1. $e^y(1+x^2)dy - 2x(1+e^y)dx = 0$ – ур-ие с разделенными переменными

$e^y(1+x^2)dy = 2x(1+e^y)dx \ \Big|:(e^y+1)(1+x^2)$

$$\frac{e^y}{1+e^y}dy = \frac{2x}{x^2+1}dx$$

$$\int\frac{d(e^y+1)}{e^y+1} = \int\frac{d(x^2+1)}{x^2+1}$$

$\ln|e^y+1| = \ln|x^2+1| + C, \ \forall C$

$\ln|e^y+1| = \ln(x^2+1)\cdot C_1, \ \forall C_1>0$

$e^y+1 = C_2(x^2+1), \ \forall C_2>0$

$y = \ln(C_2(x^2+1)-1)$,

Ответ: $y = \ln(C_2(x^2+1)-1)$, $\forall C_2>0$

2. $\frac{dy}{dx} = \frac{y}{x} + \frac{1}{\arcsin\frac{y}{x}}$ – однородное ур-ие

$u=\frac{y}{x}; \ y=ux;$ ОДЗ: $x\neq 0$

$y' = u'x + u$

$u'x + u = u + \frac{1}{\arcsin u}$

$u'x = \frac{1}{\arcsin u}$

$u'\arcsin u = \frac{1}{x}$

$\arcsin u\,du = \frac{dx}{x}$

$\int\arcsin u\,du = \int\frac{dx}{x}$

$$\int\arcsin u\,du = \left|\begin{array}{l} z=\arcsin u \\ dv = du \\ v = u \\ dz = \frac{1}{\sqrt{1-u^2}}du \end{array}\right| =$$

$$= u\arcsin u + \frac{1}{2}\int\frac{d(1-u^2)}{\sqrt{1-u^2}} = u\arcsin u + \sqrt{1-u^2} + C$$

$\ln|x| = u\arcsin u + \sqrt{1-u^2} + C, \ \forall C$

$\ln|x| = \frac{y}{x}\arcsin\frac{y}{x} + \sqrt{1-\left(\frac{y}{x}\right)^2} + C, \ \forall C$

Ответ: $\ln|x| - \frac{y}{x}\arcsin\frac{y}{x} - \sqrt{1-\frac{y^2}{x^2}} = C, \ \forall C = const.$

3. $\begin{cases} dx = (2y + x\,tg y - y^2 tg y)\,dy \\ y(0) = \pi \end{cases}$

$dx = (2y + x\,tg y - y^2 tg y)\,dy$

$x' - x\,tg y = 2y - y^2 tg y$ — ЛНДУ

$x = uv,$

$x' = u'v + v'u$

$u'v + v'u - uv\,tg y = 2y - y^2 tg y$

$v(u' - u\,tg y) = 2y - y^2 tg y - v'u$

$u' = u\,tg y$

$\frac{du}{u} = tg y\,dy, \quad u \neq 0$

$\ln|u| = \ln\left|\frac{1}{\cos y}\right| + C, \ \forall C$

$u = \frac{C_1}{\cos y}, \quad \forall C_1 \neq 0$

$u = \frac{1}{\cos y};$

$2y - y^2 tg y - \frac{v'}{\cos y} = 0$

$(2y - y^2 tg y)\cos y\,dy = dv$

$(2y\cos y - y^2 \sin y)\,dy = dv$

$C + \int 2y\cos y\,dy + y^2\cos y - \int 2y\cos y\,dy = v$

$\int 2y\cos y\,dy = \left| \begin{matrix} z = 2y \\ dz = 2dy \\ dQ = \cos y\,dy \\ Q = \sin y \end{matrix} \right| = 2y\sin y - \int 2\sin y\,dy = 2y\sin y + 2\cos y$

$\int y^2 \sin y\,dy = \left| \begin{matrix} z = y^2 \\ dz = 2y\,dy \\ dQ = \sin y\,dy \\ Q = -\cos y \end{matrix} \right| = -y^2\cos y + \int 2y\cos y\,dy$

$y^2\cos y + C_2 = v, \quad \forall C_2$

$x = uv$

$x = \frac{y^2\cos y + C_2}{\cos y}$

$y(0) = \pi \Rightarrow 0 = \frac{\pi^2\cos\pi + C_2}{\cos\pi}$

$C_2 = \pi^2$

Ответ: $x = y^2 + \frac{\pi^2}{\cos y}$

4. $\begin{cases} y' + y = e^{\frac{x}{2}}\sqrt{y} \\ y(0) = \frac{9}{4} \end{cases}$

$y' + y = e^{\frac{x}{2}}\sqrt{y}$ – ур-ие Бернулли

$y = uv$; $y' = u'v + v'u$

$u'v + v'u + uv = e^{\frac{x}{2}}\sqrt{uv}$

$v(u' + u) = e^{\frac{x}{2}}\sqrt{uv} - v'u$

$u' = -u$

$\frac{du}{dx} = -u$

$\frac{du}{u} = -dx$

$\ln|u| = -x + C$, $\forall C$

$u = e^{-x+C}$, $\forall C$

$u = e^{-x}$

$e^{\frac{x}{2}}\sqrt{e^{-x}v} - v'e^{-x} = 0$

$e^{\frac{x}{2}} \cdot e^{-\frac{x}{2}} \cdot \sqrt{v} = v'e^{-x}$

$\sqrt{v} = e^{-x}\frac{dv}{dx}$

$\frac{dx}{e^{-x}} = \frac{dv}{\sqrt{v}}$

$\int e^{x}dx = \int \frac{dv}{\sqrt{v}}$

$e^{x} + C = 2\sqrt{v}$

$v = \frac{(e^{x}+C)^2}{4}$, $\forall C$

$y = uv$; $y = \frac{e^{-x}(e^{x}+C)^2}{4}$

$y(0) = \frac{9}{4}$; $\frac{9}{4} = \frac{e^{0}(e^{0}+C)^2}{4}$

$C = 2$ (т.к $e^{x} + C \geq 0$) Ответ: $y = \frac{e^{-x}(e^{x}+2)^2}{4}$