Комплект 1

N1

$$\int \frac{dx}{x\ln^2 x} = \int \frac{d(\ln x)}{\ln^2 x} = -\frac{1}{\ln x} + C$$ ✓

N2

$$\int x^2 \sin x\, dx = \left\{\begin{matrix} u = x^2 & \Rightarrow du = 2x\,dx \\ dv = \sin x\,dx & \Rightarrow v = -\cos x \end{matrix}\right\} =$$

$$= -x^2\cos x + 2\int x\cos x\,dx = \left\{\begin{matrix} u = x & du = dx \\ dv = \cos x\,dx & v = \sin x \end{matrix}\right\} =$$

$$= -x^2\cos x + 2\cdot\left(x\sin x - \int \sin x\,dx\right) = -x^2\cos x +$$

$$+ 2x\sin x + 2\cos x + C.$$ ✓

N3

$$\int \sin^2 x\cos^4 x\,dx = \int \frac{1-\cos 2x}{2}\cdot\left(\frac{1+\cos 2x}{2}\right)^2 dx =$$

$$= \frac{1}{8}\int (1-\cos 2x)(1+2\cos 2x + \cos^2 2x)\,dx =$$

$$= \frac{1}{8}\int \left(1 + 2\cos 2x + \cos^2 2x - \cos 2x - 2\cos^2 2x - \cos^3 2x\right)dx =$$

$$= -\frac{1}{8}\int \left(\cos^3 2x + \cos^2 2x - \cos 2x - 1\right)dx =$$

1) $\int \cos^3 2x\, dx = \frac{1}{2}\int \cos^2 2x\, d(\sin 2x) =$

$= \frac{1}{2}\int (1-\sin^2 2x)\, d(\sin 2x) = \frac{1}{2}\sin 2x - \frac{\sin^3 2x}{6}$

2) $\int \cos^2 2x\, dx = \frac{1}{2}\int \cos^2 2x\, d(2x) = \frac{1}{4}\int (1+\cos 4x)\, d2x =$

$= \frac{1}{8}\int (1+\cos 4x)\cdot d(4x) = \frac{x}{2} + \frac{\sin 4x}{8}$

3) $-\int \cos 2x\, dx = -\frac{1}{2}\sin 2x$

4) $-\int dx = -x$

$\ominus\ -\frac{1}{8}\left(\cancel{\frac{\sin 2x}{2}} - \frac{\sin^3 2x}{6} + \frac{x}{2} + \frac{\sin 4x}{8} - \cancel{\frac{\sin 2x}{2}} - x\right) =$

$= \frac{\sin^3 2x}{48} - \frac{\sin 4x}{64} - \frac{x}{16} + C$ √

N4

$$\int \frac{dx}{3-2\sin x+\cos x} = \left\{ \begin{matrix} t = tg\frac{x}{2}, & dx = \frac{2dt}{1+t^2} \\ \sin x = \frac{2t}{1+t^2}, & \cos x = \frac{1-t^2}{1+t^2} \end{matrix} \right\} =$$

$$= \int \frac{2\,dt}{\left(3 - \frac{4t}{1+t^2} + \frac{1-t^2}{1+t^2}\right)(1+t^2)} = \int \frac{2\,dt}{(3+3t^2-4t+1-t^2)} =$$

$$= 2\int \frac{dt}{2t^2-4t+4} = \int \frac{dt}{t^2-2t+2} = \int \frac{dt}{(t-1)^2+1} =$$

$$= \int \frac{d(t-1)}{1+(t-1)^2} = \operatorname{arctg}(t-1) + C = \left\{ t = \operatorname{tg}\frac{x}{2} \right\} =$$

$$= \operatorname{arctg}\left(\operatorname{tg}\frac{x}{2} - 1\right) + C \quad \checkmark$$

N5

$$\int \frac{dx}{x\sqrt{x^2-2x-1}} = \left\{ \begin{array}{l} t = \frac{1}{x} \\ x = \frac{1}{t},\ dx = -\frac{dt}{t^2} \end{array} \right\} =$$

$$= -\int \frac{dt}{\cancel{t^2} \cdot \frac{1}{\cancel{t}} \sqrt{\frac{1}{\cancel{t^2}} - \frac{2t}{\cancel{t^2}} - 1 \cdot \frac{t^2}{\cancel{t^2}}}} = -\int \frac{dt}{\sqrt{1-2t-t^2}} =$$

$$= -\int \frac{dt}{\sqrt{-(t^2+2t-1)}} = \left\{ \begin{array}{l} m = t+1 \quad \big| dm = dt \\ t = m-1 \\ t^2+2t-1 = m^2-2 \end{array} \right\} =$$

$$= -\int \frac{dm}{\sqrt{2-m^2}} = -\arcsin\frac{m}{\sqrt{2}} + C =$$

$$= \left\{ m = t+1 = \frac{1}{x}+1 \right\} = -\arcsin\left(\frac{x+1}{x\sqrt{2}}\right) + C \checkmark$$

N6

$$\int \sqrt{x^2-2x+5}\, dx = \int \sqrt{(x-1)^2+4}\, d(x-1) =$$

$$= \{ t = x-1 \} = \int \sqrt{4+t^2}\, dt = \left\{ \begin{array}{l} u = \sqrt{4+t^2} \\ dv = dt \end{array} \right. \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} du = \frac{t\,dt}{\sqrt{t^2+4}} \\ v = t \end{array} \right\} = t\sqrt{4+t^2} - \int \frac{t^2 \overset{+4-4}{} \, dt}{\sqrt{t^2+4}} =$$

$$= t\sqrt{4+t^2} - \int \sqrt{t^2+4}\, dt + 4\int \frac{dt}{\sqrt{t^2+4}}$$

$$\int \sqrt{t^2+4}\, dt = \frac{t}{2}\sqrt{4+t^2} + 2\ln\left|t + \sqrt{t^2+4}\right| + C =$$

$$= \{ t = x-1 \} = \frac{x-1}{2}\sqrt{x^2-2x+5} + 2\ln\left|x-1+\sqrt{x^2-2x+5}\right| + C$$

$\checkmark$

№7

$$\int \frac{x^2-1}{x^3+x}\,dx \ominus$$

$$\frac{x^2-1}{x^3+x} = \frac{(x-1)(x+1)}{x(x^2+1)}$$

$$x^2-1 = A(x^2+1) + (Bx+C)x$$

$$\begin{cases} 1 = A+B \\ -1 = A \\ 0 = C \end{cases} \Rightarrow \begin{cases} A=-1 \\ B=2 \\ C=0 \end{cases}$$

$$\ominus -\int \frac{dx}{x} + 2\int \frac{x\,dx}{x^2+1} = -\ln|x| + \ln|x^2+1| + C$$ ✓

№8

$$\int \frac{e^{\operatorname{arctg} x} + x\ln(1+x^2) + 1}{1+x^2}\,dx = \int e^{\operatorname{arctg} x}\,d(\operatorname{arctg} x) +$$

$$+ \int \frac{x\ln(1+x^2)}{1+x^2}\,dx + \int \frac{dx}{1+x^2} = e^{\operatorname{arctg} x} +$$

$$+ \frac{\ln^2(1+x)}{4} + \operatorname{arctg} x + C$$ ✓

N9

$$\int \sin\sqrt{x}\,dx = \{\sqrt{x}=u\} = \int \sin u\, du^2 = 2\int u \sin u\, du =$$

$$= \begin{cases} f = u & \to df = du \\ dg = \sin u\, du \to & -\cos u \end{cases} \Rightarrow 2\left(-u\cos u + \int \cos u\, du\right) =$$

$$= -2\sqrt{x}\cos\sqrt{x} + 2\cdot\sin\sqrt{x} + C \quad \checkmark$$

N10

$$\int \frac{x\,dx}{x^3-3x+2} = \int \frac{x\,dx}{(x-1)^2(x+2)} \circledcirc$$

$$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

$$x = A(x^2+x-2) + B(x+2) + C(x^2-2x+1)$$

$$\begin{cases} 0 = A + C \\ 1 = A + B - 2C \\ 0 = -2A + 2B + C \end{cases} \Rightarrow \begin{cases} A = \frac{2}{9} \\ B = \frac{1}{3} \\ C = -\frac{2}{9} \end{cases}$$

$$\circledcirc \frac{2}{9}\int \frac{dx}{x-1} + \frac{1}{3}\int \frac{dx}{(x-1)^2} - \frac{2}{9}\int \frac{dx}{x+2} =$$

$$= \frac{2}{9}\ln|x-1| - \frac{1}{3}\cdot\frac{1}{x-1} - \frac{2}{9}\ln|x+2| + C \quad \checkmark$$

