\sin x}}_{I_2,\ \to 0}$$

$$\int_1^{+\infty}\frac{x}{x^3+\sin x}\,dx \geq \int_1^{+\infty}\frac{x}{x^3+1}\,dx$$ для $\forall\, x \in [1; +\infty)$

$$x\cdot 0\int_0^{+\infty}\frac{x}{x^3+1}\,dx = \frac{\sqrt{3}\pi + 3\ln(2)}{9} \approx 0{,}83$$

25

$$\int_2^3 \frac{x-2}{x^3-3x^2+4}\,dx \text{ — расходится}$$

↳ несобственный интеграл II-го рода

$$\lim_{a\to 2^+} \int_a^3 \frac{x-2}{x^3-3x^2+4}\,dx \overset{*}{=} \lim_{a\to 2^+}\left(\frac{\ln\left|\frac{x-2}{x+1}\right|}{3}\right) = +\infty$$

$$* \int_2^3 \frac{\cancel{x-2}}{(x-2)^{\cancel{2}}(x+1)}\,dx = \int_2^3 \frac{dx}{(x-2)(x+1)} = \int_2^3 \left(\frac{1}{3(x-2)} - \frac{1}{3(x+1)}\right)dx =$$

$$\frac{1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

$A = \frac{1}{3}$; $x = 2$

$B = -\frac{1}{3}$; $x = -1$

$$= \frac{1}{3}\int_2^3 \frac{dx}{(x-2)} - \frac{1}{3}\int_2^3 \frac{dx}{x+1} = \frac{\ln|x-2| - \ln|x+1|}{3}\Bigg|_2^3 =$$

$$= \frac{\ln\left|\frac{x-2}{x+1}\right|}{3}\Bigg|_2^3$$

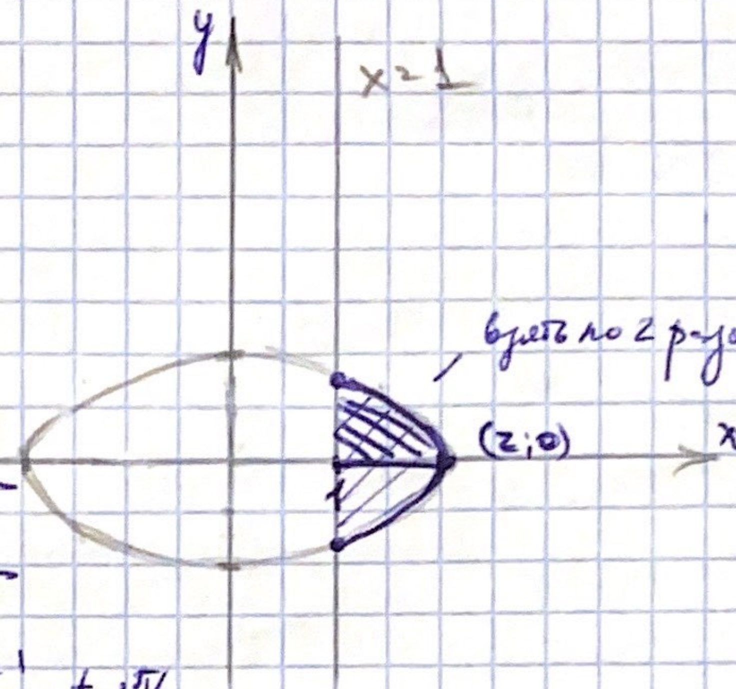
Билет 12

№1

$x = 2\cos t$

$y = \sin t$

t	0	$\pi/2$	π	$3\pi/2$
x	2	0	-2	0
y	0	1	0	-1



$\pi/3$ * =>

* $x=1$

$t = \arccos\frac{1}{2} = \pi/3$

$$S = 2\int_0^{\pi/3} y(t)\, x'(t)\,dt = -2\cdot 2\int_0^{\pi/3} \sin t \cdot \sin t\, dt =$$

$$= -4\int_0^{\pi/3} \underbrace{\sin^2 t}_{\frac{1-\cos 2t}{2}}\, dt = -2\int_0^{\pi/3} (1-\cos 2t)\,dt = -2t\Big|_0^{\pi/3} +$$

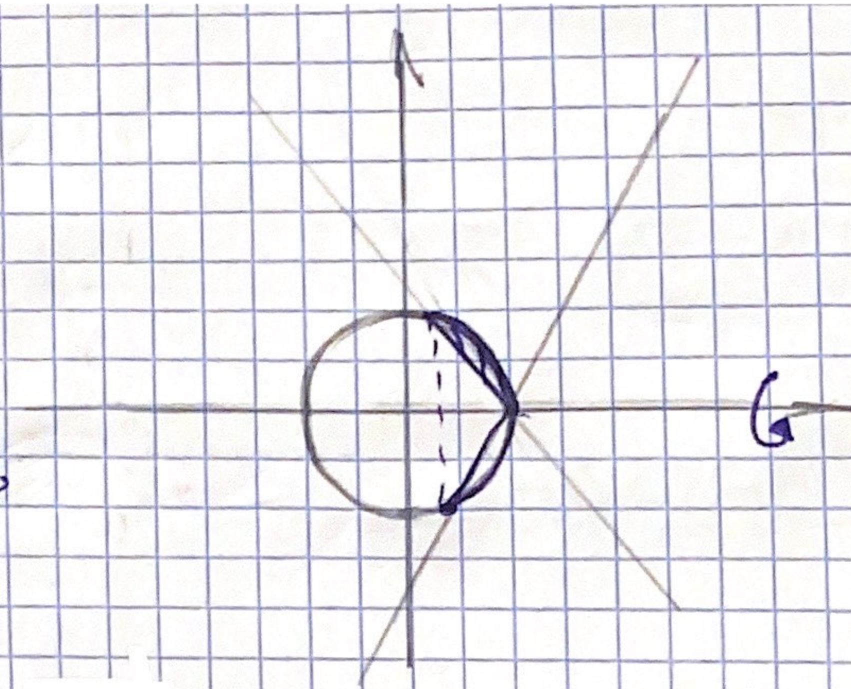
$$+ \int_0^{\pi/3} \cos 2t\, d(2t) = -2t\Big|_0^{\pi/3} + \sin 2t\Big|_0^{\pi/3} =$$

$= ?$ (<0)

52

$y^2 = 1 - x^2$

$y^2 = 2(x-1)^2$



① $2(x^2-2x+1) = 1-x^2$

$2x^2+x^2-4x+2-1=0$

$3x^2-4x-1=0$

$(3x-1)\cdot(x-1)=0$

$x_1 = 1$; $x_2 = \frac{1}{3}$;

$V_{0x} = V_1 - V_2$

$V_1 = \pi\int_{1/3}^{1}(1-x^2)\,dx = \pi\left(x\Big|_{1/3}^{1} - \frac{1}{3}x^3\Big|_{1/3}^{1}\right) =$

$= \frac{2\pi}{3}^{(27} - \frac{26\pi}{81} = \frac{51\pi-26\pi}{81} = \frac{25\pi}{81}$

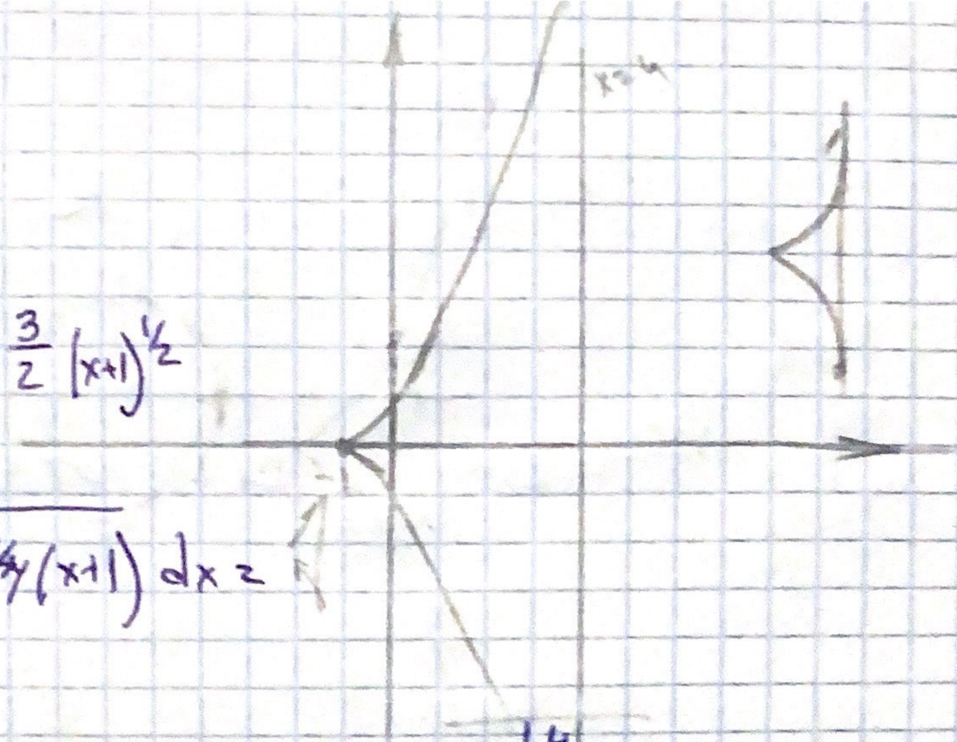
$V_2 = 2\pi\int_{1/3}^{1}(x-1)^2\,dx = 2\pi\int_{1/3}^{1}(x^2-2x+1)\,dx =$

