Федоренко Д. Вариант 15.

1

1. $xy'' = 2x(1 - e^{-(y'/x)}) + y'$

$y'' = 2(1 - e^{-(y'/x)}) + \frac{y'}{x}$

$y' = p(x)$.

$\cancel{p' = 2(1 - e^{-(p'/x)}) + \frac{p}{x}}$ $p' = 2(1 - e^{-\frac{p}{x}}) + \frac{p}{x}$

$u = \frac{p}{x}$; $p = ux$; $p' = u'x + u$

$u'x + u = 2 - 2e^{-u} + u$

$u'x = 2(1 - e^{-u})$

$\frac{du}{2(1 - e^{-u})} = \frac{dx}{x}$

$\int \frac{du}{2(1 - e^{-u})} = \int \frac{dx}{x}$

$\frac{\ln|1 - e^{-u}|}{2} = \ln|x| + C$, $\forall C$

$\ln|-1 + e^{+u}| = \ln x^2 + C_1$ $\forall C$

$|-1 + e^{+u}| = x^2 C_1$, $\forall C_1 > 0$

$-1 + e^{+u} = x^2 C_1$, $\forall C_1 \neq 0$

$e^u = x^2 C_1 + 1$ $\forall C_1 \neq 0$

$u = \ln|x^2 C_1 + 1|$, $\forall C_1 \neq 0$

$p = ux = x \ln|x^2 C_1 + 1|$

$y' = x \ln|x^2 C_1 + 1|$, $\forall C_1$

$y = \int x \ln|x^2 C_1 + 1| \, dx = \left| \begin{array}{l} u = x^2 C_2 + 1 \\ x = \sqrt{\frac{u-1}{C_2}} \\ dx = \frac{1}{\sqrt{C_2}} d\sqrt{u-1} = \frac{du}{2\sqrt{u-1} \cdot \sqrt{C_2}} \end{array} \right| = \int \frac{\sqrt{u-1} \ln u}{\sqrt{C_2} \cdot 2\sqrt{C_2} \sqrt{u-1}} du =$

$= \frac{1}{2C_2} \int \ln u \, du = \frac{1}{2C_2}(u \ln u - u) + C_3 = \frac{(x^2 C_2 + 1)\ln(x^2 C_2 + 1) - (x^2 C_2 + 1)}{2C_2} + C_3$, $\forall C_3 =$

$= \frac{(x^2 C_2 + 1)(\ln(x^2 C_2 + 1) - 1)}{2C_2} + C_3$, $\forall C_3$

$y'(1) = \ln 3$

$\ln 3 = 1 \ln(C_1+1)$

$\ln 3 = \ln(C_1+1)$

$3 = C_1 + 1$

$C_1 = 2$

$y(1) = -\frac{1}{2}$

$$-\frac{1}{2} = \frac{(C_2+1)(\ln(C_1+1)-1)}{2C_2} + C_3$$

$$C_1 = 2 \Rightarrow \quad -\frac{1}{2} = \frac{3(\ln 3 - 1)}{4} + C_3$$

$-2 = 3(\ln 3 - 1) + 4C_3$

$-2 = 3\ln 3 - 3 + 4C_3$

$\frac{1 - 3\ln 3}{4} = C_3$

$$y = \frac{(2x^2+1)(\ln(2x^2+1)-1)}{4} + \frac{1-3\ln 3}{4}$$ – Ответ

2. $y''\cos^2 y + (y')^2 \sin 2y = 0$

$$\begin{cases} y' = p(y) \\ y'' = p \cdot p' \end{cases}$$

$p \cdot p' \cos^2 y + p^2 \sin 2y = 0 \quad | : p^2\cos^2 y, \; p \neq 0, \cos^2 y \neq 0$

$$\frac{p'}{p} + \frac{\sin 2y}{\cos^2 y} = 0$$

$$\int \frac{dp}{p} = -\int \frac{2\sin y}{\cos y}\, dy$$

$\ln|p| = +2\ln|\cos y| + C, \; \forall C$

$|p| = C_1 |\cos y|^2, \; \forall C_1 > 0$

$p = C_2\cos^2 y, \; \forall C_2 \neq 0$

Пот. решение: $p = 0, y' = 0, y'' = 0 \Rightarrow 0 = 0$ - верно $y = \tilde{C}, \forall \tilde{C} = const$

$\cos y = 0$

$y = \frac{\pi}{2} + \pi n, n \in Z$

$y' = 0, y'' = 0 \Rightarrow 0 = 0$ - верно

$y' = C_2\cos^2 y$

~~[illegible]~~ $\frac{dy}{C_2\cos^2 y} = dx$

$\frac{tg\, y}{C_2} = x + C_3, \; \forall C_2, C_3$

$y = arctg(C_2(x + C_3)), \; \forall C_2, C_3$

Ответ:
$y = arctg(C_2(x + C_3)), \; \forall C_2, C_3$
$y = \tilde{C}, \; \forall \tilde{C} = const$

