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№1. $y'' = \operatorname{sh}\frac{y'}{x} + \frac{y'}{x}$; $y\left(\frac{1}{2}\right) = \frac{1}{2}$; $y'\left(\frac{1}{2}\right) = \frac{1}{2}\ln 3$

$$\begin{cases} y' = p(x) \\ y'' = p' \end{cases} \qquad p' = \operatorname{sh}\frac{p}{x} + \frac{p}{x}$$

$$\frac{p}{x} = u \Rightarrow p = u \cdot x \qquad p' = u'x + u$$

$$u'x + u = \operatorname{sh} u + u \Rightarrow u'x = \operatorname{sh} u$$

$$\frac{du}{dx} = \frac{\operatorname{sh} u}{x} \qquad \int \frac{du}{\operatorname{sh} u} = \int \frac{dx}{x}$$

$$\ln|e^{-u} - 1| - \ln|e^{-u} + 1| = \ln|x| + c, \forall c$$

$$\ln\left|\frac{e^{-p/x} - 1}{e^{-p/x} + 1}\right| = \ln|x| + c$$

$$\frac{e^{-p/x} - 1}{e^{-p/x} + 1} = x + c_1, \forall c_1$$

$$\frac{e^{-\ln 3} - 1}{e^{-\ln 3} + 1} = \frac{1}{2} + c_1$$

$$-\frac{1}{2} = \frac{1}{2} + c_1 \Rightarrow c_1 = -1.$$

$$e^{-u} - 1 = e^{-u}x + e^{-u} + x + 1$$

$$e^{-u}x = -x - 2$$

$$e^{-u} = -\frac{x+2}{x}$$

$$-u = \ln -\frac{x+2}{x} \qquad u = \ln\frac{-x}{x+2} \qquad y' = x\ln\frac{-x}{x+2}$$

$$\int dy = \int x \ln \frac{-x}{x+2}\, dx = \left[u = x \right.$$

$$= 2\ln|x+2| + \frac{x^2 \ln\left(-\frac{x}{x+2}\right)}{2} - x - 2 + C_2, \forall C_2$$

$$= 2\ln|x+2| + \frac{x^2 \ln\left(-\frac{x}{x+2}\right)}{2} - x + C_2, \forall C_2$$

$$y\left(\frac{1}{2}\right) = \frac{1}{2} \Rightarrow \frac{1}{2} = 2\ln\frac{5}{2} + \frac{\ln -\frac{1}{5}}{8} \frac{-1}{+2} + C_2$$

$$C_2 = 1 - 2\ln\frac{5}{2} - \frac{\ln -\frac{1}{5}}{8} = 1 - 2\ln\frac{5}{2}\left(\frac{1}{5}\right)^{1/8} =$$

$$= 1 - \cancel{2}\ln\left(\frac{25}{4}\cdot\left(\frac{1}{5}\right)^{1/4}\right).$$

№2. $y'' + (y')^2 \operatorname{tg} y = 0$

$$\begin{cases} y' = p(y) \\ y'' = p' \cdot p \end{cases} \qquad p' \cdot p + p^2 \operatorname{tg} y = 0 \qquad p \neq 0$$

$$p' + p \operatorname{tg} y = 0$$

$$\frac{dp}{dy} = -p \operatorname{tg} y \qquad \int \frac{dp}{p} = -\int \operatorname{tg} y\, dy$$

$$\ln|p| = \ln|\cos y| + c, \forall c$$

$$p = \cos y \cdot C_2, \forall C_2 \qquad y' = \cos y \cdot C_2$$

$$\frac{dy}{dx} = \cos y \cdot C_2 \qquad \frac{dy}{\cos y} = C_2\, dx$$

$$\ln\left|\operatorname{tg}\left(\frac{\pi}{4} + \frac{y}{2}\right)\right| = C_2 x + C_3, \forall C_2 \forall C_3$$

Ответ: $\ln\left|\operatorname{tg}\left(\frac{\pi}{4} + \frac{y}{2}\right)\right| - C_2 x - C_3 = 0.$

№4. $y'' - 7y' + 10y = -3e^{2x} - 10x + 17$; $y(0) = 4$; $y'(0) = 0$.

$k^2 - 7k + 10 = 0$ $\qquad k_1 = 2$; $k_2 = 5$.

$ФСР := \{e^{2x}; e^{5x}\}$

$y_{oo} = C_1 e^{2x} + C_2 e^{5x}$

$f = -3e^{2x} - 10x + 17$

$f_1 = -3e^{2x} \Rightarrow \alpha = 2 = k_1 \Rightarrow r = 1; n = 0$

$y_{ч1} = Ae^{2x} \cdot x$

$f_2 = -10x + 17 = (17 - 10x)e^{0x} \Rightarrow \alpha = 0 \Rightarrow r = 0$

$n = 1$

$y_{ч2} = (Bx + C)e^{0x} \cdot x^0 = Bx + C$

$y_{чн} = y_{ч1} + y_{ч2} = Ae^{2x} \cdot x + Bx + C$

$y'_{чн} = Ae^{2x} + 2Ae^{2x}x + B$

$y''_{чн} = 2Ae^{2x} + 2Ae^{2x} + 4Ae^{2x}x$

$2Ae^{2x} + 2Ae^{2x} + 4Ae^{2x}x - 7Ae^{2x} - 14Ae^{2x}x - 7B$
$+ 10Ae^{2x}x + 10Bx + C = -3e^{2x} - 10x + 17$