$= 2\pi\left(\frac{1}{3}x^3\Big|_{1/3}^{1} - x^2\Big|_{1/3}^{1} + x\Big|_{1/3}^{1}\right) = 2\pi\cdot\frac{8}{81} = \frac{16\pi}{81}$

$V_{0x} = \frac{25\pi-16\pi}{81} = \frac{9\pi}{81} = \frac{\pi}{9}$

б3

$$\begin{cases} y^2 = (x+1)^3 \\ x = 4 \end{cases}$$



$y = \sqrt{(x+1)^3}$; $y' = \frac{3}{2}(x+1)^{1/2}$

$$l = 2\cdot\int_{-1}^{4} \sqrt{1 + \tfrac{9}{4}(x+1)}\,dx =$$

$$= \int_{-1}^{4} \sqrt{13+9x}\,dx = \frac{2}{27}(13+9x)^{3/2}\Big|_{-1}^{4} =$$

$$= \frac{2}{27}\left(7^3 - 2^3\right) = \frac{670}{27}$$

б4 — ? не сходится с ответом с калькулят.

$$\int_{1}^{+\infty} \frac{3\cos^2 2x}{\sqrt{x^3+1}}\,dx$$

Особые точки: $x = +\infty$

$0 \le \cos^2 2x \le 1$, тогда $\int_{1}^{+\infty} \frac{3\cos^2 2x}{\sqrt{x^3+1}}\,dx \le \int_{1}^{+\infty} \frac{3}{\sqrt{x^3+1}}\,dx \sim \int_{1}^{+\infty} \frac{3dx}{x^{3/2}}$

$\lim_{x\to+\infty} \Rightarrow 3\cdot(-2)\frac{1}{\sqrt{x}}\Big|_{1}^{+\infty} = -6\cdot 0 + 6 = 6$

$\Rightarrow$ сходится

б)

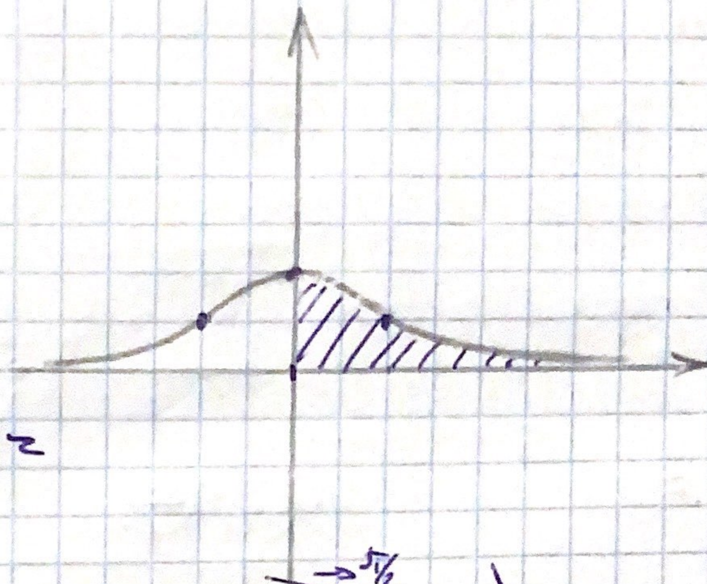
$$\int_0^1 \frac{\ln(1+\sin x)}{x\sqrt{x}}\,dx = \left\{\begin{array}{l} \sin x \sim x, \\ \quad x \to 0 \\ \ln(1+x) \sim x, \\ \quad x \to 0 \end{array}\right\} \sim \int_0^1 \frac{\cancel{x}}{\cancel{x}\sqrt{x}}\,dx =$$

$$\sim \int_0^1 \frac{dx}{\sqrt{x}} = 2\sqrt{x}\Big|_0^1 = 2 \Rightarrow \underline{\text{сходится}}$$

Билет 13

№1

$$\begin{cases} y = \dfrac{1}{1+x^2} \\ y = 0 \end{cases}$$



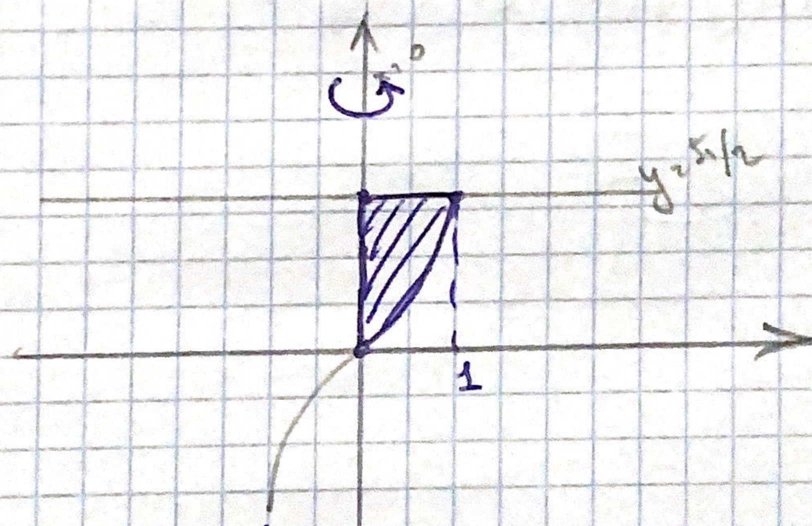
$$S = 2\cdot\int_0^{+\infty} \frac{1}{1+x^2}\,dx =$$

$$= 2\cdot \operatorname{arctg} x \Big|_0^{+\infty} = 2\Big(\lim_{x\to+\infty} \overset{\to \pi/2}{\operatorname{arctg} x} - 0\Big) =$$

$$= 2\cdot \pi/2 = \pi$$

№2.

$$\begin{cases} y = \arcsin x \\ y = \dfrac{\pi}{2} \\ x = 0 \end{cases}$$



$* x = \sin y$

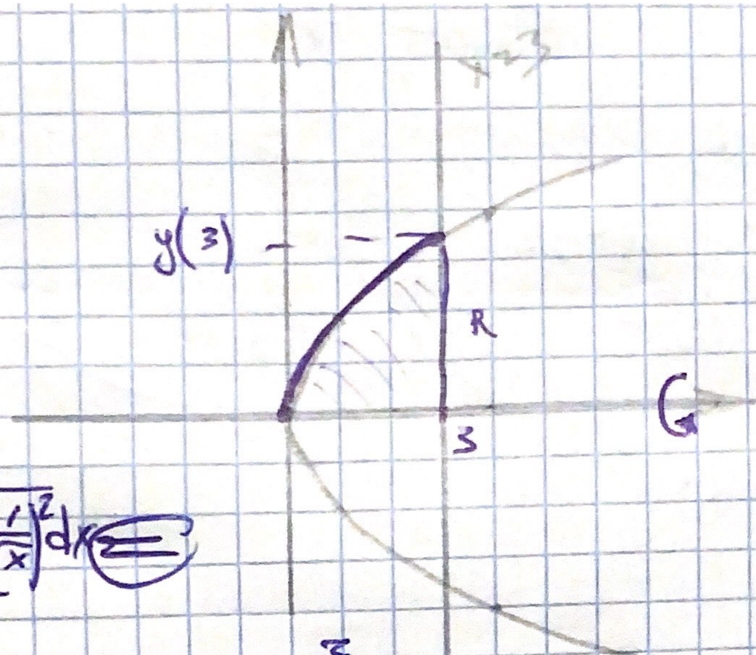
$$V_{oy} = \pi\int_0^1 \sin^2 y\,dy = \frac{\pi}{2}\int_0^1 (1-\cos 2y)\,dy =$$

$$= \frac{\pi}{2} - \frac{\pi}{4}\int_0^1 \cos 2y\, d(2y) = \frac{\pi}{2} - \frac{\pi}{4}\sin 2y\Big|_0^1 =$$

$$= \frac{\pi}{2}\left(1 - \frac{\sin 2}{2}\right)$$

13

$$\begin{cases} y^2 = 4x \\ x = 3 \end{cases}$$



$y = 2\sqrt{x}$; $y' = \frac{1}{\sqrt{x}}$

$$S = 4\pi \int_0^3 \sqrt{x} \cdot \sqrt{1 + \left(\frac{1}{\sqrt{x}}\right)^2}\, dx =$$

$$\cancel{= 4\pi \int_0^3 \sqrt{x} + \sqrt{x}\, dx =} = 4\pi \int_0^3 \sqrt{x+1}\, dx =$$

$$= 4\pi \int_0^3 \sqrt{x+1}\, d(x+1) = 4\pi \cdot \frac{2}{3} (x+1)^{3/2} \Big|_0^3 =$$

$$= \frac{56}{3}\pi$$

№4

$$\int_1^{+\infty} \frac{\operatorname{arctg} 5x}{\sqrt[4]{x^3+1}}\,dx$$ — расходится

$$\lim_{x\to+\infty} \left|\frac{x^k \operatorname{arctg} 5x}{\sqrt[4]{x^3+1}}\right| = \lim_{x\to+\infty} \frac{x^k \operatorname{arctg} 5x}{x^{3/4}\cdot\sqrt[4]{1+\frac{1}{x^3}}} = \operatorname{arctg} 5x,$$ при $k=1$, $x\to+\infty$

$$\frac{3}{4}-0=\frac{3}{4} \Rightarrow g(x)=\frac{1}{x^{3/4}}$$

$$\int_1^{+\infty} \frac{1}{\sqrt[4]{x^3}}\,dx$$ — расходится; $p=-\frac{3}{4}>-1$

