№1.

$xy'' = 2x\,tg\frac{y'}{x} + y' \quad y\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{24};\ y'\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi - \sqrt{2}}{12}$

$\begin{cases} p = y' \\ p' = y'' \end{cases} \Rightarrow xp' = 2x\,tg\frac{p}{x} + p \,/ : x \neq 0$

$p' = 2\,tg\frac{p}{x} + \frac{p}{x}$

Замена: $p = ux \Rightarrow u'x + u = p'$

$u'x + u = 2\,tg u + u$

$u'x = 2\,tg u$

$\frac{du}{tg u} = \frac{2dx}{x}$

$\ln|\sin u| = 2\ln|x| + C,\ \forall C$

$\sin u = x^2 \cdot C_1,\ \forall C_1 > 0$

$u = \arcsin(C_1 \cdot x^2)$

$\frac{dy}{dx} = x\arcsin(C_1 x^2) \quad (1)$

$dy = x\arcsin(C_1 \cdot x^2)dx$

$y = \frac{1}{2C_1}\int \arcsin(C_1 x^2)\,d(C_1 x^2)$

$y = C_1 x^2 \arcsin(C_1 x^2) + \sqrt{1-(C_1 x)^2} + \tilde{C},\ \forall\tilde{C} \quad (2)$

общее решение

Подставим $y'\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi - \sqrt{2}}{12}$ в (1):

$\frac{\pi - \sqrt{2}}{12} = \frac{1}{\sqrt{2}}\arcsin\left(\frac{C_1}{2}\right) \Rightarrow C_1 = 1$

Подставим C_1 и $y\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{24}$ в (2):

$$\frac{\pi}{24}=\frac{1}{2}\arcsin\frac{1}{2}+\frac{\sqrt{3}}{2}+\tilde{C}$$

$$\frac{\pi}{24}=\frac{\pi}{12}+\frac{\sqrt{3}}{2}+\tilde{C}$$

$$\tilde{C}=\frac{\pi}{24}-\frac{\pi}{12}-\frac{\sqrt{3}}{2}=-\frac{\pi}{24}-\frac{\sqrt{3}}{2}$$

$$y=x^2\arcsin x^2+\sqrt{1-x^4}-\frac{\pi}{24}-\frac{\sqrt{3}}{2}$$ ответ

№2.

$$y''=y'\sqrt{1-(y')^2}$$

$$\begin{cases} y'=p \\ y''=p'\cdot p \end{cases} \qquad p'p=p\sqrt{1-p^2}$$

$$p'=\sqrt{1-p^2}$$

$$\frac{dp}{\sqrt{1-p^2}}=dy$$

$$y=\arcsin p+C,\ \forall C$$

$$\arcsin p=y+C$$

$$p=\sin(y+C)$$

$$y'=\sin(y+C)$$

$$\frac{dy}{\sin(y+C)}=dx$$

$$\ln\left|tg\left(\frac{y+C}{2}\right)\right|=x+C_1,\ \forall C_1$$

$tg\left(\frac{y+C}{2}\right) = \cancel{3}e^{x+C_1}$

$tg\left(\frac{y+C}{2}\right) = \tilde{C}e^x, \ \forall \tilde{C} > 0$

$\frac{y+C}{2} = arctg\, \tilde{C}e^x$

$y = 2arctg\tilde{C}e^x + C$ – <u>общее решение</u>

№ 3.

$y^{IV} + 2y^{III} + y'' = e^{-x} + 1 + xe^{-x} - x^2 + e^{-x}\sin 3x + x\cos x$

$y^{IV} + 2y''' + y'' = 0$ ЛОДУ

$\lambda^4 + 2\lambda^3 + \lambda^2 = 0$ характеристическое ур-е ЛОДУ

$\lambda_1 = \lambda_2 = 0 \quad \lambda_3 = \lambda_4 = -1$

<u>$y_{oo} = C_1 + C_2 x + C_3 e^{-x} + C_4 x e^{-x}$ общее реш.</u>

1) $f_1(x) = e^{-x} + xe^{-x} = (1+x)e^{-x}$

$\alpha = -1; \ \beta = 0 \Rightarrow \alpha \pm \beta = -1$ корень хар. ур-я

$P_k(x) = 1 + x \Rightarrow k = 1$

$\underline{y_{1чн}} = x^2 e^{-x}(A_1 x + A_2)$

2) $f_2(x) = 1 - x^2$

$\alpha = 0, \ \beta = 0 \Rightarrow \alpha \pm \beta i = 0$ корень хар. ур.

$P_k(x) = 1 - x^2 \Rightarrow k = 2$

$\underline{y_{2чн}} = x^2(A_3 x^2 + A_4 x + A_5)$

3) $f_3(x) = e^{-x}\sin 3x$

$\alpha = -1; \ \beta = 3 \Rightarrow \alpha \pm \beta i = 1 \pm 3i$ не явл. корнем хар. ур-я

$P_k(x) = 1 \Rightarrow k = 0$

$\underline{y_{3чн}} = e^{-x}(A_6\cos 3x + A_7 \sin 3x)$

4) $f_4(x) = x\cos x$

$\alpha = 0,\ \beta = 1 \Rightarrow \alpha \pm \beta i = \pm i$ не явл. корнем х. ур.

