ИиДУ Репников Ф. ИУ1-21 ДЗ 2

Вариант 13

1. $\begin{cases} y'' + (y')^2 tg\, y = y' \sec y \\ y(0) = \frac{\pi}{3} \\ y'(0) = \frac{\sqrt{3}}{2} \end{cases}$

$y'' + (y')^2 tg\, y = y' \sec y$

Замена $\begin{cases} y' = p(y) \\ y'' = p' \cdot p \end{cases}$

$p'p + p^2 tg\, y = \frac{p}{\cos y} \quad | : p \neq 0$

$p' + p\, tg\, y = \frac{1}{\cos y}$

Замена $p = uv$; $p' = u'v + uv'$

$u'v + uv' + uv\, tg\, y = \frac{1}{\cos y}$

$u'v + u(v' + v\, tg\, y) = \frac{1}{\cos y}$

$v' + v\, tg\, y = 0$

$\frac{dv}{dy} = -v\, tg\, y$

$\int \frac{dv}{v} = -\int tg\, y\, dy$

$\ln|v| = \ln|\cos y|$

$v = \cos y$

$u'v = \frac{1}{\cos y}$

$u' = \frac{1}{v \cos y}$

$u' = \frac{1}{\cos^2 y}$

$u = tg\, y + C_1$

$y' = p$ - ~~ur~~ $= (tg\,y + C_1)\cos y$

т.к $y(0) = \frac{\pi}{3}$; $y'(0) = \frac{\sqrt{3}}{2} \Rightarrow \frac{\sqrt{3}}{2} = (\sqrt{3} + C_1)\cdot\frac{1}{2}$

$C_1 = 0$;

$y' = tg\,y \cdot \cos y = \sin y$

$\frac{dy}{dx} = \sin y$; $\int \frac{dy}{\sin y} = \int dx$; $\ln|tg(\frac{y}{2})| = x + \ln|C_2|$

$tg(\frac{y}{2}) = C_2 e^x$; $y = 2\,arctg(C_2 e^x)$

т.к $y(0) = \frac{\pi}{3} \Rightarrow \frac{\pi}{3} = 2\,arctg(C_2) \Rightarrow C_2 = \frac{\sqrt{3}}{3}$

Ответ: $y = 2\,arctg(\frac{\sqrt{3}}{3}e^x)$

2. $x^2 y'' + 2(x^2 + (y')^2)\,arcctg\frac{y'}{x} = xy'$

Замена $y' = p(x)$; $y'' = p'(x)$

$x^2 p' + 2(x^2 + p^2)\,arcctg\frac{p}{x} = x\cdot p$

$p' + 2(1 + (\frac{p}{x})^2)\,arcctg\frac{p}{x} = \frac{p}{x}$

Замена $\frac{p}{x} = u$; $p = ux$; $p' = u'x + u$

$u'x + u + 2(1+u^2)\,arcctg\,u = u$

$u'x = -2(1+u^2)\,arcctg\,u$

$\frac{du}{dx}x = -2(1+u^2)\,arcctg\,u$

$$\int \frac{du}{(1+u^2)\operatorname{arcctg} u} = -2\int \frac{dx}{x}$$

$$-\int \frac{d(\operatorname{arcctg} u)}{\operatorname{arcctg} u} = -2\int \frac{dx}{x}$$

$$\ln|\operatorname{arcctg} u| = 2\ln|x| + \ln|C_1|$$

$$\operatorname{arcctg} u = C_1 x^2$$

$$u = \operatorname{ctg}(C_1 x^2)$$

$$y' = p = ux = x\operatorname{ctg}(C_1 x^2)$$

$$y = \int x\operatorname{ctg}(C_1x^2)\,dx = \frac{1}{2C_1}\int \operatorname{ctg}(C_1x^2)\,d(C_1x^2) =$$