$0\le g(x)\le f(x)$ $\Rightarrow$ $f(x)$ — расходится по признаку сравнения

№5

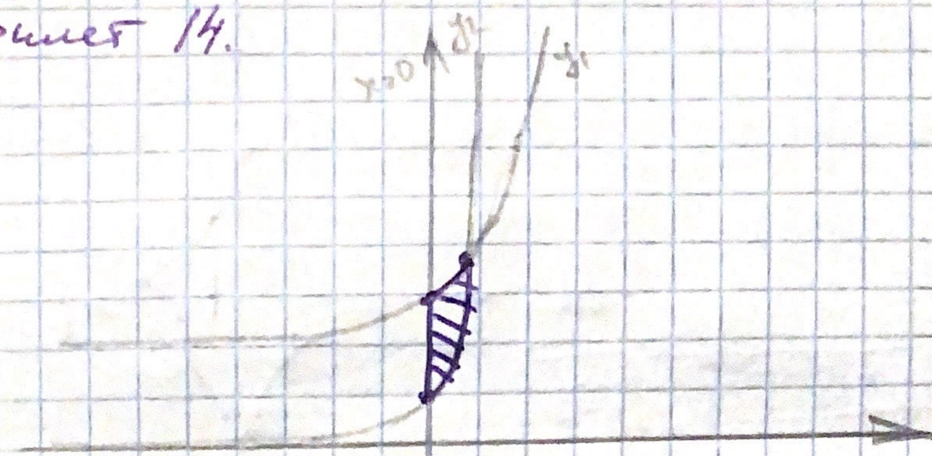
$$\int_0^1 \frac{dx}{\sqrt{x}\,(e^{x^2}-1)} = \left\{\begin{matrix} e^{x^2}-1\sim x^2, \\ x\to 0\end{matrix}\right\} = \int_0^1 \frac{dx}{\sqrt{x^5}} =$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x^5}} = +\infty$$

$\Rightarrow$ расходится

Билет 14.

№1



$$\begin{cases} y_1 = e^{2x} \\ y_2 = e^{x}+2 \\ x = 0 \end{cases}$$

$e^{2x} - e^{x} - 2 = 0$;

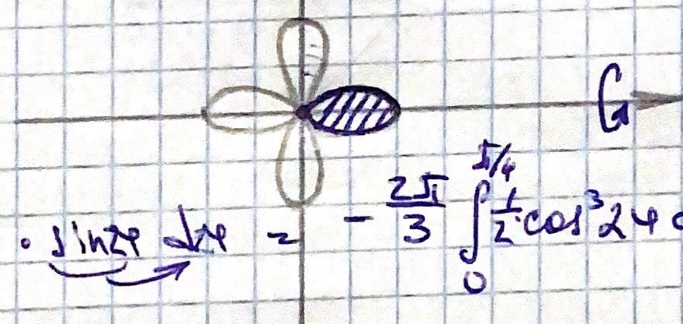
$$S = \int_0^{\ln(2)} \left(-e^{2x} + e^{x} + 2\right) dx = -\frac{e^{2x}}{2}\Big|_0^{\ln(2)} + e^{x}\Big|_0^{\ln(2)} + 2x\Big|_0^{\ln(2)} =$$

$$= -\frac{3}{2} + 1 + 2\ln(2) = -\frac{1}{2} + 2\ln(2)$$

№2 $-$? $(2\varphi \,//\, \varphi)$?

$\rho = \cos 2\varphi$

$\varphi_1 = \pi/4$; $\varphi_2 = 0$



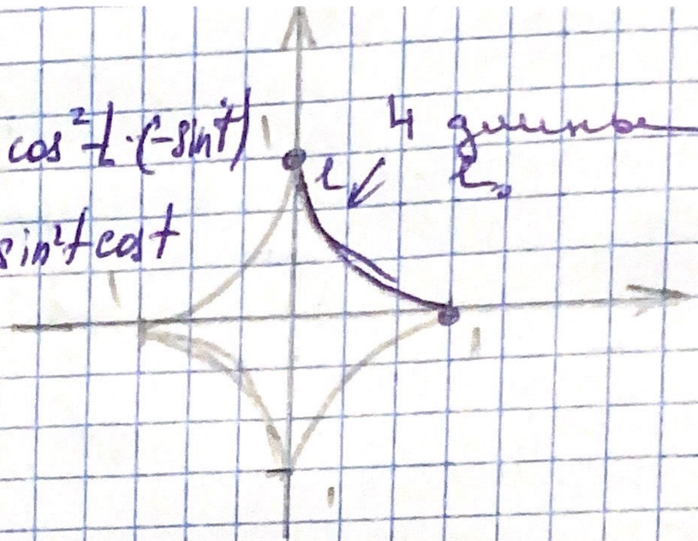
$$V = \frac{2\pi}{3}\int_0^{\pi/4} \cos^3 2\varphi \cdot \sin 2\varphi \, d\varphi = -\frac{2\pi}{3}\int_0^{\pi/4} \frac{1}{2}\cos^3 2\varphi \, d(\cos 2\varphi) =$$

$$= -\frac{\pi}{3}\cdot\frac{\cos^4 2\varphi}{4}\Big|_0^{\pi/4} = \underbrace{\frac{\pi\cdot\cos^4 0}{3\cdot 4}} - \underbrace{\frac{\pi\cdot\cos^4 \frac{\pi}{2}}{3}}_{0} =$$

$$= \frac{\pi}{12}$$

№3.

$$\begin{cases} x = \cos^3 t \\ y = \sin^3 t \end{cases} \quad ; \quad x' = 3\cos^2 t \cdot (-\sin t) \quad y' = 3\sin^2 t \cos t$$



t	0	$\pi/2$	π	$3\pi/2$
x	1	0	-1	0
y	0	1	0	-1

$$L = 4 \cdot 3 \int_0^{\pi/2} \sqrt{\cos^4 t \cdot \sin^2 t + \sin^4 t \cos^2 t}\, dt =$$

$$= 12 \cdot \int_0^{\pi/2} \sqrt{\cos^2 t \sin^2 t \underbrace{(\cos^2 t + \sin^2 t)}_{1}}\, dt =$$

$$= 12 \int_0^{\pi/2} |\cos t| \cdot |\sin t|\, dt = 12 \int_0^{\pi/2} \sin t \, d(\sin t) =$$

$$= 6 \cdot \frac{\sin^2 t}{1} \Bigg|_0^{\pi/2} = 6$$

№4 (Билет 14)

$$\int_1^{+\infty} \frac{\ln x}{x}\, dx \qquad f(x) \geq 0, \text{ на } x \in [1; +\infty)$$

$$g(x) = \ln(x) \geq f(x)$$

$$\lim_{x \to +\infty} g(x) = +\infty \text{ - расходится}$$

$$\text{сл} \Rightarrow \int_1^{+\infty} f(x) \text{ - расходится}$$

№5

$$\int_0^{\pi/2} \frac{1-\cos x}{x^3}\, dx = \left\{ \begin{array}{l} 1-\cos x \sim \frac{x^2}{2} \\ x \to 0 \end{array} \right\} \text{ Особ. точки: } x = 0$$

$$= \int_0^{\pi/2} \frac{x^2}{2x^3}\, dx = \int_0^{\pi/2} \frac{dx}{2x} =$$

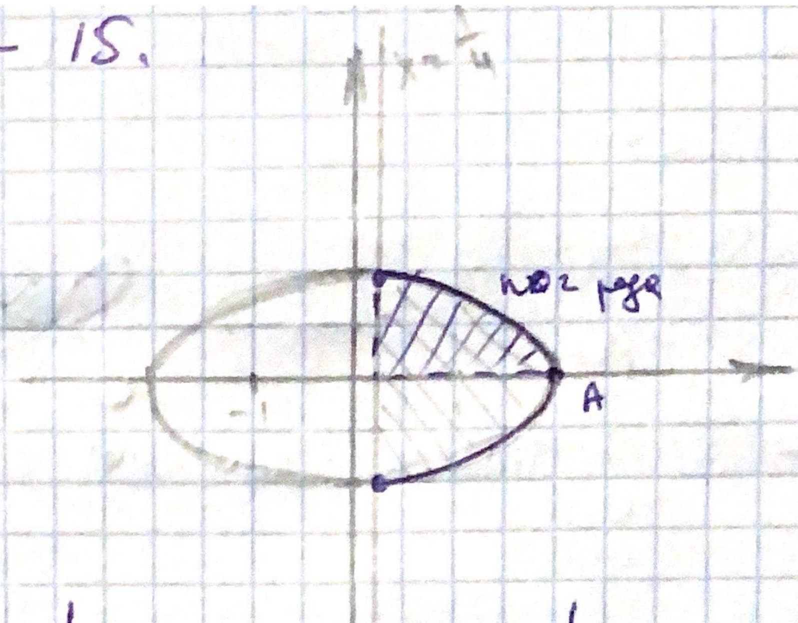
$$= \frac{1}{2}\ln(x)\Big|_0^{\pi/2} = \frac{\ln \pi/2}{2} + \infty = +\infty$$

$\Rightarrow$ расходится

Билет 15.

