Вариант 13. Решников Д. ИУ1-21.

Задача 1.

$\rho = \sqrt{6}\cos\varphi$

$\rho^2 = 9\cos 2\varphi$

$\rho = \sqrt{6}\cos\varphi$

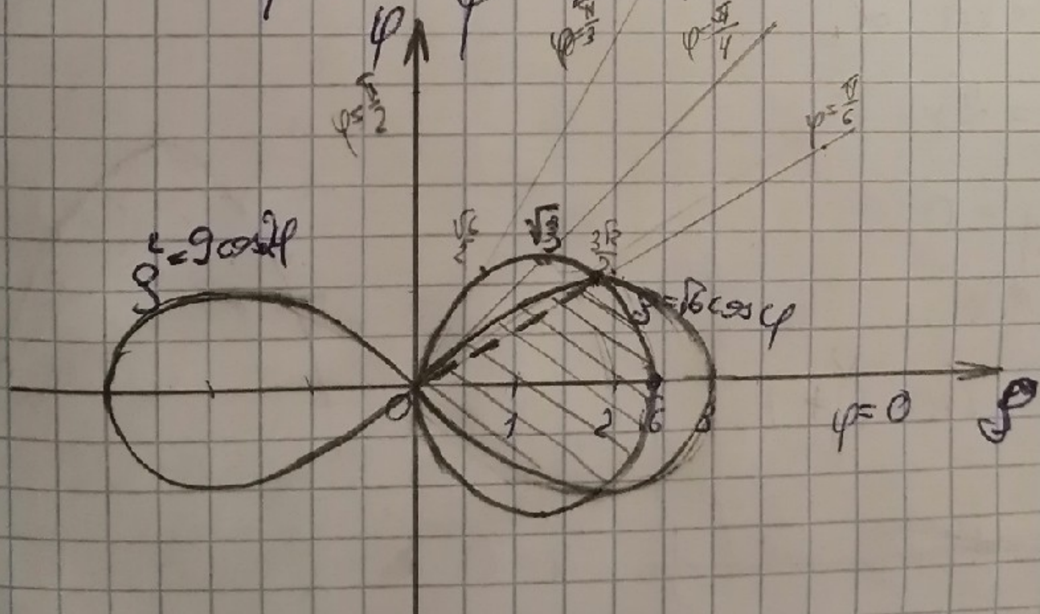
φ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
ρ	$\sqrt{6}$	$\frac{3\sqrt{2}}{2}$	$\sqrt{3}$	$\frac{\sqrt{6}}{2}$	0	$-\frac{\sqrt{6}}{2}$	$-\frac{3\sqrt{2}}{2}$	$-\sqrt{6}$	0	$\sqrt{6}$

$\rho = 3\sqrt{\cos 2\varphi}$; $\cos(2\varphi) \geq 0$

φ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$
ρ	3	$\frac{3\sqrt{2}}{2}$	0

$-\frac{\pi}{2} + 2\pi n \leq 2\varphi \leq \frac{\pi}{2} + 2\pi n,\ n \in \mathbb{Z}$

$-\frac{\pi}{4} + \pi n \leq \varphi \leq \frac{\pi}{4} + \pi n,\ n \in \mathbb{Z}$



$S = \frac{1}{2}\int_{\alpha}^{\beta} \rho^2(\varphi)\,d\varphi$ $\qquad \sqrt{6}\cos\left(\frac{\pi}{6}\right) = \frac{3\sqrt{2}}{2}$; $3\sqrt{\cos\left(2\cdot\frac{\pi}{6}\right)} = \frac{3\sqrt{2}}{2}$.