$$= \frac{\ln|\sin(C_1x^2)|}{2C_1} + C_2$$

Ответ: $y = \frac{\ln|\sin(C_1x^2)|}{2C_1} + C_2$

3. $y^{IV} + 4y''' + 5y'' = (\cos x)e^{-2x} + e^{4x}x^2 + x^2 -$
$- x\sin x \cdot e^{-2x} + \cos 2x + 5$

$$k^4 + 4k^3 + 5k^2 = 0$$

$$k^2(k^2 + 4k + 5) = 0$$

$$k_{1,2} = 0;\ k_{3,4} = -2 \pm i$$

$$y_{oo} = \tilde{C}_1 + \tilde{C}_2 x + e^{-2x}(\tilde{C}_3 \sin x + \tilde{C}_4 \cos x)$$

$$f = \cos(x)e^{-2x} + e^{4x}x^2 + x^2 - x\sin x \cdot e^{-2x} + \cos 2x + 5$$

$f_1 = x^2 + 5$; $\alpha = 0$; $r = 2$; $n = 2$

$y_{r_1} = x^2(A_1 x^2 + B_1 x + C_1)$

$f_2 = e^{4x} x^2$; $\alpha = 4$; $r = 0$; $n = 2$

$y_{r_2} = e^x(A_2 x^2 + B_2 x + C_2)$

$f_3 = \cos 2x$; $\beta = 2$; $\alpha = 0$; $r = 0$; $n = 0$

$y_{r_3} = A_3 \sin 2x + B_3 \cos 2x$

$f_4 = (\cos x) e^{-2x} - x \sin x e^{-2x}$; $\alpha = -2$; $\beta = \pm 1$

$r = 1$; $n = 1$

$y_{r_4} = x e^{-2x}((A_4 x + B_4) \sin x + (A_5 x + B_5) \cos x)$

$y_{rH} = x^2(A_1 x^2 + B_1 x + C_1) + e^x(A_2 x^2 + B_2 x + C_2) +$
$+ A_3 \sin 2x + B_3 \cos 2x + x e^{-2x}((A_4 x + B_4) \sin x +$
$+ (A_5 x + B_5) \cos x)$

4 $\begin{cases} y'' - 6y' + 5y = 11 - 5x - 8x^x \\ y(0) = 5 \\ y'(0) = 1 \end{cases}$

$k^2 - 6k + 5 = 0$

$\begin{cases} k_1 = 1 \\ k_2 = 5 \end{cases}$

$y_{oo} = C_1 e^x + C_2 e^{5x}$

$$y_{он} = C_1(x)e^x + C_2(x)e^{5x}$$

$$\begin{cases} C_1'(x)e^x + C_2'(x)e^{5x} = 0 \\ C_1'(x)e^x + 5C_2'(x)e^{5x} = 11 - 5x - 8e^x \end{cases}$$

$$C_2'(x) = -C_1'(x)e^{-4x}$$

$$C_1'(x)e^x + 5(-C_1'(x)e^{-4x})e^{5x} = 11 - 5x - 8e^x$$

$$-4C_1'(x)e^x = 11 - 5x - 8e^x$$

$$C_1'(x) = \frac{11 - 5x - 8e^x}{-4e^x} = 2 + \frac{5}{4}xe^{-x} - \frac{11}{4}e^{-x}$$

$$C_1(x) = \int\left(2 + \frac{5}{4}xe^{-x} - \frac{11}{4}e^{-x}\right)dx =$$

$$= 2x + \frac{11}{4}e^{-x} - \frac{5}{4}\int x\,de^{-x} = 2x + \frac{11}{4}e^{-x} - \frac{5}{4}\left(xe^{-x} - \right.$$

$$\left. - \int e^{-x}dx\right) = 2x + \frac{11}{4}e^{-x} - \frac{5}{4}xe^{-x} - \frac{5}{4}e^{-x} + C_1 =$$

$$= 2x + \frac{6}{4}e^{-x} - \frac{5}{4}xe^{-x} + C_1$$

$$C_2'(x) = -e^{-4x}\left(2 + \frac{5}{4}xe^{-x} - \frac{11}{4}e^{-x}\right) = -2e^{-4x} - \frac{5}{4}xe^{-5x} +$$