№1

$$\begin{cases} x = 2\cos t \\ y = \sin t \\ x = \frac{1}{4} \end{cases} ; \quad A = (2, 0)$$



t	0	$\pi/2$	π	$3\pi/2$
x	2	0	-2	0
y	0	1	0	1

$\frac{1}{4} = 2\cos t \Rightarrow t_2 = \arccos\frac{1}{8}$

$$S = -2 \cdot 2 \cdot \int_0^{t_2} \sin t \cdot \sin t \, dt = -4 \int_0^{t_2} \underbrace{\sin^2 t}_{\frac{1-\cos 2t}{2}} \, dt =$$

$$= -2 \int_0^{t_2} (1 - \cos 2t) \, dt = -2t \Big|_0^{t_2} + \sin 2t \Big|_0^{t_2} =$$

$$= -2 \cdot \arccos\frac{1}{8} + \overset{*}{\sin\left(2 \cdot \arccos\frac{1}{8}\right)} = \boxed{\frac{\sqrt{63}}{32} - 2\arccos\frac{1}{8}}$$

$$* \ \sin\left(2\arccos\frac{1}{8}\right) = 2 \cdot \underbrace{\sin\left(\arccos\frac{1}{8}\right)}_{\sin\alpha = \sqrt{1-\cos\alpha}} \cdot \underbrace{\cos\left(\arccos\frac{1}{8}\right)}_{\frac{1}{8}} =$$

$$= \cancel{2} \cdot \frac{1}{\cancel{8}_4} \cdot \sqrt{1 - \left(\frac{1}{8}\right)^2} =$$

$$= \frac{1}{4} \cdot \frac{\sqrt{63}}{8} = \frac{\sqrt{63}}{32}$$

№4 (билет 15)

$$\int_1^{+\infty} \frac{\sin\frac{1}{x}}{\sqrt{x+1}} = \left\{ \begin{array}{l} \sin\frac{1}{x} \sim \frac{1}{x}, \\ x \to +\infty \end{array} \right\} = \int_1^{+\infty} \frac{1}{x\sqrt{x+1}}\,dx =$$

$$\lim_{x\to+\infty} \frac{x^k}{x^{3/2}\sqrt{1+\frac{1}{x}}} = 1, \text{ при } k = 3/2, \ x \to +\infty$$

$$= \int_1^{+\infty} \frac{dx}{x^{3/2}} = -\frac{2}{\sqrt{x}}\Big|_1^{+\infty} = 0+2 = 2$$

$\Rightarrow$ сходится

№5

$$\int_0^1 \frac{\ln(1+\sqrt[5]{x^3})}{e^x-1}\,dx = \left\{ \begin{array}{l} e^x - 1 \sim x, \\ x \to 0 \\ \ln(1+x^{3/5}) \sim x^{3/5}, \\ x^{3/5} \to 0 \end{array} \right\} =$$

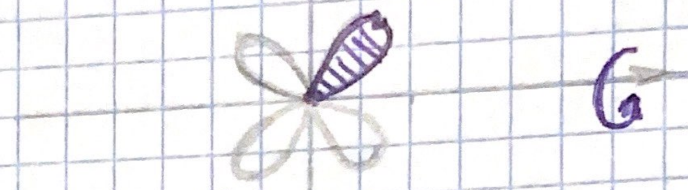
$$= \int_0^1 \frac{x^{3/5}}{x}\,dx = \int_0^1 \frac{dx}{x^{2/5}} = \frac{5}{3}x^{3/5}\Big|_0^1 =$$

$$= \frac{5}{3} - 0 = \frac{5}{3} \Rightarrow \text{сходится}$$

52

$\rho = \sin 2\varphi$

$\varphi_1 = 0 \ ; \ \varphi_2 = \pi/4$



$$V = \frac{2\pi}{3}\int_0^{\pi/4} \sin^3 2\varphi \cdot \sin 2\varphi \, d(2\varphi) =$$

$$= \frac{2\pi}{3}\int_0^{\pi/4} \frac{(1-\cos 4\varphi)^2}{4}\, d(2\varphi) = \frac{\pi}{6}\int_0^{\pi/4} (1 - 2\cos 4\varphi + \cos^2 4\varphi)\, d2\varphi =$$

$$= \frac{\pi}{12}\int_0^{\pi/4} (1 - 2\cos 4\varphi + \cos^2 4\varphi)\, d(4\varphi) = \frac{\pi\cdot\varphi}{3}\Big|_0^{\pi/4} - \underbrace{\frac{\pi \sin 4\varphi}{6}\Big|_0^{\pi/4}}_{0}$$

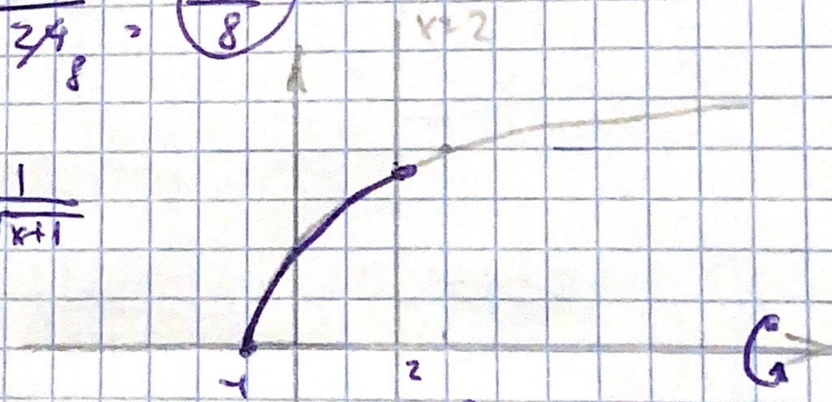
$$+ \frac{\pi}{12}\int_0^{\pi/4} \underbrace{\cos^2 4\varphi}_{1+\cos 8\varphi} (4\varphi) = \frac{\pi^2}{12} + \frac{\pi^2}{24} + 0 =$$

$$= \frac{2\pi^2 + \pi^2}{24} = \frac{3\pi^2}{24_8} = \frac{\pi^2}{8}$$

53 −? $(x = -1)$?

$y = 2\sqrt{x+1} \ ; \ y' = \frac{1}{\sqrt{x+1}}$

$x = 2$



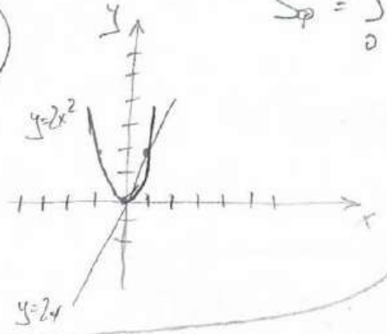
$$S_{ox} = 2\cdot 2\pi\int_{-1}^{2} \sqrt{x+1}\cdot\sqrt{1+\frac{1}{x+1}}\, dx = 4\pi\int_{-1}^{2} \cancel{\sqrt{x+1}}\cdot\frac{\sqrt{x+2}}{\cancel{\sqrt{x+1}}}\, dx =$$

$$= 4\pi\int_{-1}^{2} \sqrt{x+2}\, d(x+2) = \frac{2\cdot 4\pi (x+2)^{3/2}}{3}\Big|_{-1}^{2} = \frac{56\pi}{3}$$

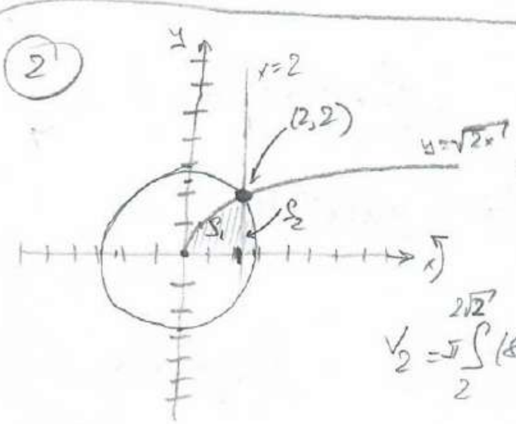
Вар. 16.