N11

$$\int \frac{dx}{e^x\sqrt{4e^{2x}-1}} = \left\{ \begin{array}{l} u=e^x \\ du=e^x dx \end{array} \right\} = \int \frac{du}{u^2\sqrt{4u^2-1}} =$$

$$= \left\{ v=\frac{1}{u},\ u=\frac{1}{v},\ du=-\frac{dv}{v^2} \right\} = -\int \frac{\cancel{v^2}\,dv}{\cancel{v^2}\sqrt{4\cdot\frac{1}{v^2}-1}} =$$

$$= -\int \frac{v\,dv}{\sqrt{4-v^2}} = \frac{1}{2}\int \frac{d(4-v^2)}{\sqrt{4-v^2}} = \sqrt{4-v^2}+C =$$

$$= \left\{ v=\frac{1}{u}=\frac{1}{e^x} \right\} = \sqrt{4-\frac{1}{e^{2x}}}+C \quad \checkmark$$

N12

$$\int \frac{3x+2}{x^3} e^{-3x}\,dx = \int\left(\frac{3e^{-3x}}{x^2}+\frac{2e^{-3x}}{x^3}\right)dx =$$

$$= \int \frac{3dx}{e^{3x}\cdot x^2} + \int \frac{2dx}{e^{3x}\cdot x^3} \ominus$$

$$\int \frac{2dx}{e^{3x}x^3} = \left\{ \begin{array}{ll} u=\frac{1}{e^{3x}} & du=-\frac{3}{e^{3x}}dx \\ dv=\frac{2dx}{x^3} & v=-\frac{2}{2}\cdot\frac{1}{x^2}=-\frac{1}{x^2} \end{array} \right\} =$$

$$= -\frac{1}{x^2e^{3x}} - \int \frac{3}{x^2e^{3x}}dx$$

$(=) \int \frac{3\cancel{dx}}{\cancel{e^{3x} \cdot x^2}} - \frac{1}{x^2 e^{3x}} - \int \frac{3}{\cancel{x^2 e^{3x}}} dx = -\frac{1}{x^2 e^{3x}} + C$ √

N13

$$\int \frac{(\ln^2 x)\sqrt[4]{1+\ln^3 x}}{x} dx = \int \ln^2 x \sqrt[4]{1+\ln^3 x}\, d\ln x =$$

$$= \frac{1}{3} \int (1+\ln^3 x)^{\frac{1}{4}} d(1+\ln^3 x) =$$

$$= \frac{4}{3} \cdot \frac{(1+\ln^3 x)^{\frac{5}{4}}}{5} + C = \frac{4(1+\ln^3 x)^{\frac{5}{4}}}{15} + C$$ √

N14

$$\int \cos^3 x \sin x \ln(\sin x)\, dx = \int \cos^2 x \sin x \ln(\sin x)\, d\sin x =$$

$$= \int (\sin x - \sin^3 x) \ln(\sin x)\, d\sin x = \{\sin x = t\}$$

$$= \int (t - t^3) \ln t\, dt = \int t \ln t\, dt - \int t^3 \ln t\, dt \; (=)$$

1) $\int t \ln t\, dt = \begin{Bmatrix} u = \ln t & du = \frac{dt}{t} \\ dv = t\,dt & v = \frac{t^2}{2} \end{Bmatrix} =$

$$= \frac{t^2}{2}\ln t - \frac{1}{2}\int t\,dt = \frac{t^2}{2}\ln t - \frac{t^2}{4}$$

$$2)\ \int t^3 \ln t\,dt = \left\{ \begin{matrix} u = \ln t & du = \frac{dt}{t} \\ dv = t^3 dt & v = \frac{t^4}{4} \end{matrix} \right\} =$$

$$= \frac{t^4}{4}\ln t - \frac{1}{4}\int t^3 dt = \frac{t^4}{4}\ln t - \frac{t^4}{16}$$

$$⑤\ \frac{t^4}{16} - \frac{t^4}{4}\ln t + \frac{t^2}{2}\ln t - \frac{t^2}{4} + C =$$

$$= \frac{\sin^4 x}{16} - \frac{\sin^4 x}{4}\ln\sin x + \frac{\sin^2 x}{2}\ln\sin x - \frac{\sin^2 x}{4} + C \checkmark$$

N15

$$\int \frac{dx}{x\sqrt{x^2-3x+2}} = \left\{ t = \frac{1}{x},\ x = \frac{1}{t},\ dx = -\frac{dt}{t^2} \right\} =$$

$$= -\int \frac{dt}{\cancel{t^2}\cdot\frac{1}{\cancel{t}}\sqrt{\frac{1}{\cancel{t^2}} - \frac{3t}{\cancel{t^2}} + \frac{2t^2}{\cancel{t^2}}}} = -\int \frac{dt}{\sqrt{2}\sqrt{t^2 - \frac{3}{2}t + \frac{1}{2}}} =$$

$$= \left\{ \begin{matrix} p = t - \frac{3}{4},\ t = p + \frac{3}{4} \\ t^2 - \frac{3}{2}t + \frac{1}{2} = p^2 - \frac{1}{16} \end{matrix} \right\} = -\frac{1}{\sqrt{2}}\int \frac{dp}{\sqrt{p^2 - \frac{1}{16}}} =$$

$$= -\frac{1}{\sqrt{2}}\cdot \ln\left|p + \sqrt{p^2 - \frac{1}{16}}\right| + C = \left\{ p = t - \frac{3}{4} = \frac{1}{x} - \frac{3}{4} \right\}$$

$$= -\frac{1}{\sqrt{2}}\ln\left|\frac{1}{x}-\frac{3}{4}+\sqrt{\frac{1}{x^2}-\frac{3}{2x}+\frac{1}{2}}\right|+C =$$

$$= -\frac{1}{\sqrt{2}}\ln\left|\frac{4-3x}{4x}+\sqrt{\frac{x^2-3x+2}{x^2}}\right|+C \quad \sqrt{/x}\ ?$$

N16

$$\int\frac{dx}{\sin^2 x-\sin x\cos x-\cos^2 x} =$$

$$= \int\frac{dx}{\cos^2 x\left(\frac{\sin^2 x}{\cos^2 x}-\frac{\sin x}{\cos x}-1\right)} = \int\frac{d\,tg x}{tg^2 x-tg x-1} =$$

$$= \int\frac{d\,tg x}{\left(tg x-\frac{1}{2}\right)^2-\frac{5}{4}} = -\frac{1}{\sqrt{5}}\ln\left|\frac{\frac{\sqrt{5}}{2}+tg x-\frac{1}{2}}{\frac{\sqrt{5}}{2}-tg x+\frac{1}{2}}\right|+C$$

$$= \frac{1}{\sqrt{5}}\ln\left|\frac{2tg x-1-\sqrt{5}}{2tg x-1+\sqrt{5}}\right|+C \quad \sqrt{}$$