$y''' - 5y'' + 9y' - 5y = xe^{2x} + x + x^2 \sin x - e^{2x}\sin x - 6$ (3)

$y''' - 5y'' + 9y' - 5y = 0$ - ЛОДУ

Характерист. ур-ие

$k^3 - 5k^2 + 9k - 5 = 0$

$(k-1)(k^2 - 4k + 5) = 0$

$k_1 = 1 \quad k^2 - 4k + 5 = 0$

$D = 16 - 20 = -4$

$k_{2,3} = \frac{4 \pm 2i}{2} = 2 \pm i$

ФСР ЛОДУ $= \{e^x, e^{2x}\cos x, e^{2x}\sin x\}$

$y_{oo} = C_1 e^x + C_2 e^{2x}\cos x + C_3 e^{2x}\sin x$

Правая часть

$f(x) = xe^{2x} + x + x^2\sin x - e^{2x}\sin x - 6$

$f_1(x) = e^{2x} \cdot x \Rightarrow \alpha = 2,\ n = 1,\ r = 0$

$f_2(x) = x - 6 \Rightarrow \alpha = 0,\ n = 1,\ r = 0$

$f_3(x) = x^2 \sin x \Rightarrow \alpha = 0,\ n = 2,\ \beta = 1,\ r = 0$

$f_4(x) = -e^{2x}\sin x \Rightarrow \alpha = 2,\ \beta = 1,\ n = 0 \quad \alpha \pm \beta i = 2 \pm i = k_2 = k_3 \Rightarrow r = 1$

$y_{1чн} = (A_1x + B_1)e^{2x} \cdot x^0 = e^{2x}(A_1x + B_1)$

$y_{2чн} = e^0 (A_2x + B_2) x^0 = A_2x + B_2$

$y_{3чн} = e^0 \cdot x^0 ((A_3x^2 + B_3x + D_3)\sin x + (A_4x^2 + B_4x + D_4)\cos x) =$

$= (A_3x^2 + B_3x + D_3)\sin x + (A_4x^2 + B_4x + D_4)\cos x$

$y_{4чн} = e^{2x} \cdot x^1 (A_5 \sin x + A_6 \cos x)$

$y_{он} = y_{oo} + y_{чн}$

$y_{он} = C_1e^x + C_2e^{2x}\cos x + C_3e^{2x}\sin x + e^{2x}(A_1x + B_1) + A_2x + B_2 + (A_3x^2 + B_3x + D_3)\sin x$

$+ (A_4x^2 + B_4x + D_4)\cos x + e^{2x} \cdot x(A_5 \sin x + A_6 \cos x)$

$y''-8y'+16y=2e^{4x}+16x+8$

$y''-8y'+16y=0$

$k^2-8k+16=0$

$k_1=k_2=4$

ФСР ЛОДУ $=\{e^{4x},\ xe^{4x}\} \Rightarrow y_{oo}=C_1e^{4x}+C_2xe^{4x}$

$y_{он}=C_1(x)e^{4x}+C_2(x)\cdot x\cdot e^{4x}$

$$\begin{cases} C_1'e^{4x}+C_2'xe^{4x}=0 \\ 4C_1'e^{4x}+C_2'e^{4x}+4C_2'xe^{4x}=2e^{4x}+16x+8 \end{cases}$$

$C_2'e^{4x}=2e^{4x}+16x+8$

$C_2'=2+16xe^{-4x}+8e^{-4x}$

$C_2=2x+\int 16xe^{-4x}dx-2e^{-4x}=e^{-4x}(2xe^{4x}-4x-3)+\tilde{C}_2,\ \forall \tilde{C}_2=const$

$$\int 16xe^{-4x}dx=\left|\begin{matrix}u=x\\ du=dx\\ v=-\frac{e^{-4x}}{4}\end{matrix}\right|=\frac{16\cdot x(-e^{-4x})}{4}-16\int\frac{-e^{-4x}}{4}dx=-4xe^{-4x}+e^{-4x}$$

$C_1'=-C_2'x$

$C_1=-\int(2x+16x^2e^{-4x}+8e^{-4x})dx \ominus$

$$\int 16x^2e^{-4x}=\left|\begin{matrix}u=x^2\\ du=2x\,dx\\ v=\frac{e^{-4x}}{-4}\end{matrix}\right|=\frac{16x^2e^{-4x}}{-4}-16\int\frac{e^{-4x}\cdot 2x\,dx}{-4}=-4x^2e^{-4x}+8\int xe^{-4x}dx=$$

$$=\left|\begin{matrix}u=x\\ du=dx\\ v=\frac{e^{-4x}}{-4}\end{matrix}\right|=-4x^2e^{-4x}+\frac{8\cdot x\cdot e^{-4x}}{-4}-8\int\frac{e^{-4x}dx}{-4}=-4x^2e^{-4x}-2xe^{-4x}+\frac{e^{-4x}}{2}$$