$P_k(x) = x \Rightarrow k = 1$

$\underline{y_{4чн}} = (A_8 x + A_9)\cos x + (A_{10}x + A_{11})\sin x$

$\underline{y_{чн}} = y_{1чн} + y_{2чн} + y_{3чн} + y_{4чн} =$

$= x^2 e^{-x}(A_1 x + A_2) + x^2(A_3 x^2 + A_4 x + A_5) +$

$+ e^{-x}(A_6 \cos 3x + A_7 \sin 3x) + (A_8 x + A_9)\cos x +$

$+ (A_{10}x + A_{11})\sin x$

№4.

$y'' - 3y' + 2y = e^{2x} + 2x - 5 \quad y(0) = 0,\ y'(0) = 2$

$y'' - 3y' + 2y = 0$ ЛОДУ

$\lambda^2 - 3\lambda + 2 = 0$ характеристическое ур-е

$\lambda_1 = 1 \quad \lambda_2 = 2$

$y_{оо} = C_1 e^x + C_2 e^{2x}$

1) $f_1(x) = e^{2x}$

$\alpha = 2,\ \beta = 0 \Rightarrow \alpha \pm \beta i = 2$ - корень хар. ур-я

$y_{1чн} = Axe^{2x}$

$y'_{1чн} = Ae^{2x} + 2Axe^{2x}$

$y''_{1чн} = 2Ae^{2x} + 2Ae^{2x} + 4Axe^{2x} = 4Ae^{2x} + 4Axe^{2x}$

$4Ae^{2x} + \cancel{4Axe^{2x}} - 3Ae^{2x} - \cancel{6Axe^{2x}} + \cancel{2Axe^{2x}} = e^{2x}$

$A = 1 \Rightarrow y_{1чн} = xe^{2x}$

2) $f_2(x) = 2x - 5$

$\alpha = 0, \beta = 0 \Rightarrow \alpha \pm \beta i = 0$ не корень хар. ур-я

$P_k(x) = 2x - 5 \Rightarrow k\,1$

$y_{2чн} = Bx + C$

$y'_{2чн} = B$

$y''_{2чн} = 0$

$-3B + 2Bx + 2C = 2x - 5$

$\begin{cases} -3B + 2C = -5 \\ 2B = 2 \end{cases} \quad \begin{cases} 2C = -2 \\ B = 1 \end{cases} \quad \begin{cases} C = -1 \\ B = 1 \end{cases}$

$y_{2чн} = x - 1$

$y_{он} = y_{оо} + y_{чн} - C_1e^{x} + C_2e^{2x} + xe^{2x} + x - 1$

$y' = C_1e^{x} + 2C_2e^{2x} + e^{2x} + 2xe^{2x} + 1$

$\begin{cases} y(0) = C_1 + C_2 - 1 = 0 \\ y'(0) = C_1 + 2C_2 + 1 + 1 = 2 \end{cases} \quad \begin{cases} C_1 + C_2 = 1 \\ C_1 + 2C_2 = 0 \end{cases} \quad \begin{cases} C_1 = 2 \\ C_2 = -1 \end{cases}$

$y = 2e^{x} - e^{2x} + xe^{2x} + x - 1$

Ответ: $y = 2x + (x-1)e^{2x} + x - 1$

№5.

$2xy'' - (1+4x)y' + (1+2x)y = 3\sqrt{x}e^x, \quad y_1 = e^x$

Замена: $y = e^x p$

$y' = e^x p + e^x p' = (p + p')e^x$

$y'' = (p' + p'')e^x + (p + p')e^x = (p + 2p' + p'')e^x$

$2x(p + 2p' + p'')e^x - (1+4x)(p+p')e^x + (1+2x)e^x p =$

$= 3\sqrt{x}e^x$

$2xp'' - p' = 3\sqrt{x}$

Замена: $p'(x) = z(x), \quad p'' = z'$

ЛНДУ: $2xz' - z = 3\sqrt{x} \quad (*)$

$2xz' - z = 0$ ЛОДУ

$z' = \frac{z}{2x}$

$\frac{dz}{z} = \frac{dx}{2x}$ (учтем реш. $z=0$)

$\ln|z| = \frac{1}{2}\ln|x| + \ln|C| = \ln|C\sqrt{x}|$

$z_0 = C\sqrt{x}$

$z = C(x)\sqrt{x}$

$z' = C'(x)\sqrt{x} + \frac{1}{2}C(x)\frac{1}{\sqrt{x}}$

Подставим в $(*)$:

$2x\left(C'(x)\sqrt{x} + \frac{1}{2}C(x)\frac{1}{\sqrt{x}}\right) - C(x)\sqrt{x} = 3\sqrt{x}$

$2xC'(x)\sqrt{x} = 3\sqrt{x}$

$$c'(x) = \frac{3\cancel{z}}{2x}$$

$$c(x) = \frac{3}{2}\int\frac{dx}{x} = \frac{3}{2}\ln|x| + C_1, \quad \forall C_1$$

$$z = \left(\frac{3}{2}\ln|x| + C_1\right)\sqrt{x}$$

$$p' = z$$

$$p = \int\left(\frac{3}{2}\ln|x| + C_1\right)\sqrt{x}\,dx = \frac{3}{2}\int\ln|x|\sqrt{x}\,dx +$$

$$+ C_1\int\sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}x^{\frac{3}{2}} + C_2 \quad + \frac{2}{3}x^{\frac{3}{2}}$$

Ответ: $y = e^x p = x^{\frac{3}{2}}e^x\left(\ln(x) - \frac{2}{3} + \frac{2}{3}C_1\right) + C_2$