N17

$$\int\frac{2x+3}{(2x+1)^4}\,dx =$$

$$2x+3 = A(2x+1)^3 + B(2x+1)^2 + C(2x+1) + D$$

$$\begin{cases} 0 = 8A \\ 0 = 12A + 4B \\ 2 = 6A + 4B + 2C \\ 3 = A + B + C + D \end{cases} \Rightarrow \begin{cases} A = 0 \\ B = 0 \\ C = 1 \\ D = 2 \end{cases}$$

$$\textcircled{=} \int \frac{dx}{(2x+1)^3} + 2\int \frac{dx}{(2x+1)^4} = -\frac{1}{4(2x+1)^2} - \frac{1}{3(2x+1)^3} + C =$$

$$= -\frac{6x+7}{12(2x+1)^3} + C \quad \checkmark$$

N18

$$\int \frac{x+3}{\sqrt{x^2-4x+5}}\,dx = \int \frac{(2x-4)\cdot\frac{1}{2}+5}{\sqrt{x^2-4x+5}}\,dx =$$

$$= \frac{1}{2}\int \frac{2x-4}{\sqrt{x^2-4x+5}}\,dx + 5\int \frac{dx}{\sqrt{x^2-4x+5}} =$$

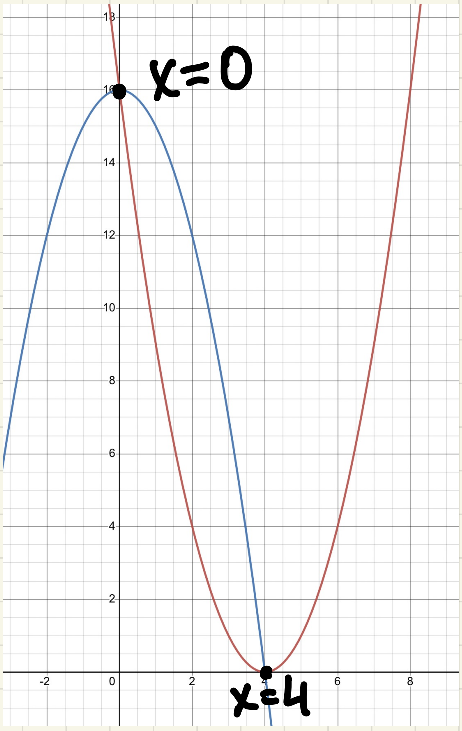
$$= \frac{1}{2}\int \frac{d(x^2-4x+5)}{\sqrt{x^2-4x+5}} + 5\cdot\int\frac{dx}{\sqrt{(x-2)^2+1}} =$$

$$= \sqrt{x^2-4x+5} + 5\ln\left|x-2+\sqrt{(x-2)^2+1}\right| + C \checkmark$$

Комплект 2

№1

Выч-ть $S_{фиг.}$, огр. $y=(x-4)^2$; $y=16-x^2$



$$S_{фиг.} = S_{син.} - S_{красн.} =$$

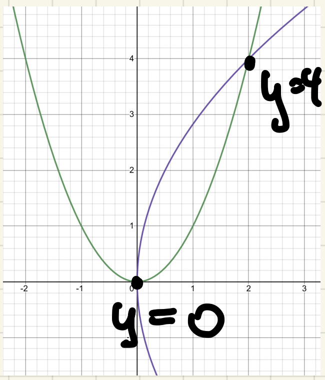
$$= \int_0^4 (\cancel{16}-x^2-x^2+8x-\cancel{16})\,dx =$$

$$= \int_0^4 (8x-2x^2)\,dx = \left(4x^2-\frac{2x^3}{3}\right)\Big|_0^4 =$$

(тут $\Big|_a^b$ очевидны и без рисунка) $= 64-\frac{128}{3} = \frac{64}{3}$ V

№2

Выч-ть $V_{тела}$, обр. вращением вокруг Oy фиг., огр-й $y=x^2$, $y^2=8x$



$x,y\geq 0 \Rightarrow x=\sqrt{y}$, $x=\frac{y^2}{8}$

$$V_{т.} = \pi\int_0^4 y\,dy - \pi\int_0^4 \frac{y^4}{64}\,dy =$$

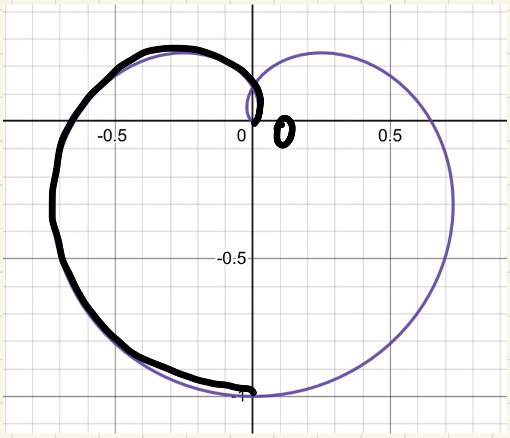
(—||—)

$$= \pi\cdot\frac{y^2}{2}\Big|_0^4 - \pi\cdot\frac{y^5}{64\cdot 5}\Big|_0^4 = 8\pi - \frac{16}{5}\pi = \frac{24}{5}\pi \;✓$$

№3

Выч-ть длину дуги:

$r = a\sin^3\frac{\varphi}{3}, \ \varphi\in(0;3\pi)$



$$l_{дуги} = 2\int_0^{\frac{3\pi}{2}} \sqrt{r^2+(r'_\varphi)^2}\,d\varphi =$$

$$= 2\int_0^{\frac{3\pi}{2}} a\sin^2\frac{\varphi}{3}\,d\varphi =$$

$$= 6a\int_0^{\frac{3\pi}{2}} \sin^2\frac{\varphi}{3}\,d\left(\frac{\varphi}{3}\right) =$$

$$= \frac{6a}{2\cdot 2}\int_0^{\frac{3\pi}{2}} \left(1-\cos\frac{2\varphi}{3}\right)d\left(\frac{2\varphi}{3}\right) =$$

$$= \frac{3a}{2}\left(\frac{2\varphi}{3} - \sin\frac{2\varphi}{3}\right)\Big|_0^{\frac{3\pi}{2}} =$$

$$= \frac{3a}{2}(\pi - 0) = \frac{3\pi a}{2} \;✓$$

Отдельно:

$r^2 = a^2\sin^6\frac{\varphi}{3}$

$r'_\varphi = a\cdot \cancel{3}\sin^2\frac{\varphi}{3}\cdot\cos\frac{\varphi}{3}\cdot\frac{1}{\cancel{3}}$

$(r'_\varphi)^2 = a^2\sin^4\frac{\varphi}{3}\cos^2\frac{\varphi}{3}$

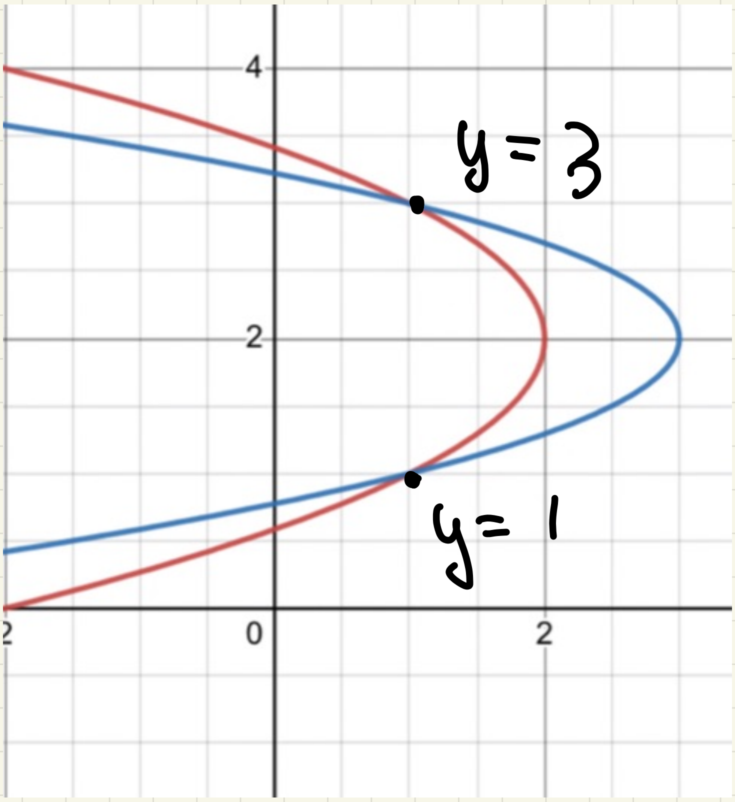
$$\sqrt{a^2\sin^6\frac{\varphi}{3} + a^2\sin^4\frac{\varphi}{3}\cos^2\frac{\varphi}{3}} = a\sin^2\frac{\varphi}{3}\sqrt{\sin^2\frac{\varphi}{3}+\cos^2\frac{\varphi}{3}} =$$