①

$$S_\varphi = \int_0^1 2x\,dx - \int_0^1 2x^2\,dx = x^2\Big|_0^1 - \frac{2}{3}x^3\Big|_0^1 = 1 - \frac{2}{3} = \underline{\underline{\frac{1}{3}}}$$



②



$$V = V_1 + V_2$$

$$V_1 = \int_0^2 \pi\,|2x|\,dx = \pi x^2\Big|_0^2 = 4\pi$$

$$V_2 = \pi\int_2^{2\sqrt{2}} (8 - x^2)\,dx = 8\pi x\Big|_2^{2\sqrt{2}} - \frac{\pi x^3}{3}\Big|_2^{2\sqrt{2}}$$

$y^2 = 2x \quad x^2 + y^2 = 8$

$y^2 = 8 - x^2 \quad x^2 + 2x - 8 = 0$

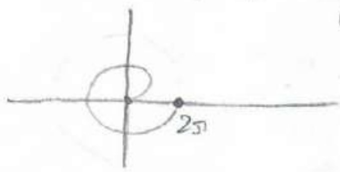
$(x+1)^2 = 9$

$x + 1 = 3 \quad x = 2$

$$V = 4\pi + 16\pi\sqrt{2} - 16\pi - \frac{16\sqrt{2}\pi}{3} + \frac{8\pi}{3} = \frac{12\pi + 48\pi\sqrt{2} - 48\pi - 16\sqrt{2}\pi + 8\pi}{3} = \frac{32\sqrt{2}\pi - 28\pi}{3}$$

$$= \underline{\underline{\frac{32\sqrt{2}\pi - 28\pi}{3}}}$$

③ $\rho = \varphi$



$$l = a^2\int_0^{2\pi} \sqrt{\rho^2 + 1}\,d\rho = a\left(\frac{\rho\sqrt{1+\rho^2}}{2}\Big|_0^{2\pi} + \frac{1}{2}\ln\left|\sqrt{1+\rho^2} + \rho\right|\Big|_0^{2\pi}\right) =$$

$$= \underline{\underline{a\left(\pi\sqrt{1+4\pi^2} + \frac{1}{2}\ln\left|\sqrt{1+4\pi^2} + 2\pi\right|\right)}}$$

④ $\displaystyle\int_1^{+\infty} \frac{\sqrt{\operatorname{arctg} x}}{4+x^2}\,dx$ по признаку сх-ти $\displaystyle\frac{\sqrt{\operatorname{arctg} x}}{4+x^2} < \frac{2\pi}{4+x^2}$ $\quad \displaystyle\int_1^{+\infty} \frac{1}{4+x^2}\,dx =$

$$= \lim_{b\to+\infty}\left(\int_1^b \frac{1}{4+x^2}\right) = \lim_{b\to+\infty}\left(\pi\operatorname{arctg}\left(\frac{b}{2}\right) - \pi\operatorname{arctg}\left(\frac{1}{2}\right)\right) = \ldots \Rightarrow \int_1^{+\infty} \frac{2\pi}{4+x^2}\,dx$$ схо-

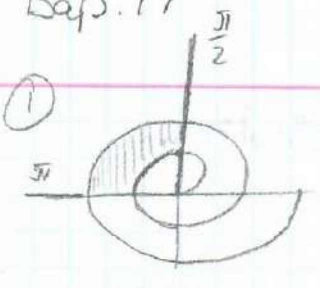
дится => по сх-ся и исходный

⑤ $\displaystyle\int_0^1 \frac{\sqrt[3]{\operatorname{tg} x}}{\ln(1+x^2)} \sim \int_0^1 \frac{\sqrt[3]{\operatorname{tg} x}\,dx}{x^2},\ x\to 0 \quad \frac{\sqrt[3]{\operatorname{tg} x}}{x^2} < \frac{2}{x^2} \quad \int_0^1 \frac{2}{x^2}\,dx = \frac{-2}{x}\Big|_0^1 = -2 \Rightarrow$

=> сх-ся и исходный

Вар. 17

(1)



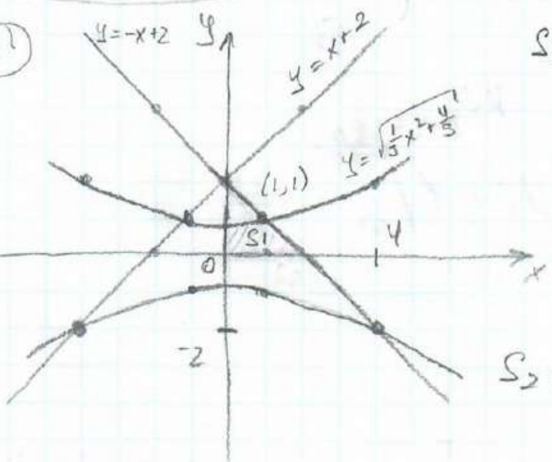
$$\cancel{S = \int_{\frac{5}{2}\pi}^{3\pi} \sqrt{\rho^2+1}\,d\rho - \int_{\frac{\pi}{2}}^{\pi} \sqrt{\rho^2+1}\,d\rho} =$$

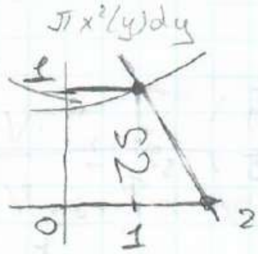
$$S = \int_{\frac{5}{2}\pi}^{3\pi} \frac{\rho^2}{2}\,d\varphi - \int_{\frac{\pi}{2}}^{\pi} \frac{\rho^2}{2}\,d\rho = \frac{1}{6}\rho^3\Big|_{\frac{5\pi}{2}}^{3\pi} - \frac{1}{6}\varphi^3\Big|_{\frac{\pi}{2}}^{\pi} =$$

$$= \frac{27\pi^3}{6} - \frac{125\pi^3}{6\cdot 8} - \frac{\pi^3}{6} + \frac{\pi^3}{6\cdot 4} = \cancel{54\pi^3}\ \frac{27\cdot 8\pi^3 - 125\pi^3 - 8\pi^3 + 2\pi^3}{48} =$$

$$= \frac{85\pi^3}{48}$$

(2)

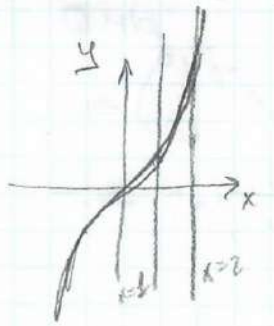




$$S_1 = \pi\int_{\frac{2}{\sqrt5}}^{1} (5y^2-4)\,dy = \frac{5\pi y^3}{3}\Big|_{\frac{2}{\sqrt5}}^{1} - 4\pi y\Big|_{\frac{2}{\sqrt5}}^{1} = \frac{5\pi}{3} - \frac{8\pi}{3\sqrt5} - 4\pi + \frac{8\pi}{\sqrt5} = \frac{5\sqrt5\pi - 8\pi - 4\sqrt5\pi + 24\pi}{3\sqrt5} = \frac{16\pi + \sqrt5\pi}{3\sqrt5}$$

$$S_2 = \pi\int_0^1 (y-2)^2\,dy = \frac{7\pi}{3} \qquad S = S2 - S1 < 0\ ?$$

(3)



$y = x^3 \qquad \int \pi y^2(x)\,dx$

$$V = \int_1^2 \pi x^6\,dx = \frac{\pi x^7}{7}\Big|_1^2 = \frac{128\pi - \pi}{7} = \frac{127}{7}\pi$$

(4) $\int_1^{+\infty} \frac{\sin x}{x\sqrt{x}+1}\,dx \qquad \frac{\sin x}{x\sqrt{x}+1} < \frac{1}{x\sqrt{x}+1}$ при $x \to +\infty$ $f(x) = \frac{1}{x\sqrt{x}+1}$ − бм φ.

$$\lim_{x\to+\infty} \frac{\frac{1}{x\sqrt{x}+1}}{\left(\frac{1}{x}\right)^k} = \frac{1\cdot x^k}{x^{\frac32}\left(1+\frac{1}{x^{\frac32}}\right)} = 1, \text{ при } k = \frac{3}{2}$$

$$\int_1^{+\infty} \frac{dx}{x^{\frac32}} \quad \lim_{b\to+\infty}\left(-2x^{-\frac12}\Big|_1^b\right) = \lim_{b\to+\infty}\left(\frac{-2}{\sqrt{x}} + 2\right) = 2 \Rightarrow$$ сходится и исходной

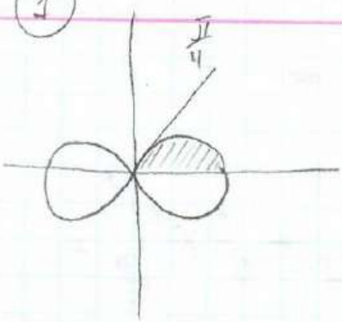
(5) $\int_0^1 \frac{2^x - 1}{\sin^2 x}\,dx$ особые точки − $x = 0$

$$\sim \int_0^1 \frac{x\ln 2}{x^2}\,dx \qquad \int_0^1 \frac{\ln 2}{x}\,dx = \ln 2 \ln x\Big|_0^1$$ расх-ся ⇒