$S = 2\cdot\frac{1}{2}\left(\int_0^{\frac{\pi}{6}} (\sqrt{6}\cos\varphi)^2\,d\varphi + \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (9\cos(2\varphi)\,d\varphi)\right) =$

(1) $\int_0^{\frac{\pi}{6}} 6\cos^2\varphi\, d\varphi + \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 9\cos(2\varphi)\,d\varphi = \int_0^{\frac{\pi}{6}} 6\cos^2\varphi\, d\varphi + \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (18\cos^2\varphi - 9)\,d\varphi$

$= 6\cdot\int_0^{\frac{\pi}{6}} \frac{1+\cos(2\varphi)}{2}\,d\varphi + 9\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} ((1+\cos(2\varphi))-1)\,d\varphi =$

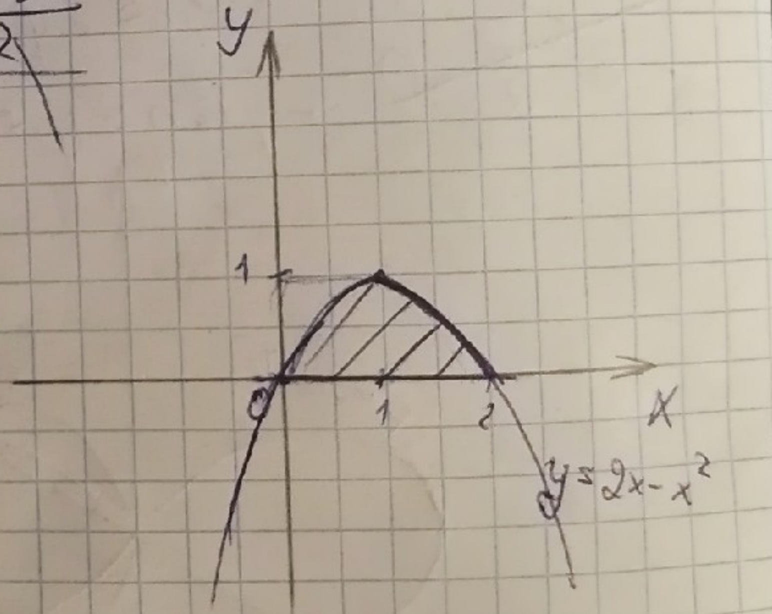
$= 3\left(\varphi + \frac{\sin 2\varphi}{2}\right)\Big|_0^{\frac{\pi}{6}} + 9\left(\frac{\sin 2\varphi}{2}\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}}\right) = 3\left(\frac{\pi}{6} + \frac{\sin\frac{\pi}{3}}{2} - \frac{\sin 0}{2}\right) + 9\left(\frac{\sin\frac{\pi}{2}}{2} - \frac{\sin\pi}{2}\right) = \frac{\pi}{2} + \frac{3\sqrt{3}}{4} + \frac{9}{2} - \frac{9\sqrt{3}}{4} =$

$= \frac{9}{2} + \frac{\pi}{2} - \frac{6\sqrt{3}}{4} = \frac{\pi}{2} - \frac{3\sqrt{3}}{2} + \frac{9}{2} = \frac{\pi}{2} + \frac{9-3\sqrt{3}}{2}$

Ответ: $S = \frac{\pi}{2} + \frac{9-3\sqrt{3}}{2}$

Задача 2.