$$= a\sin^2\frac{\varphi}{3}$$

№4

Выч-ть S фиг., огр. $2y^2+x-8y+5=0$,
$y^2+x-4y+2=0$

$$2(y^2-4y+4)-8+x+5=0 \Rightarrow 2(y-2)^2+x=3$$
$$(y^2-4y+4)-4+x+2=0 \Rightarrow (y-2)^2+x=2$$



$$x = 3-2(y-2)^2$$
$$x = 2-(y-2)^2$$

$$S = 2\int_1^2 \left(3-2(y-2)^2-2+(y-2)^2\right)dy =$$

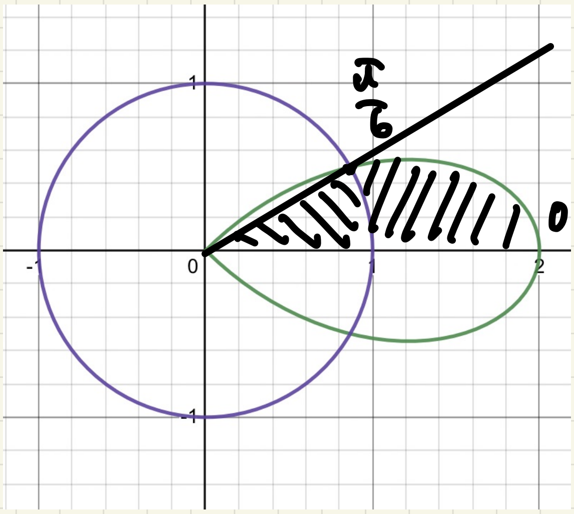
$$= 2\int_1^2 \left(1-(y-2)^2\right)dy =$$

$$= 2\cdot\left(y-\frac{(y-2)^3}{3}\right)\Bigg|_1^2 =$$

$$= 2\cdot\left(2-0-1+\frac{-1}{3}\right) = \frac{4}{3}$$

№5

Выч-ть S фиг., огр-й $r=2a\cos 2\varphi$, $|\varphi| \le \frac{\pi}{4}$ и лежащей вне $r=a$.



$$2a\cos 2\varphi = a$$
$$\cos 2\varphi = \frac{1}{2}$$
$$2\varphi = \frac{\pi}{3}$$
$$\varphi = \frac{\pi}{6}$$

$$S = S_{\text{сег.}} - S_{\text{фиол.}}$$

$$S_1 = \frac{1}{2}\int_0^{\frac{\pi}{6}} 4a^2\cos^2 2\varphi \, d\varphi = 2a^2 \int_0^{\frac{\pi}{6}} \cos^2 2\varphi \, d\varphi =$$

$$= \frac{a^2}{4}\int_0^{\frac{\pi}{6}} (1+\cos 4\varphi)\, d(4\varphi) = \frac{a^2}{4}\left(4\varphi + \sin 4\varphi\right)\Big|_0^{\frac{\pi}{6}} =$$

$$= \frac{a^2}{4}\left(\frac{2\pi}{3} + \frac{\sqrt{3}}{2} - 0\right) = \frac{\pi a^2}{6} + \frac{a^2\sqrt{3}}{8}$$

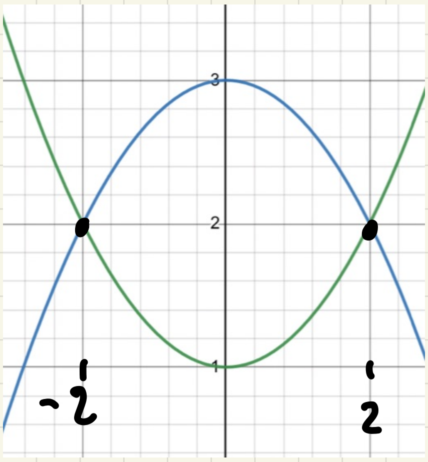
$$S_2 = \frac{1}{2}\int_0^{\frac{\pi}{6}} a^2 d\varphi = \frac{1}{2}a^2(\varphi)\Big|_0^{\frac{\pi}{6}} = \frac{\pi a^2}{12}$$

$$S = 2\left(\frac{\pi a^2}{12} + \frac{a^2\sqrt{3}}{8}\right) = \frac{\pi a^2}{6} + \frac{a^2\sqrt{3}}{4}$$

№6

Выч-ть $V_{тела}$, обр. вр. вокруг Ox фиг., огр.

$y = 3 - x^2$, $y = 1 + x^2$



$$V_{т.} = V_{цил.} - V_{зел.}$$

$$V_{т.} = 2\pi \int_0^1 \left((3-x^2)^2 - (1+x^2)^2 \right) dx =$$

$$= 2\pi \int_0^1 \left(9 - 6x^2 + \cancel{x^4} - 1 - 2x^2 - \cancel{x^4} \right) dx =$$

$$= 2\pi \int_0^1 (8 - 8x^2)\, dx = 16\pi \int_0^1 (1 - x^2)\, dx =$$

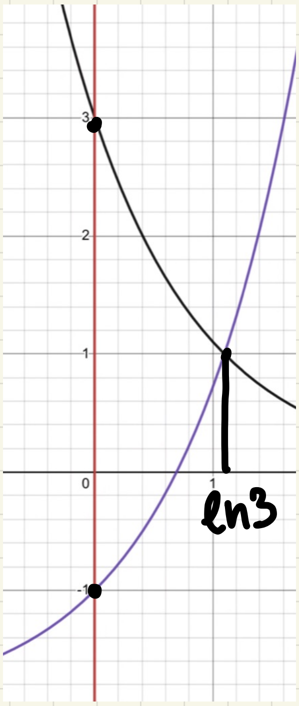
$$= 16\pi \cdot \left(x - \frac{x^3}{3} \right) \Big|_0^1 = 16\pi \cdot \left(1 - \frac{1}{3} \right) =$$

$$= \frac{32\pi}{3}$$

№7

Выч-ть $S_{фиг.}$, огр-й $y = e^x - 2$, $y = 3e^{-x}$, $x = 0$

$$e^x - 2 = \frac{3}{e^x} \Rightarrow x = \ln 3$$



$$S = \int_0^{\ln 3} (3e^{-x} - e^{x} + 2)\,dx =$$

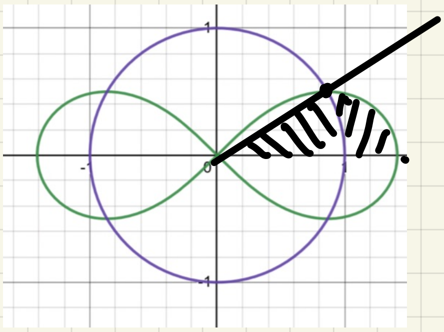
$$= \left(-3e^{-x} - e^{x} + 2x\right)\Big|_0^{\ln 3} =$$

$$= -\frac{3}{3} - 3 + 2\ln 3 - (-3-1) =$$

$$= 2\ln 3 = \ln 9.$$

№8

Выч-ть $S_{фиг.}$, огр. $r=1$ и $r^2 = 2\cos 2\varphi$ (вне ω и внутри лемнискаты)



$$S = 4S_{част.};\quad S_{част.} = S_{Л} - S_{\omega}$$

$$\cos 2\varphi = \frac{1}{2}$$

$$2\varphi = \frac{\pi}{3} \to \varphi = \frac{\pi}{6}$$

$$S_{Л} = \frac{1}{2}\int_0^{\frac{\pi}{6}} 2\cos 2\varphi\, d\varphi = \frac{1}{2}\cdot \sin 2\varphi\Big|_0^{\frac{\pi}{6}} =$$