⇒ расх-ся и исходной

Вар. 18. $\rho^2 = \cos 2\varphi$

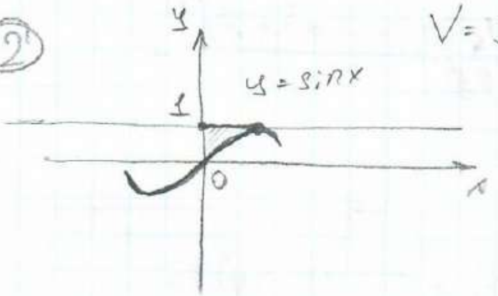
(1)



$$\int_0^{\frac{\pi}{4}} \frac{\rho^2}{2}\,d\varphi = \int_0^{\frac{\pi}{4}} \frac{\cos 2\varphi}{2}\,d\varphi = \frac{1}{4}\int_0^{\frac{\pi}{4}} \cos 2\varphi\, d2\varphi =$$

$$= \frac{1}{4}\sin 2\varphi \Big|_0^{\frac{\pi}{4}} = \underline{\underline{\frac{1}{4}}}$$

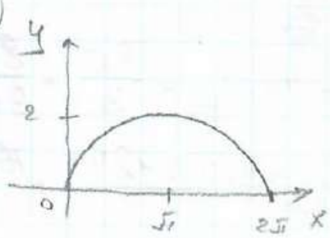
(2)



$$V = \pi \int_0^1 \arcsin y\, dy = \pi\left(\arcsin y \cdot y \Big|_0^1 + \sqrt{1-y^2}\Big|_0^1\right) =$$

$$= \underline{\underline{\frac{\pi^2}{2} - 1}}$$

(3)



$$l = \int_0^{\frac{3\pi}{2}} \sqrt{\cos^2 t + \sin^2 t}\, dt = t\Big|_0^{\frac{3\pi}{2}} = \underline{\underline{\frac{3\pi}{2}}}$$

(4) $\int_1^{+\infty} \frac{\operatorname{arctg} x}{\sqrt{x^3+5}}\,dx$ $\quad \frac{\operatorname{arctg} x}{\sqrt{x^3+5}} \le \frac{\frac{\pi}{2}}{\sqrt{x^3+5}}$ $\quad f(x) = \frac{\frac{\pi}{2}}{\sqrt{x^3+5}} \to 0$ при $x \to +\infty \Rightarrow$ БМФ.

$\frac{\frac{\pi}{2}}{\sqrt{x^3+5}} \sim \frac{1}{\sqrt{x^3}}$, $x \to +\infty$ $\quad \int_1^{+\infty} \frac{dx}{\sqrt{x^3}}$ $\quad \lim_{b \to +\infty}\left(-2x^{-\frac{1}{2}}\Big|_1^b\right) =$

$= \lim_{b\to+\infty}\left(\frac{-2}{\sqrt{+\infty}} + \frac{2}{\sqrt{1}}\right) = 2 \Rightarrow$

$\Rightarrow$ сходится и исходный

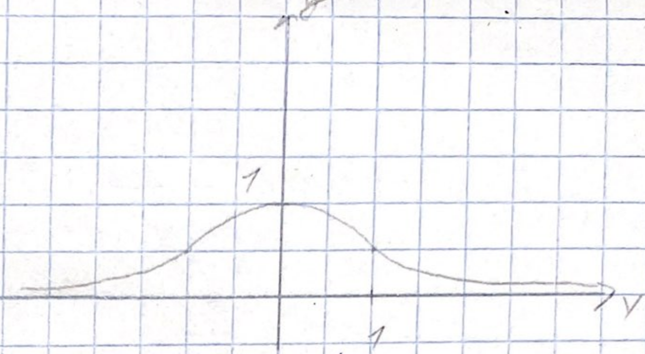
(5) Особая точка: $x = 0$

$\int_0^{\frac{\pi}{2}} \frac{dx}{x\sqrt{\sin x}} \sim \int_0^{\frac{\pi}{2}} \frac{dx}{x\sqrt{x}}$ при $x \to 0$ $\quad \int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{x^3}}$ $\quad \lim_{b\to 0+}\left(\int_b^{\frac{\pi}{2}} \frac{dx}{\sqrt{x^3}}\right) =$

$= \lim_{b \to 0+}\left(\frac{-2}{\sqrt{x}}\Big|_b^{\frac{\pi}{2}}\right) = \lim_{b\to 0+}\left(\frac{-2}{\sqrt{\frac{\pi}{2}}} + \frac{2}{\sqrt{b}}\right) = +\infty \Rightarrow$ расходится и начальный

Билет 26

№1



$$\begin{cases} y = \frac{1}{1+x^2} \\ y = 0 \end{cases}$$

$$S = \int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = 2\int_0^{\infty} \frac{1}{x^2+1}\,dx = \lim_{b\to\infty} \int_0^{b} \frac{1}{x^2+1}\,dx =$$

$$= 2\lim_{b\to\infty} F(x)\Big|_0^{a} = 2\cdot\frac{\pi}{2} = \pi$$

$F(x) = arctg(x) + c$

$\lim\limits_{a\to\infty} F(a) = \frac{\pi}{2}$

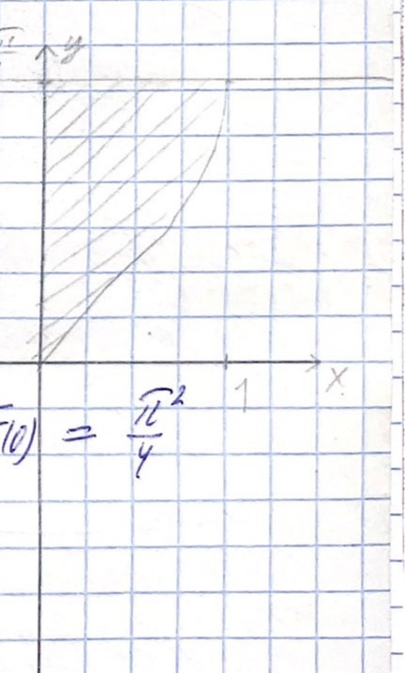
$F(0) = 0$

Ответ: π

№2

$x = \sin y$

$x^2 = \sin^2 y$



$$V_y = \pi \int_c^d x^2\,dy$$

$y = arcsin x \quad y = \frac{\pi}{2} \quad x = 0$

$$V_y = \pi \int_0^{\frac{\pi}{2}} \sin^2 y\,dy = \pi F(y)\Big|_0^{\frac{\pi}{2}} = \pi F(\tfrac{\pi}{2}) - F(0) = \frac{\pi^2}{4}$$

$$F(y) = \int \sin^2(y)\,dy = \frac{y}{2} - \frac{\sin 2y}{4} + c$$

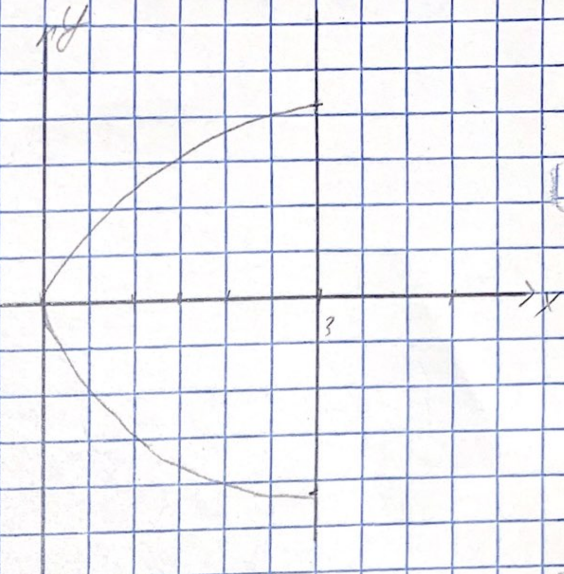
$F(\frac{\pi}{2}) = \frac{\pi}{4} \quad F(0) = 0$

Ответ: $\frac{\pi^2}{4}$

№3

$y^2 = 4x \quad y = \cancel{2}\,3$

$y = \sqrt{4x}$



$S = 2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$

$S = 2\pi \int_0^3 \sqrt{4x}\sqrt{1+\frac{1}{x}}\,dx =$

$= \lim_{\delta \to 0} \int_{0+\delta}^{3} f(x)\,dx =$

$f(x) = 4\pi\sqrt{\frac{1}{x}+1}\sqrt{x}$

разрыв

$\frac{1}{x} \to x = 0$

$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a) = \frac{56\pi}{3}$

$F(x) = \int 4\pi\sqrt{\frac{1}{x}+1}\sqrt{x}\,dx = \frac{8\pi x\sqrt{x+1}}{3} + \frac{8\pi\sqrt{x+1}}{3} + C$

$\int 4\pi\sqrt{\frac{1}{x}+1}\sqrt{x}\,dx = 4\pi\int\sqrt{x}\sqrt{\frac{x+1}{x}}\,dx = 4\pi\int\sqrt{x+1}\,dx =$