$-3Ae^{2x} - 7B + 10Bx + C = -3e^{2x} - 10x + 17$

$\begin{cases} A = 1 \\ B = -1 \\ C = 10 \end{cases}$ $\qquad y_{чн} = e^{2x} \cdot x - x + 10$

$y_{он} = y_{oo} + y_{чн}$

$y_{он} = C_1 e^{2x} + C_2 e^{5x} + xe^{2x} - x + 10$

$y(0)=4;\ y'(0)=0$

при $x=0$, $y_{чн}=0 \Rightarrow \quad C_1+10=4$

$C_1=-6$

$y'_{он}=2C_1e^{2x}+5C_2e^{5x}+e^{2x}+2xe^{2x}-1$

при $x=0$, $y'_{он}=-1$

$$2\cdot -6+5C_2+1-\cancel{1}=-\cancel{1}$$

$$C_2=\frac{12-1}{5}=2{,}2$$

Ответ: $y_{чн}=-6e^{2x}+2{,}2e^{5x}+xe^{2x}-x+10.$

№3. $y'''-4y''-y'+4y=(3\cos x)e^{4x}+x^3e^x\cos 2x+x^2-5x\sin x\, e^{4x}-$

$-e^x\sin 2x-e^{-x}-4$

Хар-е ур-е: $k^3-4k^2-k+4=0$

$k=1;\ k=-1;\ k=4;\ y_{oo}=C_1e^x+C_2e^{-x}+C_3e^{4x}$

$f_1=e^{4x}(3\cos x-5x\sin x)\Rightarrow \alpha=4,\ n=1,\ r=0$

$y_{чн_1}=e^{4x}((A_1x+B_1)\cos x+(A_2x+B_2)\sin x)x^0$

$f_2=e^{x}(x^3\cos 2x-\sin 2x)\Rightarrow \alpha=1,\ n=3,\ r=0$

$y_{чн_2}=e^x((A_3x^3+B_3x^2+C_3x+D)\cos 2x+$

$+(A_4x^3+B_4x^2+C_4x+D_4)\sin 2x)$

$f_3=x^2-4\Rightarrow \alpha=0,\ n=2,\ r=0$

$y_{чн_3}=e^0(A_5x^2+B_5x+C_5)x^0$

$f_4=-e^{-x}\Rightarrow \alpha=-1;\ n=0;\ r=1$

$y_{чн_4}=A_6e^{-x}\cdot x$

$y_{он}=y_{oo}+y_{чн_1}+y_{чн_2}+y_{чн_3}+y_{чн_4}.$

№5. $(x^2-1)y'' + (3x-1)y' + y = \frac{2}{x-1}$; $y_1 = \frac{1}{1+x}$

$$y'' + \frac{3x-1}{x^2-1}y' + \frac{y}{x^2-1} = \frac{2}{(x^2-1)(x-1)}$$

$$y_1 = \frac{1}{1+x} \; ; \; y_2 = y_1 \int \frac{e^{-\int P(x)\,dx}}{y_1^2}\,dx$$

$$e^{-\int P(x)\,dx} = e^{-\int \frac{3x-1}{x^2-1}\,dx} \circledequal$$

$$\frac{3x-1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{Ax+A+Bx-B}{x^2-1}$$

$$\begin{cases} 3 = A+B \\ -1 = A-B \end{cases} \Rightarrow \begin{matrix} A=1 \\ B=2 \end{matrix} \quad \circledequal \; e^{-\left(\int \frac{dx}{x-1} + \int \frac{2dx}{x+1}\right)} =$$

$$= e^{-\left(\ln|x-1| + \ln(x+1)^2\right)} = \frac{1}{|x-1|(x+1)^2}$$

$$y_2 = \frac{1}{1+x}\int \frac{(1+x)^2}{(x-1)(1+x^2)}\,dx = \frac{\ln|x-1|}{x+1}$$

$$y_{oo} = C_1 y_1 + C_2 y_2$$

$$\begin{cases} \frac{C_1'}{x+1} + C_2' \cdot \frac{\ln(x-1)}{x+1} = 0 \quad \Big| \cdot \frac{1}{x+1} \quad "+" \\ \frac{-C_1'}{(x+1)^2} + C_2' \cdot \frac{\left(\frac{x+1}{x-1} - \ln|x-1|\right)}{(x+1)^2} = \frac{2}{(x-1)(x^2-1)} \end{cases}$$

$$\frac{C_2' \ln|x-1|}{(1+x)^2} + C_2' \frac{\left(\frac{x+1}{x-1} - \ln(x-1)\right)}{(x+1)^2} = \frac{2}{(x-1)(x^2-1)}$$

$$C_2'\left(\frac{\cancel{x+1}}{(\cancel{x-1})(\cancel{x+1})^2}\right) = \frac{2}{(x-1)(\cancel{x-1})(\cancel{x+1})} \Rightarrow C_2' = \frac{2}{x-1}$$

$$C_2 = 2\ln|x-1|$$

$$\frac{C_1'}{x+1} + \frac{2\ln(x-1)}{(x+1)(x-1)} = 0 \qquad C_1' = \frac{-2\ln(x-1)}{x-1} \Rightarrow$$

$$C_1 = -\ln^2(x-1)$$

$$y_{он} = C_1 y_1 + C_2 y_2$$