$$= \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

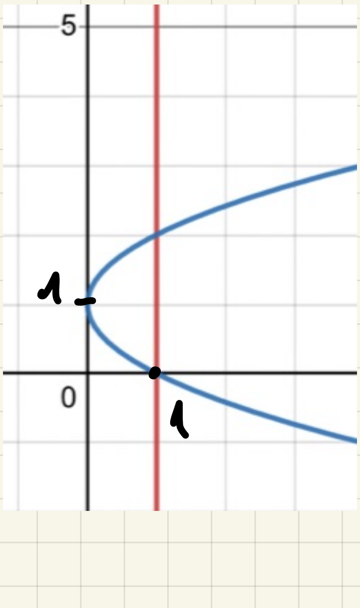
$$S_\omega = \frac{1}{2}\int_0^{\frac{\pi}{6}} d\varphi = \frac{1}{2} \cdot \frac{\pi}{6} = \frac{\pi}{12}$$

$$S_{част} = \frac{\sqrt{3}}{4} - \frac{\pi}{12}$$

$$S = \sqrt{3} - \frac{\pi}{3}$$

№9

Вычислить $V_{т.}$, обр. вр. вокруг Ox фиг., огр.
$x = y^2 - 2y + 1, x = 1$



$$(y-1)^2 = x$$

$$y - 1 = \sqrt{x} \quad (y \geq 1)$$

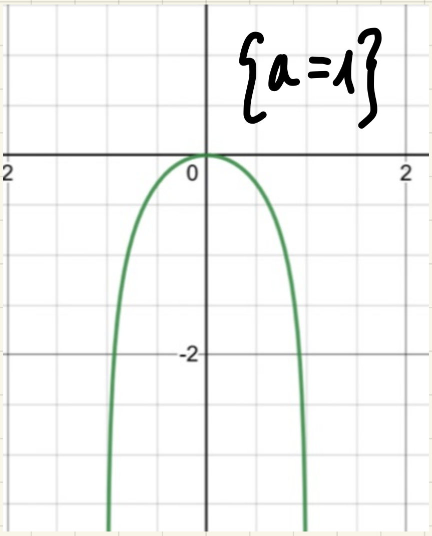
$$y = \sqrt{x} + 1$$

$$V = 2\pi \int_0^1 (\sqrt{x}+1)^2 dx = 2\pi \int_0^1 (x + 2\sqrt{x} + 1) dx =$$

$$= 2\pi \left(\frac{x^2}{2} + \frac{4x\sqrt{x}}{3} + x\right)\Big|_0^1 = \frac{17\pi}{3}$$

№10

Выч-ть $l_{\text{дуги}}$ $y = a\ln(a^2 - x^2)$ от $x_1 = 0$ до $x_2 = a/2$.



$$l = \int_0^{\frac{a}{2}} \sqrt{1 + (y')^2}\, dx$$

$$y' = a \cdot \frac{1}{a^2 - x^2} \cdot (-2x) = -\frac{2ax}{a^2 - x^2}$$

$$y'^2 = \frac{4a^2x^2}{(a^2 - x^2)^2}$$

$$\sqrt{1 + (y'^2)} = \sqrt{\frac{(a^2 - x^2)^2 + 4a^2x^2}{(a^2 - x^2)^2}} = \sqrt{\frac{a^4 + 2a^2x^2 + x^4}{(a^2 - x^2)^2}} =$$

$$= \frac{a^2 + x^2}{a^2 - x^2}, \text{ т.к. } x \le \frac{a}{2}$$

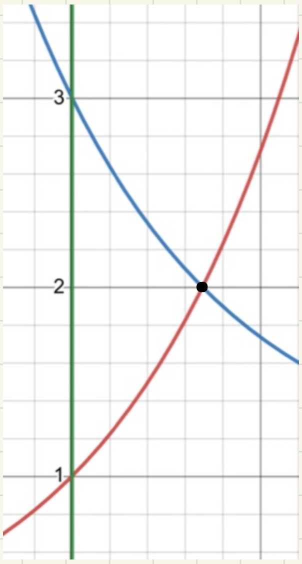
$$l = \int_0^{\frac{a}{2}} \frac{a^2 + x^2}{a^2 - x^2}\, dx = 2a^2 \int_0^{\frac{a}{2}} \frac{dx}{a^2 - x^2} - \int_0^{\frac{a}{2}} dx =$$

$$= 2a^2 \cdot \left(\frac{1}{2a} \ln\left| \frac{a + x}{a - x} \right| \right)\Bigg|_0^{\frac{a}{2}} - x\Big|_0^{\frac{a}{2}} =$$

$$= \cancel{2}a^{\cancel{2}} \left(\frac{1}{\cancel{2a}} \ln 3 \right) - \frac{a}{2} = a\left(\ln 3 - \frac{1}{2}\right) \checkmark$$

№11

Вычислить $V_т$, обр. вращ. вокруг Ox фиг., огр.
$y=e^x$; $y=1+2e^{-x}$, $x=0$



$$e^x = 1+2e^{-x} \Rightarrow x=\ln 2$$

$$V_т = \pi\int_0^{\ln 2}\left((1+2e^{-x})^2 - e^{2x}\right)dx =$$

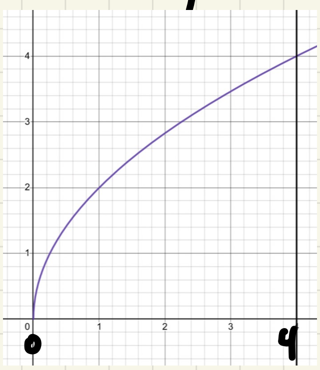
$$= \pi\int_0^{\ln 2}\left(1+4e^{-x}+4e^{-2x}-e^{2x}\right)dx =$$

$$= \pi\left(x-4e^{-x}-2e^{-2x}-\frac{e^{2x}}{2}\right)\Big|_0^{\ln 2} = \pi\cdot\left(\ln 2-\frac{4}{2}-\frac{2}{4}-\frac{4}{2}-0+4+2+\frac{1}{2}\right)=$$

$$= \pi(\ln 2+2)$$ ✓

№12

Вычислить $S_{пов.}$, обр. вращ. вокруг Ox дуги
$y=2\sqrt{x}$, отсеч. $x=4$



$$Q_{ox} = 2\pi\int_0^4 y\sqrt{1+(y')^2}\,dx$$

$$y' = \frac{1}{\sqrt{x}}$$

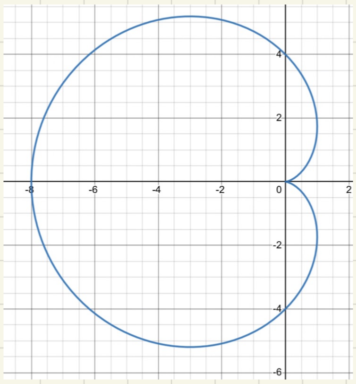
$$y\sqrt{1+(y')^2} = 2\sqrt{x}\sqrt{1+\frac{1}{x}} = 2\sqrt{x+1}$$

$$Q_{0x} = 4\pi\int_0^4 \sqrt{x+1}\,dx = \frac{2\cdot 4\pi\,(x+1)^{\frac{3}{2}}}{3}\Big|_0^4 =$$

$$= \frac{8\pi}{3}\cdot\left(5\sqrt{5}-1\right)$$

№13

Выч-ть длину кардиоиды $r = 4(1-\cos\varphi)$



$$l = 2\int_0^{\pi} \sqrt{r^2+(r')^2}\,d\varphi$$

$$r' = 4\sin\varphi,\quad (r')^2 = 16\sin^2\varphi$$

$$\sqrt{r^2+(r')^2} = \sqrt{16-32\cos\varphi+\underbrace{16\cos^2\varphi+16\sin^2\varphi}_{=16}} =$$

$$= \sqrt{32-32\cos\varphi} = 4\sqrt{2}\sqrt{1-\cos\varphi}$$

$$\frac{1}{2}l = 4\sqrt{2}\int_0^{\pi}\sqrt{1-\cos\varphi}\,d\varphi = 8\int_0^{\pi}\sin\frac{\varphi}{2}\,d\varphi =$$