$\begin{cases} y = 2x - x^2 \\ y = 0 \end{cases}$



$y = -(x-1)^2 + 1$

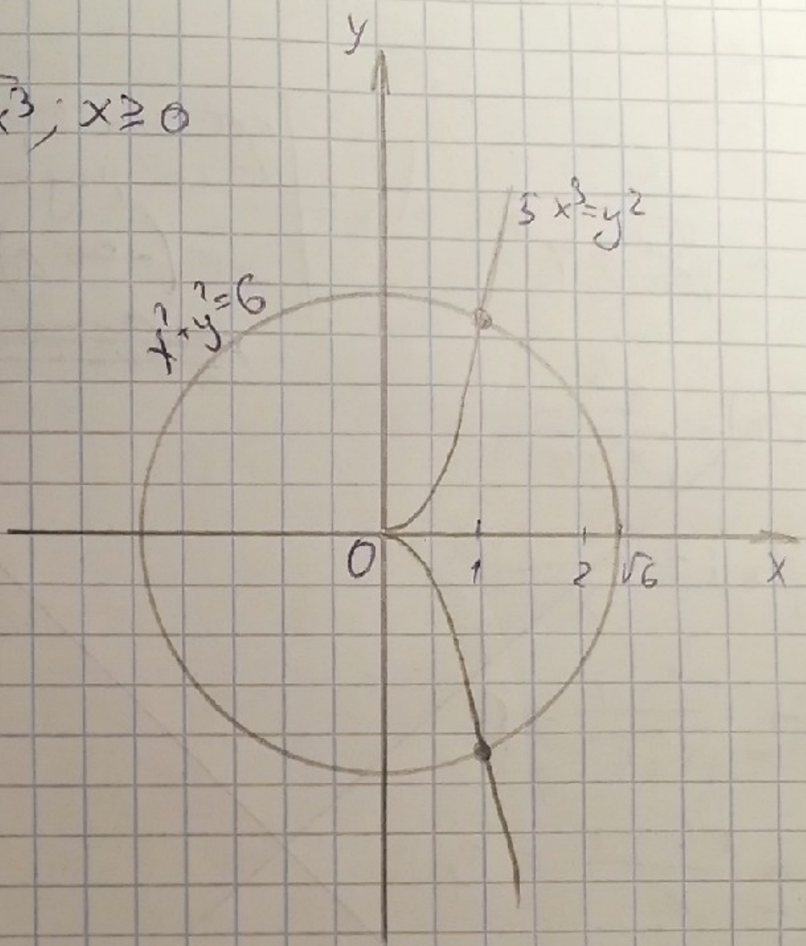
$a = 0;\ b = 2$

$V_{oy} = 2\pi\int_0^2 x\cdot(2x - x^2)\,dx = 2\pi\int_0^2 (2x^2 - x^3)\,dx = 2\pi\left(\frac{2x^3}{3} - \frac{x^4}{4}\right)\Big|_0^2 =$

$= 2\pi\left(\frac{2\cdot 8}{3} - \frac{16}{4}\right) = 2\pi\cdot\frac{4}{3} = \frac{8\pi}{3}$. Ответ: $V_{oy} = \frac{8\pi}{3}$

Задача 3.

$$\begin{cases} 5x^3 = y^2 \\ x^2 + y^2 = 6 \end{cases} ; \quad y = \pm\sqrt{5x^3}; \quad x \geq 0$$



$$x^2 + 5x^3 = 6$$

$$5x^3 + x^2 - 6 = 0$$

$$x = 1$$

$$\begin{array}{l|l} 5x^3 + x^2 - 6 & x - 1 \\ \underline{5x^3 - 5x^2} & \overline{5x^2 + 6x + 6} \\ \quad 6x^2 - 6 & \\ \quad \underline{6x^2 - 6x} & \\ \qquad 6x - 6 & \\ \qquad \underline{6x - 6} & \\ \qquad\quad 0 & \end{array}$$

$$(x-1)(\overset{>0}{5x^2 + 6x + 6}) = 0$$

$$\underline{x = 1}$$

$$y = \sqrt{5x^3}$$

$$y' = \frac{3\sqrt{5}}{2} \cdot x^{\frac{1}{2}}$$

$$(y')^2 = \frac{45}{4} x$$

$$l = 2 \cdot \int_0^1 \sqrt{1 + \frac{45}{4} x}\, dx = \frac{8}{45} \left(1 + \frac{45}{4} x\right)^{\frac{3}{2}} \cdot \frac{2}{3} \Big|_0^1 =$$

$$= \frac{16}{135} \left(\frac{7}{2}\right)^3 = \frac{686}{135} = 5\frac{11}{135}$$

Ответ: $l = 5\frac{11}{135}$

Задача 4.