$\ominus\ -x^2+4x^2e^{-4x}+4e^{-4x}x+e^{-4x}+\tilde{C}_1$

$y_{он}=e^{4x}(-x^2+4x^2e^{-4x}+4e^{-4x}\cdot x+e^{-4x}+\tilde{C}_1)+xe^{4x}(e^{-4x}(2xe^{4x}-4x-3)+\tilde{C}_2)$

$y_{он}=-x^2e^{4x}+4x^2+4x+1+e^{4x}\tilde{C}_1+x(2xe^{4x}-4x-3)+xe^{4x}\tilde{C}_2$

$y(0)=3 \Rightarrow 3=C_1+1 \Rightarrow C_1=2$

$y'(0)=12 \Rightarrow 12=4+4C_1-3+C_2 \Rightarrow C_2=3$

$y_{он}=-x^2e^{4x}+4x^2+4x+1+2e^{4x}+2x^2e^{4x}-4x^2-3x+3xe^{4x}$

$y_{он}=x^2e^{4x}+3xe^{4x}+x+2e^{4x}+1=e^{4x}(x^2+3x+2)+x+1$

Ответ: $y_{он}=e^{4x}(x^2+3x+2)+x+1$

5. $y'' - 4y'\operatorname{tg}x + (2\operatorname{tg}^2 x - 1)y = 5\sec^5 x$

$y_1 = \sec x$

$P_1(x) = -4\operatorname{tg}x$

$f(x) = 5\sec^5 x$

$$y_2 = y_1 \cdot \int \frac{e^{-\int P_1(x)dx}}{y_1^2}\,dx$$

$$\int P_1(x)dx = \int -\frac{4\sin x}{\cos x}dx = \int 4\frac{d(\cos x)}{\cos x} = 4\ln|abs x| = \ln\cos^4 x$$

$$e^{-\int P_1(x)dx} = e^{-\ln\cos^4 x} = \cancel{\ln\left(\frac{1}{\cos^4 x}\right)}\ \frac{1}{\cos^4 x} = \sec^4 x$$

$$\cancel{y_2 = \sec x\int\frac{\ln\frac{1}{\cos^4 x}}{\sec^2 x}dx = \sec x\int\frac{-\ln\cos^4 x}{\frac{1}{\cos^2 x}}dx = \pm\sec x\int \ln(\cos x)\cos^2 x\,dx =}$$

$$\cancel{= \sec x\int\frac{2\ln\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x}}dx = 2\sec x\int\frac{\ln\sec^2 x}{\sec^2 x}}dx$$

$$y_2 = \sec\int\frac{\sec^4 x}{\sec^2 x}dx = \sec\int\frac{1}{\cos^2 x}dx = \operatorname{tg}x\sec x$$ – второе частное решение

$$\begin{cases} y_1 = \sec x \\ y_2 = \operatorname{tg}x\sec x \end{cases}$$

$y_{oo} = C_1 y_1(x) + C_2 y_2(x) = C_1\sec x + C_2\operatorname{tg}x\sec x$

$y_{on} = C_1(x)\sec x + C_2(x)\operatorname{tg}x\sec x$

$$\begin{cases} C_1'\sec x + C_2'\operatorname{tg}x\sec x = 0 \quad |\cdot(-\operatorname{tg}x) \\ C_1'\operatorname{tg}x\sec x + C_2'\left(\frac{1+\sin^2 x}{\cos^3 x}\right) = 5\sec^5 x \end{cases} \quad "+"$$

$$C_2' = \frac{5}{\cos^2 x}$$

$$C_2 = 5\operatorname{tg}x + \tilde{C}_2$$

$$\frac{C_1'}{\cos x} + \frac{5\sin x}{\cos^3 x} = 0$$

$$C_1' = -\frac{5\sin x}{\cos^3 x}$$

$$C_1 = \int -\frac{5\sin x}{\cos^3 x}dx = \int\frac{5d(\cos x)}{\cos^3 x} = \frac{5\cos^{-2}x}{-2} + \tilde{C}_1$$

$$y_{on} = \left(-\frac{5\cos^{-2}x}{2} + \tilde{C}_1\right)\sec x + (5\operatorname{tg}x + \tilde{C}_2)\operatorname{tg}x\sec x$$ – ответ