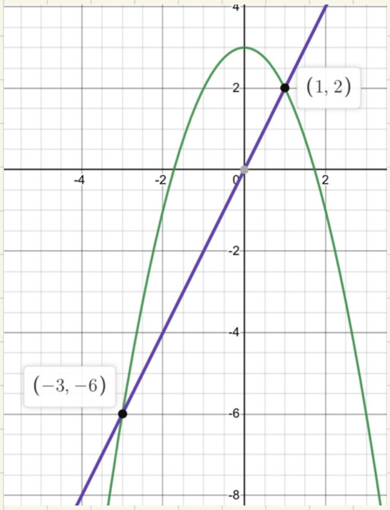
$$= 16 \int_0^{\pi} \sin\frac{\varphi}{2}\, d\frac{\varphi}{2} = 16\cdot\left(-\cos\frac{\varphi}{2}\right)\Big|_0^{\pi} =$$

$$= -16\cdot(0-1) = 16$$

$l = 32$ ✓

N14

Выч-ть $S_{\text{фиг.}}$, огр-й $y = 3-x^2$, $y = 2x$



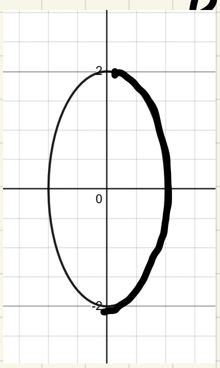
$$S = \int_{-3}^{1} (3-x^2-2x)\,dx = \left(3x-\frac{x^3}{3}-x^2\right)\Big|_{-3}^{1} =$$

$$= 3-\frac{1}{3}-1-(-9+\cancel{9}-9) = 10\frac{2}{3}$$ ✓

N15

Выч-ть V_T, обр. вращ. вокруг Oy фиг., огр-й

$4x^2+y^2=4$



$$\frac{x^2}{1^2}+\frac{y^2}{2^2}=1$$

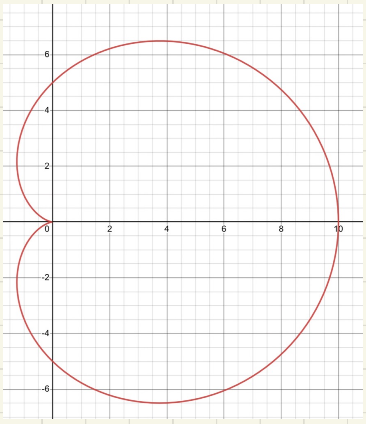
$x = \sqrt{1-\frac{y^2}{4}}$ – нужн. часть

$$V_T = \pi \int_{-2}^{2} \left(1-\frac{y^2}{4}\right)dy = 2\pi\int_{0}^{2}\left(1-\frac{y^2}{4}\right)dy =$$

$$= 2\pi\cdot\left(y-\frac{y^3}{12}\right)\Big|_0^2 = 2\pi\cdot\left(2-\frac{2}{3}\right) = \frac{8\pi}{3}$$

№16

Вычислить длину дуги кривой $r = 5(1+\cos\varphi)$



$$l_{дуги} = 2l = 2\cdot\int_0^{\pi}\sqrt{r^2+(r')^2}\,d\varphi$$

$$r' = -5\sin\varphi\ ;\ (r')^2 = 25\sin^2\varphi$$

$$r^2 = 25+25\cos^2\varphi+50\cos\varphi$$

$$\sqrt{r^2+(r')^2} = \sqrt{25+50\cos\varphi+25} =$$

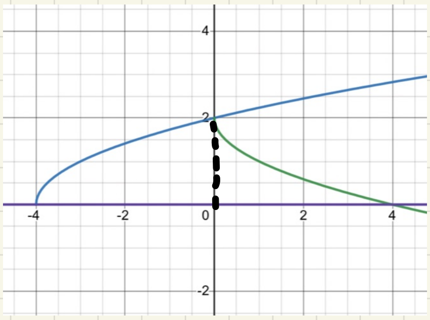
$$= 5\sqrt{2}\,\sqrt{1+\cos\varphi} = 10\cos\frac{\varphi}{2}$$

$$l = 20\int_0^{\pi}\cos\frac{\varphi}{2}\,d\varphi = 40\int_0^{\pi}\cos\frac{\varphi}{2}\,d\frac{\varphi}{2} = 40\cdot\sin\frac{\varphi}{2}\Big|_0^{\pi} =$$

$= 40.$ ✓

№17

Вычислить $S_{фиг.}$, огр-й $y=\sqrt{x+4}$, $y=2-\sqrt{x}$, $y=0$.



$$S_{фиг.} = S_1 + S_2$$

$$S_1 = \int_{-4}^{0} \sqrt{x+4}\,dx = \frac{2}{3}(x+4)^{\frac{3}{2}}\Big|_{-4}^{0} = \frac{16}{3}$$

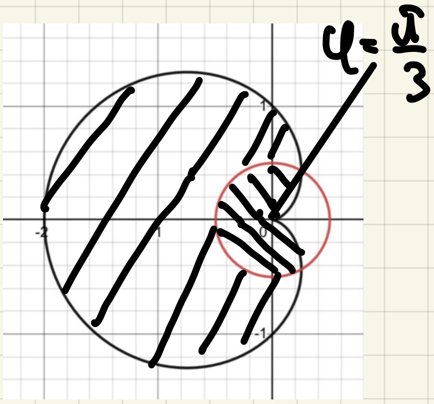
$$S_2 = \int_{0}^{4} 2-\sqrt{x}\,dx = \left(2x - \frac{2}{3}x^{\frac{3}{2}}\right)\Big|_{0}^{4} = 8-\frac{16}{3} = \frac{8}{3}$$

$S = 8$ ✓

№18

Вычислить $S_{фиг.}$, огр-й $r = 1-\cos\varphi$, $r = \frac{1}{2}$

(внутри 1, вне 2)



$$1 - \cos\varphi = \frac{1}{2}$$

$$\cos\varphi = \frac{1}{2}$$

$$\varphi = \frac{\pi}{3}$$

$$S_{фиг.} = S_{кард.} - S_{окр.}$$

$$S = \cancel{2}\cdot\left(\cancel{\frac{1}{2}}\int_{\frac{\pi}{3}}^{\pi}(1-\cos\varphi)^2\,d\varphi - \cancel{\frac{1}{2}}\int_{\frac{\pi}{3}}^{\pi}\frac{1}{4}\,d\varphi\right) =$$

$$= \left(\varphi - 2\sin\varphi + \frac{\varphi}{2} + \frac{\sin 2\varphi}{4} - \frac{1}{4}\varphi\right)\Bigg|_{\frac{\pi}{3}}^{\pi} =$$

$$= \left(\frac{5}{4}\varphi - 2\sin\varphi + \frac{\sin 2\varphi}{4}\right)\Bigg|_{\frac{\pi}{3}}^{\pi} =$$

$$= \frac{5\pi}{4} - 0 + 0 - \frac{5\pi}{12} + \sqrt{3} - \frac{\sqrt{3}}{8} = \frac{5\pi}{6} + \frac{7\sqrt{3}}{8}$$

Комплект 3

N1

$$\int_1^{+\infty} \frac{\sin x}{x^6+1}\,dx = \left\{ \begin{array}{l} \text{Н.И. 1 рода} \\ \sin x - \text{знакоперем на } x\to+\infty \\ x^6+1 \sim x^6 \end{array} \right\}$$

$$\int_1^{+\infty} \frac{|\sin x|}{x^6} - \text{сх-ся}, \alpha = 6 > 1, \text{ ш-но исх. сх-ся абс.}$$