$$+ \frac{11}{4}e^{-5x} =$$

$$C_2(x) = \int = \frac{1}{2}e^{-4x} - \frac{11}{20}e^{-5x} + \frac{1}{4}\int x\,de^{-5x} =$$

$$= \frac{1}{2}e^{-4x} - \frac{11}{20}e^{-5x} + \frac{1}{4}xe^{-5x} - \frac{1}{20}e^{-5x} + C_2 =$$

$$= \frac{1}{2}e^{-4x} - \frac{1}{2}e^{-5x} + \frac{1}{4}xe^{-5x} + C_2.$$

$$y_{oн} = 2xe^x + \frac{6}{4} - \frac{5}{4}x + C_1e^x + \frac{1}{2}e^x - \frac{1}{2} + \frac{1}{4}x + C_2e^{5x} =$$

$$= C_1e^x + C_2e^{5x} + \frac{1}{2}e^x + 2xe^x - x + 1.$$

$$y' = C_1e^x + 5C_2e^{5x} + \frac{1}{2}e^x + 2e^x + 2xe^x - 1 =$$

$$= C_1e^x + 5C_2e^{5x} + \frac{5}{2}e^x + 2xe^x - 1.$$

$$\begin{cases} y(0) = C_1 + C_2 + \frac{3}{2} = 5 \\ y'(0) = C_1 + 5C_2 + \frac{3}{2} = 1 \end{cases}$$

$$\begin{cases} C_1 = \frac{7}{2} - C_2 \\ \frac{7}{2} - C_2 + 5C_2 + \frac{3}{2} = 1 \end{cases}$$

$$4C_2 = 1 - \frac{3}{2} - \frac{7}{2}; \quad \begin{cases} C_2 = -1 \\ C_1 = \frac{9}{2} \end{cases}$$

$$y = \frac{9}{2}e^x - e^{5x} + \frac{1}{2}e^x + 2xe^x - x + 1$$

$$y = 5e^x - e^{5x} + 2xe^x - x + 1$$

Ответ: $y = 5e^x - e^{5x} + 2xe^x - x + 1.$

$$5\ \begin{cases} y'' + (tg\,x - 2)y' + (1 - tg\,x)y = 2e^x\cos^2 x \\ y_1 = e^x \end{cases}$$

$$y'' + (tg\,x - 2)y' + (1 - tg\,x)y = 0$$

$$y_2(x) = y_1(x)\int y_1^{-2}\cdot e^{-\int a_1(x)dx}dx =$$

$$= e^x\int \frac{1}{e^{2x}} e^{-\int(tg\,x - 2)dx}dx = e^x\int\frac{1}{e^{2x}}e^{2x + \ln|\cos x|}dx =$$

$$= e^x\int \cos x\,dx = e^x\sin x$$

$$y_{oo} = C_1e^x + C_2e^x\sin x$$

$$y_{он} = C_1(x)e^x + C_2(x)e^x\sin x$$

$$\begin{cases} C_1'(x)e^x + C_2'(x)e^x\sin x = 0 \\ C_1'(x)e^x + C_2'(x)(e^x\sin x + e^x\cos x) = 2e^x\cos^2 x \end{cases}$$

$$C_1'(x) = -C_2'(x)\sin x$$

$$-C_2'\sin x\cdot e^x + C_2'(x)(e^x\sin x + e^x\cos x) = 2e^x\cos^2 x$$

$$C_2'e^x\cos x = 2e^x\cos^2 x;\quad C_2' = 2\cos x$$

$$C_2 = 2\sin x + \tilde{C}_2$$

$$C_1' = -2\cos x\sin x = -\sin 2x$$

$$C_1 = \frac{1}{2}\cos 2x + \tilde{C}_1$$

$$y = \left(\frac{1}{2}\cos 2x + \tilde{C}_1\right)e^x + (2\sin x + C_2)e^x\sin x$$ Ответ.