$= \left| \begin{array}{l} u = x+1 \\ x = u-1 \\ dx = du \end{array} \right| = 4\pi\int\sqrt{u}\,du = 4\pi\cdot\frac{2u^{\frac{3}{2}}}{3} = \frac{8\pi u^{\frac{3}{2}}}{3} =$

$= \frac{8\pi x\sqrt{x+1}}{3} + \frac{8\pi\sqrt{x+1}}{3}$

$\lim_{\delta \to 0} F(0+\delta) = \frac{8\pi}{3} \qquad F(3) = \frac{64\pi}{3}$

Ответ: $\frac{56\pi}{3}$

Билет 26

№4

$$\int_1^{+\infty} \frac{\operatorname{arctg} 5x}{\sqrt[4]{x^3+1}}\,dx$$

$$\lim_{x\to+\infty}\left(\frac{x^k \operatorname{arctg} 5x}{\sqrt[4]{x^3+1}}\right) = \lim_{x\to+\infty} \frac{x^k \operatorname{arctg} 5x}{x^{\frac{3}{4}} \cdot \sqrt[4]{1+\frac{1}{x^3}}} = \operatorname{arctg} 5x \text{ при}$$

$k = 1,\ x\to+\infty$

$$g(x) = \frac{1}{x^{3/4}}$$

$\int_1^{+\infty} \frac{1}{\sqrt[4]{x^3}}\,dx$ – расходится; $\Rightarrow$

расходится и изначальный

№5

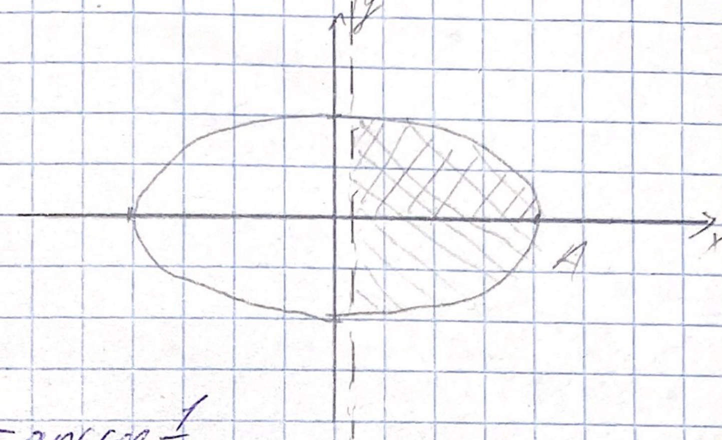
$$\int_0^1 \frac{dx}{\sqrt{x}\,(e^{x^2}-1)} = \left| \begin{matrix} e^{x^2}-1 \sim x^2 \\ x\to 0 \end{matrix} \right\} = \int_0^1 \frac{dx}{\sqrt{x^5}} =$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x^5}} = +\infty \Rightarrow \text{расходится}$$

Билет 28

№1

$x = 2\cos t$
$y = \sin t$
$x = \frac{1}{4}$



$A = (2;0)$

$\frac{1}{4} = 2\cos t \Rightarrow t_2 = \arccos\frac{1}{8}$

$$S = -2\cdot 2\int_0^{t_2} \sin t \cdot \sin t \, dt = -4\int_0^{t_2} \sin^2 t \, dt =$$

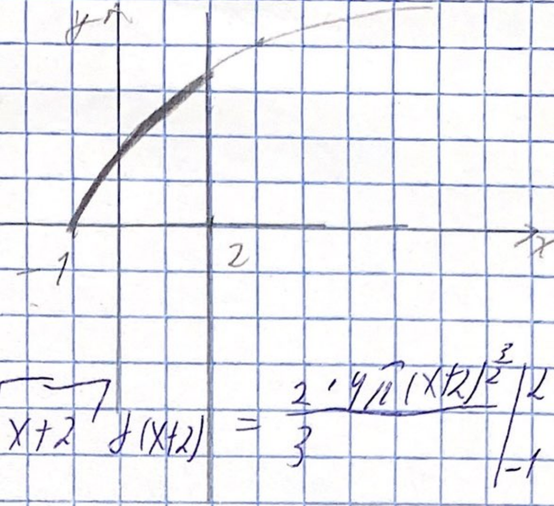
$$= -2\int_0^{t_2} (1-\cos 2t)\,dt = -2t\Big|_0^{t_2} + \sin 2t\Big|_0^{t_2} = -2\arccos\frac{1}{8} +$$

$$\sin(2\cdot\arccos\frac{1}{8}) = \frac{\sqrt{63}}{32} - 2\arccos\frac{1}{8}$$

$$\sin(2\arccos\frac{1}{8}) = 2\sin(\arccos\frac{1}{8})\cdot\cos(\arccos\frac{1}{8}) =$$

$$= 2\cdot\frac{1}{8}\cdot\sqrt{1-(\frac{1}{8})^2} = \frac{1}{4}\cdot\frac{\sqrt{63}}{8} = \frac{\sqrt{63}}{32}$$

№3



$y = 2\sqrt{x+1}$

$x = 2$ $\qquad y' = \frac{1}{\sqrt{x+1}}$

$S = 2 \cdot 2\pi \int_{-1}^{2} \sqrt{x+1} \cdot \sqrt{1 + \frac{1}{x+1}}\, dx =$

$= 4\pi \int_{-1}^{2} \sqrt{x+2}\, dx = 4\pi \int_{-1}^{2} \sqrt{x+2}\, d(x+2) = \frac{2 \cdot 4\pi (x+2)^{\frac{3}{2}}}{3} \Big|_{-1}^{2} =$

$= \frac{56\pi}{3}$

№4

$\int_{1}^{+\infty} \frac{\sin\frac{1}{x}}{\sqrt{x+1}} = \left| \begin{matrix} \sin\frac{1}{x} \sim \frac{1}{x} \\ x \to +\infty \end{matrix} \right| = \int_{1}^{+\infty} \frac{1}{x\sqrt{x+1}}\, dx = \lim_{x \to +\infty} \frac{x^{\alpha}}{x^{\frac{3}{2}}\sqrt{1+\frac{1}{x}}} =$

$= 1$, при $\alpha = \frac{3}{2}$; $x \to +\infty$

$\int_{1}^{+\infty} \frac{dx}{x^{\frac{3}{2}}} = -\frac{2}{\sqrt{x}} \Big|_{1}^{+\infty} = 0 + 2 = 2 \Rightarrow$ сходится

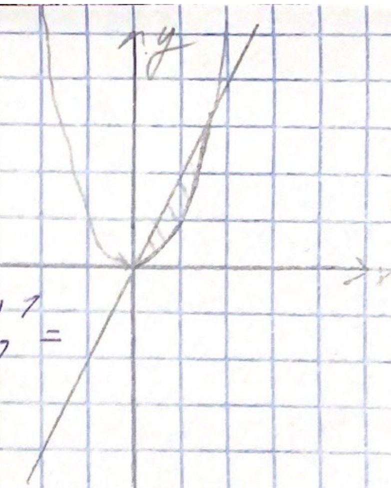
№5

$\int_{0}^{1} \frac{\ln(1+\sqrt[5]{x^3})}{e^x - 1}\, dx = \left| \begin{matrix} e^x - 1 \sim x_1 \\ x \to 0 \\ \ln(1+x^{\frac{3}{5}}) \sim x^{\frac{3}{5}} \\ x^{\frac{3}{5}} \to 0 \end{matrix} \right| = \int_{0}^{1} \frac{x^{\frac{3}{5}}}{x}\, dx =$

$= \int_{0}^{1} \frac{dx}{x^{\frac{2}{5}}} = \frac{5}{3} x^{\frac{3}{5}} \Big|_{0}^{1} = \frac{5}{3} \Rightarrow$ сходится

Билет 29

№1



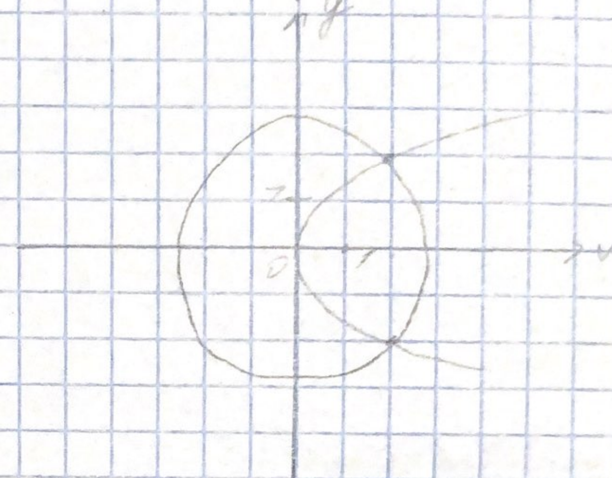
$y = 2x^2$

$y = 2x$

$S = \int_0^1 2x\,dx - \int_0^1 2x^2\,dx = x^2\Big|_0^1 - \frac{2}{3}x^3\Big|_0^1 =$