√

N2

$$\int_0^1 \frac{dx}{\sqrt[3]{x}+x^2} = \left\{ \begin{array}{l} \text{Н.И. 2 рода} \\ \sqrt[3]{x}+x^2 \sim \sqrt[3]{x}, x\to 0 \end{array} \right\} =$$

$$= \int_0^1 \frac{dx}{x^{\frac{1}{3}}} - \text{сх-ся}, \alpha < 1$$

√

N3

$$\int_1^{\infty} \frac{dx}{\sqrt{1+x}\,\sqrt[3]{1+x^2}} = \left\{ \text{Н.И. 1 рода} \right\} =$$

$$= \int_1^{\infty} \frac{dx}{\sqrt{x(1+\underbrace{\tfrac{1}{x}}_{\to 0})}\,\sqrt[3]{x^2(1+\underbrace{\tfrac{1}{x^2}}_{\to 0})}} = \int_1^{\infty} \frac{dx}{x^{7/6}} \text{ — сх-ся, } \alpha > 1 \checkmark$$

N4

{2 род}

$$\int_0^1 \frac{dx}{x^3 + x\sin x + \sqrt{x}} = \int_0^1 \frac{dx}{\sqrt{x}(\underbrace{x^{2,5}}_{\to 0} + \underbrace{\sqrt{x}\sin x}_{\to 0} + 1)} = \int_0^1 \frac{dx}{\sqrt{x}} \text{ —}$$

$$\text{— сх-ся, } \alpha = \frac{1}{2} < 1 \checkmark$$

N5

$$\int_1^{+\infty} \frac{dx}{(x+2)^4 + x^3 + x\ln x} = \{\text{н.и. 1 рода}\} =$$

$$= \int_1^{+\infty} \frac{dx}{x^4(1 + \underbrace{\tfrac{8}{x}}_{\to 0} + \underbrace{\tfrac{24}{x^2}}_{\to 0} + \underbrace{\tfrac{32}{x^3}}_{\to 0} + \underbrace{\tfrac{16}{x^4}}_{\to 0} + \underbrace{\tfrac{\ln x}{x^3}}_{\to 0})} = \int_1^{+\infty} \frac{dx}{x^4} \text{ — сх-ся, } \alpha = 4 > 1$$

$\checkmark$

№6

$$\int_1^{+\infty} \frac{\operatorname{arctg}(x)}{x(x^2+1)}\,dx = \left\{ \begin{array}{l} \text{Н.И. 1 рода} \\ \operatorname{arctg}\infty = \frac{\pi}{2} \\ x^3+x \sim x^3 \\ \quad x \to +\infty \end{array} \right\} =$$

$$= \frac{\pi}{2} \int_1^{+\infty} \frac{dx}{x^3} \doteq \text{сх-ся}, \ \alpha = 3 > 1 \quad \checkmark$$

№7

$$\int_1^2 \frac{\operatorname{arctg} x}{x(x^2-1)}\,dx = \left\{ \text{Н.И. 2 рода} \right\} =$$

$$= \int_1^2 \frac{\operatorname{arctg} x}{x\underbrace{(x-1)}(x+1)}\,dx$$

Расм:

$$\left. \begin{array}{l} f(x) = \dfrac{\operatorname{arctg} x}{x(x-1)(x+1)} \\ g(x) = \dfrac{1}{x-1} \end{array} \right\} \Rightarrow \lim_{x\to 1} \frac{f(x)}{g(x)} = \text{const}$$

$$\lim_{x\to 1} \frac{\operatorname{arctg} x}{x(x+1)} = \frac{\pi}{8} \neq 0 \Rightarrow \int_1^2 \frac{1}{(x-1)}\,dx \text{ -расх-ся}$$

$$\alpha = 1, \geq 1$$

$\checkmark$

№8

$$\int_0^1 \frac{\sqrt{x}}{\ln(1+x^2)}\,dx = \left\{ \begin{array}{l} \text{н.и. 2 рода} \\ \ln(1+x^2) \sim x^2,\ x \to 0 \end{array} \right\} =$$

$$= \int_0^1 \frac{dx}{x^{\frac{3}{2}}} - \text{расх-ся},\ \alpha = \frac{3}{2} \geqslant 1 \quad \checkmark$$

№9

$$\int_1^{+\infty} \frac{dx}{x^3 + x^{\frac{1}{3}} + 1} = \left\{ \begin{array}{l} \text{н.и. 1 рода} \\ x^3 + x^{\frac{1}{3}} + 1 \sim x^3,\ x \to \infty \end{array} \right\} =$$

$$= \int_1^{+\infty} \frac{dx}{x^3} - \text{сх-ся},\ \alpha = 3 > 1 \quad \checkmark$$

№10

$$\int_1^{+\infty} \frac{\sqrt{x}\,dx}{1+x^7} = \left\{ \begin{array}{l} \text{н.и. 1 рода} \\ 1+x^7 \sim x^7,\ x \to \infty \end{array} \right\} = \int_1^{+\infty} \frac{dx}{x^{6,5}} -$$

$$- \text{сх-ся},\ \alpha = 6{,}5 > 1. \quad \checkmark$$

№11

$$\int_1^{+\infty} \frac{dx}{x^{\frac13}+\cos^2 x} = \left\{ \text{Н.И. 1 рода} \right\} =$$

$$= \int_1^{+\infty} \frac{dx}{x^{\frac13}\Big(1+\underbrace{\frac{1}{x^{\frac13}}\cos^2 x}_{\to 0}\Big)} = \int_1^{+\infty} \frac{dx}{x^{\frac13}} - \text{расх-ся}, \alpha=\frac13 < 1$$

$$\int_3^4 \underbrace{\frac{x^2-x+1}{x^4-9x^2}}_{2\text{ род}}\,dx$$

√

№12

$$\int_1^{+\infty} \frac{x^2-x+1}{x^4-9x^2}\,dx = \underbrace{\int_1^3 \frac{x^2-x+1}{x^4-9x^2}\,dx}_{2\text{ род}} + \underbrace{\int_4^{+\infty} \frac{x^2-x+1}{x^4-9x^2}\,dx}_{1\text{ род}} =$$

1) $\displaystyle\int_1^3 \frac{x^2-x+1}{x^2(x^2-9)}\,dx:$

Рассм:

$$f(x) = \frac{x^2-x+1}{x^2(x^2-9)} \quad \Big\} \Rightarrow \lim_{x\to 3} \frac{f(x)}{g(x)} = const \neq 0$$

$$g(x) = \frac{1}{x-3}$$

$$\lim_{x\to 3} \frac{x^2-x+1}{x^2(x+3)} = \frac{9-2}{9\cdot 6} = \frac{7}{54} \neq 0$$

$$\int_1^3 \frac{dx}{(x-3)} \text{ — расх-ся}, \ \alpha = 1, \ \geq 1 \Rightarrow \text{подобные}$$

2,3) Можно не рассм. т.к. $\sum$ все равно расх-ся

Отв: расх-ся. ✓

N13

$$\int_0^1 \frac{dx}{\sqrt{x}-1} = \int_0^1 \frac{\cancel{(\sqrt{x}+1)}^{\,2}}{x-1}\,dx = 2\int_0^1 \frac{dx}{x-1} \text{ — расх-ся}, \ \alpha = 1$$ ✓

N14

$$\int_0^1 \frac{dx}{e^x - \cos 2x} = \int_0^1 \frac{dx}{\underbrace{\underbrace{e^x-1}_{=x} + \underbrace{1-\cos 2x}_{=2x^2}}_{\sim x,\ x\to 0}} = \int_0^1 \frac{dx}{x} \text{ — расх-ся}, \ \alpha = 1$$ ✓

N15 2 род

$$\int_0^1 \frac{\cancel{\sin x}^{\text{const}}}{\sqrt{1-x^2}}dx \simeq \int_0^1 \frac{dx}{\sqrt{(1-x)(1+x)}}$$

$$\left.\begin{aligned} f(x) &= \frac{1}{\sqrt{(1-x)(1+x)}} \\ g(x) &= \frac{1}{\sqrt{1-x}} \end{aligned}\right\} \Rightarrow \lim_{x\to 1} \frac{f(x)}{g(x)} = \text{const}$$

$$\lim_{x\to 1} \frac{1}{\sqrt{1+x}} = 1/\sqrt{2} \neq 0 \Rightarrow \text{подобны}$$

$$\int_0^1 \frac{1}{(1-x)^{\frac{1}{2}}}dx \text{ — сх-ся}, \alpha = \frac{1}{2} < 1$$ ✓

