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№1.
$$y' + y\cos x = \frac{1}{2}\sin 2x$$

$$y' = \frac{1}{2}\sin 2x - y\cos x \qquad \text{ЛНДУ}$$

1) $\int \frac{dy}{y} = \int -\cos x\,dx \qquad \ln|y| = -\sin x + c \quad \forall c$

$$y = \pm e^{-\sin x}\cdot e^{c} \quad \forall c; \qquad y = \pm e^{-\sin x} c_1 \quad \forall c_1 > 0$$

$$y = e^{-\sin x} c_2, \quad \forall c_2 \neq 0;$$

$$y = e^{-\sin x} c_3, \quad \forall c_3$$

2) $y_{он} = e^{-\sin x} c_3(x)$

$$\left(e^{-\sin x} c_3(x)\right)' + e^{-\sin x}\cdot c_3(x)\cdot\cos x = \frac{1}{2}\sin 2x$$

$$-\cos x\, e^{-\sin x} c_3(x) + e^{-\sin x} c_3'(x) + e^{-\sin x} c_3(x)\cos x = \frac{1}{2}\sin 2x$$

$$c_3'(x) = \frac{\sin 2x}{2\cdot e^{-\sin x}} \qquad c_3(x) = \int \frac{\sin x\, d(-\sin x)}{e^{-\sin x}} =$$

$$= \int -\sin x\, d(e^{\sin x}) = -\sin x\, e^{\sin x} - \int e^{\sin x}\, d(-\sin x) =$$

$$= -\sin x\, e^{\sin x} + e^{\sin x} = e^{\sin x}(1 - \sin x) + \tilde{c}, \quad \forall \tilde{c}$$

$$y_{он} = e^{-\sin x}\left(e^{\sin x}(1 - \sin x) + \tilde{c}\right)$$

Ответ: $e^{-\sin x}\left(e^{\sin x}(1 - \sin x) + \tilde{c}\right)$.

№2 $(1+y^2)(e^{2x}dx - e^y dy) - (1+y)dy = 0$ — с разделяющ. перем.

$$(e^{2x} + y^2 e^{2x})\,dx = (e^y + e^y y^2 + 1 + y)\,dy$$

$$\frac{(e^y + e^y y^2 + 1 + y)}{1+y^2}\,dy = e^{2x}\,dx$$

$$\int e^y\,dy + \int \frac{1+y}{1+y^2}\,dy = \frac{1}{2}\int e^{2x}\,d(2x)$$

$$e^y + \int \frac{dy}{1+y^2} + \frac{1}{2}\int \frac{d(y^2)}{1+y^2} = \frac{1}{2}e^{2x} + c \quad \forall c$$

$$e^y + \operatorname{arctg} y + \frac{1}{2}\ln|1+y^2| = \frac{1}{2}e^{2x} + c$$

$$e^y + \operatorname{arctg} y + \frac{1}{2}\ln|1+y^2| = -\frac{1}{2}e^{2x} + \tilde{c}, \quad \forall \tilde{c}$$

Ответ: $e^y + \operatorname{arctg} y + \frac{1}{2}\ln|1+y^2| + \frac{1}{2}e^{2x} = \tilde{c} \quad \forall \tilde{c}$.

~~№3.~~ $\cancel{\begin{cases} xy' - 2y = xy^3 = 0 \\ y(1) = 1 \end{cases}}$

№3. $\begin{cases} y' + \frac{1}{y} = \frac{y}{2x} \\ y(1) = -1 \end{cases}$

$y' + \frac{1}{y} = \frac{y}{2x}$ — ур-е Бернулли

$y = uv$; $y' = u'v + v'u$

$u'v + v'u + \frac{1}{uv} = \frac{uv}{2x}$ $\qquad v'u + \frac{1}{uv} = \frac{uv}{2x} - u'v$

$\cancel{v'u - \frac{uv}{2x}} \quad \cancel{v'u = \frac{uv}{2x}}$ $\qquad u' - \frac{u}{2x} = 0$; $v' = \frac{1}{u^2 v}$

$\int \frac{du}{u} = \frac{1}{2}\int \frac{dx}{x}$ $\qquad \ln|u| = \frac{1}{2}\ln|x| + c \quad \forall c$

$u = x^{\frac{1}{2}} \cdot C_1 \quad \forall C_1 \neq 0 \quad \Rightarrow \quad u = x^{1/2}$

$\cancel{v' = \frac{1}{(\sqrt{u})^2 \cdot v}} \qquad \cancel{\frac{dv}{du} = \frac{1}{uv}} \qquad \int \frac{dv}{v} = \int \frac{du}{u/v^2}$

$v' = \frac{1}{xv} \qquad \frac{dv}{dx} = \frac{1}{xv} \qquad \int v\,dv = \int \frac{dx}{x}$

$\frac{v^2}{2} = \ln|x| + c \;, \forall c \qquad \cancel{v = \sqrt{2\ln|x|} + c, \forall c}.$

$\cancel{y = uv \;;\; y = \sqrt{2x\ln|x|}}$

$v = \sqrt{2(\ln|x| + c)} \;, \forall c; \quad y = uv;$

$y = \sqrt{2x(\ln|x| + c)}$

$-1 = \sqrt{2(\ln|1| + c)}$

Ответ: $y = \sqrt{2x(\ln|x| + c)}$

№4. $\begin{cases} y' = \frac{y}{x} + \sin\frac{y}{x} \\ y(1) = \frac{\pi}{2} \end{cases}$

$y' = \frac{y}{x} + \sin\frac{y}{x}$ — ОДУ.

$t = \frac{y}{x} \;;\; y = tx \;;\; y' = t'x + t$

$t'x + \cancel{t} = \cancel{t} + \sin(t) \quad \Rightarrow \quad \frac{dt}{dx} x = \sin(t) \quad \cancel{\int dt =}$

$\int \frac{dx}{x} = \int \frac{dt}{\sin t} \qquad \ln|x| + c = \ln\left|tg\left(\frac{t}{2}\right)\right|$

$\ln|x| + c = \ln\left|tg\left(\frac{y}{2x}\right)\right| \;, \forall c.$

№2.

$$xc = tg\frac{y}{2x}, \quad \forall c. \qquad y = 2x\,arctg(xc)$$

$$\frac{\pi}{2} = 2\cdot 1\,arctg(\cancel{x}\cdot c) \qquad \frac{\pi}{2} = 2\cdot arctg(c) \qquad c = 1$$

Ответ: $y = 2x\,arctg(x)$