$= 1 - \frac{2}{3} = \frac{1}{3}$

№2



$y^2 = 2x \quad x^2 + y^2 = 8$

$y^2 = 8 - x^2 \quad x^2 + 2x - 8 = 0$

$(x+1)^2 = 9$

$V = V_1 + V_2$

$x + 1 = 3$

$x = 2$

$V_1 = \int_0^2 \pi(2x)\,dx = \pi x^2\Big|_0^2 = 4\pi$

$V_2 = \pi\int_0^{2\sqrt{2}} (8 - x^2)\,dx = 8\pi x\Big|_2^{2\sqrt{2}} - \frac{\pi x^3}{3}\Big|_2^{2\sqrt{2}} =$

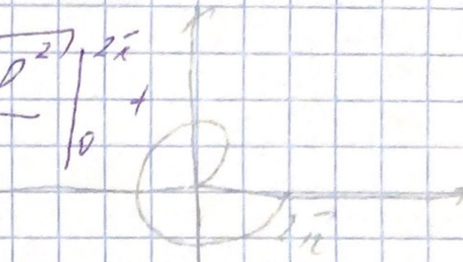
$V = 4\pi + 16\pi\sqrt{2} - 16\pi - \frac{16\sqrt{2}\pi}{3} + \frac{8\pi}{3} =$

$= \frac{12\pi + 48\pi\sqrt{2} - 48\pi - 16\sqrt{2}\pi + 8\pi}{3} = \frac{32\sqrt{2}\pi - 28\pi}{3} =$

$= \frac{32\sqrt{2}\pi - 28\pi}{3}$

№3

$$l = a^2\int_0^{2\pi}\sqrt{\rho^2+1}\,d\rho = a\left(\frac{\rho\sqrt{1+\rho^2}}{2}\Big|_0^{2\pi} +\right.$$



$$\left.+\frac{1}{2}\ln\left|\sqrt{1+\rho^2}+\rho\right|\Big|_0^{2\pi}\right) = a\left(\pi\sqrt{1+4\pi^2}+\frac{1}{2}\ln\left|\sqrt{1+4\pi^2}+2\pi\right|\right)$$

№4

$$\int_1^{+\infty}\frac{\sqrt{arctg\,x}}{4+x^2}$$

$$\frac{\sqrt{arctg\,x}}{4+x^2} < \frac{2\pi}{4+x^2}, \quad \int_1^{+\infty}\frac{1}{4+x^2}\,dx = \lim_{b\to+\infty}\left(\int_1^b\frac{1}{4+x^2}\right) =$$

$$= \lim_{b\to+\infty}\left(\pi\, arctg\left(\frac{b}{2}\right) - \pi\, arctg\left(\frac{1}{2}\right)\right) \Rightarrow \int_1^{+\infty}\frac{2\pi}{4+x^2}\,dx$$

сходится => сходится и исходный.

№5

$$\int_0^1\frac{\sqrt[3]{tg\,x}}{\ln(1+x^2)} \sim \int_0^1\frac{\sqrt[3]{tg\,x}\,dx}{x^2}, \quad x\to 0 \quad \frac{\sqrt[3]{tg\,x}}{x^2} < \frac{2}{x^2}$$

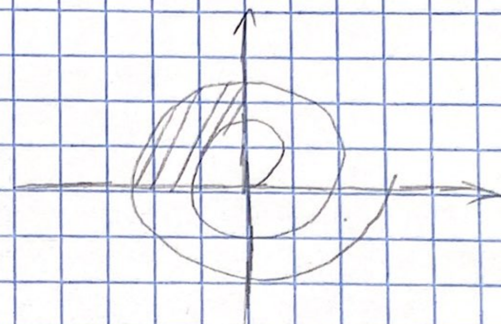
$$\int_0^1\frac{2}{x^2}\,dx = \frac{-2}{x}\Big|_0^1 = -2$$ => сходится и исходный

Билет 30

№1

$\rho = \varphi$

$\varphi = \frac{\pi}{2}$; $\varphi = \pi$



$$S = \int_{\pi}^{3\pi} \frac{\varphi^2}{2} d\varphi - \int_{\frac{\pi}{2}}^{\pi} \frac{\varphi^2}{2} d\varphi =$$

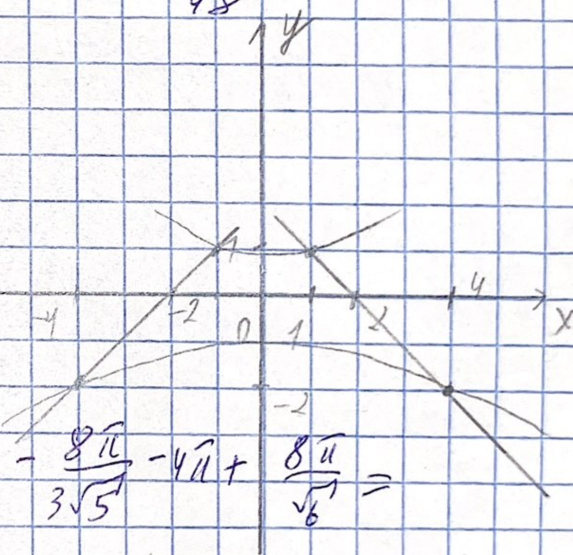
$$= \frac{1}{6}\varphi^3 \Big|_{\frac{5\pi}{2}}^{3\pi} - \frac{1}{6}\varphi^3 \Big|_{\frac{\pi}{2}}^{\pi} = \frac{27\pi^3}{6} - \frac{125\pi^3}{6\cdot 8} - \frac{\pi^3}{6} + \frac{\pi^3}{6\cdot 4} =$$

$$= \frac{27\cdot 8\pi^3 - 125\pi^3 - 8\pi^3 + 2\pi^3}{48} = \frac{85\pi^3}{48}$$

№2

$x^2 = 5y^2 - 4$

$x^2 = (y-2)^2$



$$V_1 = \pi \int_{\frac{2}{\sqrt{5}}}^{1} (5y^2 - 4)\, dy =$$

$$= \frac{5\pi y^3}{3} \Big|_{\frac{2}{\sqrt{5}}}^{1} - 4\pi y \Big|_{\frac{2}{\sqrt{5}}}^{1} = \frac{5\pi}{3} - \frac{8\pi}{3\sqrt{5}} - 4\pi + \frac{8\pi}{\sqrt{5}} =$$

$$= \frac{16\pi + \sqrt{5}\pi}{3\sqrt{5}}$$

$$V_2 = \pi \int_0^1 (y-2)^2 dy = \frac{7\pi}{3}$$

$$V = V_1 - V_2 = \frac{16\pi + \sqrt{5}\pi}{3\sqrt{5}} - \frac{7\pi}{3} = \frac{16\pi\sqrt{5} - 30\pi}{15}$$

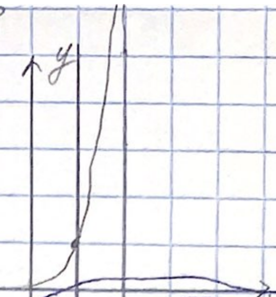
№3

$y = x^3$

$x = 1$

$x = 2$

$S = 2\pi \int_1^2 x^3 \sqrt{1+9x^4}\, dx =$



$= F(2) - F(1) \underset{2\pi}{=} \left(\frac{145\sqrt{145}}{54} - \frac{5\sqrt{10}}{27} \right) = \boxed{\frac{145\pi\sqrt{145} - 10\pi\sqrt{10}}{27}}$

$F(x) = \int x^3 \sqrt{1+9x^4}\, dx = \int \frac{1}{36} \cdot \sqrt{t}\, dt =$

$= \frac{1}{36} \int t^{\frac{1}{2}}\, dt = \frac{1}{36} \cdot \frac{2\sqrt{t}^3}{3} = \frac{1}{36} \cdot \frac{2(1+9x^4)^{\frac{3}{2}}}{3} =$

$= \frac{(9x^4+1)^{\frac{3}{2}}}{54}$

$F(1) = \frac{5\sqrt{10}}{27} \qquad F(2) = \frac{145\sqrt{145}}{54}$

№4

$\int_1^{+\infty} \frac{\sin x}{x\sqrt{x}+1}\, dx \qquad \frac{\sin x}{x\sqrt{x}+1} < \frac{1}{x\sqrt{x}+1}$ при $x \to +\infty$ $\quad P(x) = \frac{1}{x\sqrt{x}+1}$ – б.м.ф.

$\lim_{x\to+\infty} \frac{\frac{1}{x\sqrt{x}+1}}{\left(\frac{1}{x}\right)^k} = 1$ при $k = \frac{3}{2}$

$\int_1^{+\infty} \frac{dx}{x^{\frac{3}{2}}}, \quad \lim_{b\to+\infty} \left(-2x^{-\frac{1}{2}} \Big|_1^b \right) = \lim_{b\to+\infty} \left(\frac{-2}{\sqrt{x}} + 2 \right) = 2 \Rightarrow$ сходится

и исходный.

№5

$\int_0^1 \frac{2^x - 1}{\sin^2 x}\, dx$ особ. т. $x = 0$

$\sim \int_0^1 \frac{x \ln 2}{x^2} \qquad \int_0^1 \frac{\ln 2}{x}\, dx = \ln 2 \ln x \Big|_0^1$ расход $\Rightarrow$

$\Rightarrow$ расходится и исходный.