$$\int_2^{+\infty} \frac{e^{3/x}-1}{\sqrt{x^2+4}}\,dx = \left|\begin{array}{l}\text{нес. интегр.}\\ \text{1-го рода}\\ x\in(-\infty;0)\cup(0;+\infty)\end{array}\right|$$

$$f(x)=\frac{e^{3/x}-1}{\sqrt{x^2+4}}$$

$$g(x)=\frac{e^{3/x}-1}{x}$$

$$\lim_{x\to+\infty}\frac{e^{3/x}-1}{\sqrt{x^2+4}}=0$$

$$\lim_{x\to+\infty}\frac{e^{3/x}-1}{x}=0$$

$f(x)$ и $g(x)$ бмф при $x\to+\infty$

$$\lim_{x\to+\infty}\frac{f(x)}{g(x)}=\lim_{x\to+\infty}\frac{x}{\sqrt{x^2+4}}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{4}{x^2}}}=1$$

$f(x)\sim g(x)$ при $x\to+\infty$

$$\int_2^{+\infty} g(x)\,dx=\lim_{b\to+\infty}\int_2^b\frac{e^{3/x}-1}{x}\,dx$$

$$\int_2^{+\infty}\frac{e^{3/x}-1}{\sqrt{x^2+4}}\,dx=\int_2^{+\infty}\frac{e^{3/x}}{\sqrt{x^2+4}}-\int_2^{+\infty}\frac{dx}{\sqrt{x^2+4}}$$

$$f(x)=\frac{e^{3/x}-1}{\sqrt{x^2+4}};\quad g(x)=\frac{-1}{\sqrt{x^2+4}}$$

$f(x)>g(x)$ при $x\in\mathbb{R}\setminus\{0\}$

$$\lim_{x\to+\infty}\frac{e^{3/x}-1}{\sqrt{x^2+4}}=0;\quad \lim_{x\to+\infty}\frac{-1}{\sqrt{x^2+4}}=0.$$

$$\int_2^{+\infty}\frac{dx}{\sqrt{x^2+4}} = \ln\left|x+\sqrt{x^2+4}\right|\Big|_2^{+\infty} = \lim_{x\to+\infty}\ln\left|x+\sqrt{x^2+4}\right| + \ln\left|\sqrt{8}-2\right|$$

расходится

значит и $\int_2^{+\infty}\frac{e^{3/x}-1}{\sqrt{x^2+4}}dx$ расходится

Задача 5

$$\int_0^{\pi/4}\frac{\sin x}{x\sqrt{x}}dx = \left|\begin{array}{l}\text{нес. инт.}\\ \text{2-го рода}\\ x\in(0;+\infty)\end{array}\right|$$

$$\cancel{|f(x)| = \left|\frac{\sin x}{x\sqrt{x}}\right| = \frac{|\sin x|}{x\sqrt{x}} \le \frac{1}{x\sqrt{x}} - g(x)}$$

$$\cancel{\int_0^{\pi/4} g(x)dx = \int_0^{\pi/4}\frac{dx}{x\sqrt{x}} = \lim_{\varepsilon\to0}\int_0^{\frac{\pi}{4}-\varepsilon}\frac{dx}{x\sqrt{x}} = \lim_{\varepsilon\to0}\left(-\frac{2}{\sqrt{x}}\right)\Big|_0^{\frac{\pi}{4}-\varepsilon}}$$

$$= -2\lim_{\varepsilon\to0}\left(\frac{1}{\sqrt{\frac{\pi}{4}-\varepsilon}}\right.$$

$$\int_0^{\pi/4}\frac{\sin x}{x\sqrt{x}}dx$$

$$|f(x)| = \left|\frac{\sin x}{x\sqrt{x}}\right| = \frac{|\sin x|}{x\sqrt{x}} \le \frac{1}{x\sqrt{x}} - \text{сходится}$$

значит и $\int_0^{\pi/4}\frac{\sin x}{x\sqrt{x}}dx$ сходится.

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$x\ne0$