N16

$$\int_1^2 \frac{dx}{\ln x} = \left\{ \ln(1+(x-1)) \sim x-1,\ x\to 1 \right\} =$$

$$= \int_1^2 \frac{dx}{x-1} \text{ — расх-ся}, \alpha = 1,$$ ✓

N17

$$\int_0^{30} \frac{dx}{\sqrt[3]{x} + 2\sqrt[4]{x} + x^5} = \left\{ \sqrt[3]{x} + 2\sqrt[4]{x} + x^5 \sim \sqrt[4]{x} \right\} =$$

$$= \int_0^{30} \frac{dx}{x^{\frac{1}{4}}} - \text{сх-ся}, \ \alpha = \frac{1}{4} < 1 \quad \checkmark$$

N18

$$\int_0^1 \frac{\overset{\text{const}}{\sin^3 x}}{\sqrt{1-x}}\, dx \cong \int_0^1 \frac{dx}{\sqrt{1-x}} - \text{сх-ся}, \ \alpha = \frac{1}{2} \quad \checkmark$$

Комплект 4

№1

$$2y\cdot y'' - 3(y')^2 = 4y^2, \quad y(0)=1, \quad y'(0)=2$$

Явно нет x:

$$\begin{cases} y' = p(y) \\ y'' = p'\cdot p \end{cases}$$

$$2y\cdot p'\cdot p - 3p^2 = 4y^2$$

$$p' - \frac{3p^{\cancel{2}}}{2y\cancel{p}} = \frac{2\cancel{4}y^{\cancel{2}}}{\cancel{2y}p}$$

$$p' - \frac{3}{2}\cdot\left(\frac{p}{y}\right) = 2\left(\frac{y}{p}\right)$$

$$u = \frac{p}{y}, \quad p = uy, \quad p' = u'y + u$$

$$u'y + u - \frac{3}{2}u = \frac{2}{u}$$

$$u'y = \frac{2}{u} + \frac{u}{2}$$

$$\frac{du}{dy}\cdot y = \frac{4+u^2}{2u}$$

$$\frac{2u}{4+u^2}\,du = \frac{dy}{y}$$

$$\frac{du^2}{4+u^2} = \frac{dy}{y}$$

$$\ln|4+u^2| = \ln C_1 \cdot y$$

$$C_1 y = 4+u^2$$

$$C_1 y = 4+\frac{p^2}{y^2}$$

$$C_1 y = 4+\frac{(y')^2}{y^2}$$

Исп. Н.У.: $y(0)=1,\ y'(0)=2$

$$C_1 = 4+\frac{4}{1}$$

$$C_1 = 8$$

$$8y = 4+\frac{(y')^2}{y^2}$$

$$(y')^2 = 8y^3-4y^2$$

$$y' = \sqrt{8y^3-4y^2}$$

$$\frac{dy}{dx} = \sqrt{8y^3 - 4y^2}$$

$$\frac{dy}{2y\sqrt{2y-1}} = dx$$

$$\frac{1}{2}\int \frac{dy}{y\sqrt{2y-1}} = \int dx$$

$$t = 2y-1 \Rightarrow y = \frac{t+1}{2}$$

$$dt = 2dy$$

$$\int \frac{dt}{(t+1)\sqrt{t}} = 2\int \frac{ds}{s^2+1} = 2\operatorname{arctg} s$$

$$s = \sqrt{t}$$

$$ds = \frac{\cancel{d}t}{2\sqrt{t}}$$

$$2\operatorname{arctg}\sqrt{t}$$

$$2\operatorname{arctg}\sqrt{2y-1}$$

$$\operatorname{arctg}\sqrt{2y-1} = x + C$$

Исп. Н.У.: $x=0, y=1$

$$\operatorname{arctg} 1 = C$$

$$C = \frac{\pi}{4}$$

$$\operatorname{arctg}\sqrt{2y-1} = x + \frac{\pi}{4}$$ √/?

№2

$$y'' + (y')^2 = 4y'(e^y+1)^3, \quad y(0)=0, \; y'(0)=16$$

явно нет x:

$$\begin{cases} y' = p \\ y'' = p \cdot p' \end{cases}$$

$$\cancel{p} \cdot p' + p^{\cancel{2}} = 4\cancel{p}(e^y+1)^3$$

$$p' + p = 4(e^y+1)^3$$

$$p = u \cdot v$$

$$u'v + uv' + u \cdot v = 4(e^y+1)^3$$

$$v(u'+u) + uv' = 4(e^y+1)^3$$

$$\begin{cases} u' + u = 0 \\ uv' - 4(e^y+1)^3 = 0 \end{cases}$$

1) $u' = -u$

$$\frac{du}{u} = -dy$$

$$\ln|u| = -y + C_1$$

$$\ln u = -y + C_1$$

$$u = e^{-y} \cdot e^{C_1}, \text{ пусть } C_1 = 0$$

2) $e^{-y} \cdot v' - 4(e^y+1)^3 = 0$

$$\frac{dv}{dy}\cdot e^{-y} = 4(e^y+1)^3$$

$$dv = 4e^y(e^y+1)^3\,dy$$

$$dv = (4e^{4y}+12e^{3y}+12e^{2y}+4e^y)\,dy$$

$$V = e^{4y} + 4e^{3y} + 6e^{2y} + 4e^y + C_2$$

$$p = u\cdot v = e^{3y} + 4e^{2y} + 6e^y + 4 + C_2\cdot e^{-y}$$

$$y' = e^{3y} + 4e^{2y} + 6e^y + 4 + C_2\cdot e^{-y}$$

Исп. Н.У. : $y(0)=0,\ y'(0)=16$

$$16 = 1+4+6+4+C_2 \Rightarrow C_2 = 1$$

$$y' = e^{3y} + 4e^{2y} + 6e^y + 4 + e^{-y}$$

$$\frac{dy}{dx} = e^{3y} + 4e^{2y} + 6e^y + 4 + e^{-y}$$

$$\frac{e^y\,dy}{e^{4y}+4e^{3y}+6e^{2y}+4e^y+1} = dx$$

$$\frac{d(e^y)}{(e^y+1)^4} = dx$$

$$\int \frac{d(e^y)}{(e^y+1)^4} = \int dx$$

$$-\frac{1}{3(e^y+1)^3} = x + C_3$$

Исп. Н.У.: $x=0, y=0$

$$-\frac{1}{3\cdot 8} = C_3$$

$$C_3 = -\frac{1}{24}$$

$$\frac{1}{3(e^y+1)^3} + x - \frac{1}{24} = 0$$ — общ. инт-л V/?

N3

$$y'' = \frac{y'}{x} - \frac{1}{2y'}, \quad y(1) = \frac{2}{3}, \quad y'(1) = 1$$

Явно нет y

$$\begin{cases} y' = p \\ y'' = p' \end{cases}$$

$$p' = \frac{p}{x} - \frac{1}{2p}$$

$$p' - \frac{1}{x}\cdot p = -\frac{1}{2p}$$

1) ЛОДУ:

$$p' - \frac{p}{x} = 0$$

$$\frac{dp}{p} = \frac{dx}{x}$$

$$\ln|p| = \ln|c_1 x|$$

$p = c_1 x$ - общ. реш-е

2) Пусть $c_1 = c_1(x)$ - предп. реш-е.

$$p = c_1(x)x, \quad p' = c_1'(x)x + c_1(x)$$

$$c_1'(x)\cdot x + \cancel{c_1(x)} - \cancel{c_1(x)} = -\frac{1}{2c_1(x)\cdot x}$$

$$c_1'(x)\cdot x = -\frac{1}{2c_1(x)x} \Big| \cdot \frac{c_1(x)\,dx}{x}$$

$$c_1(x)\,dc_1(x) = -\frac{dx}{2x^2}$$

$$\frac{c_1^2(x)}{2} = \frac{1}{2x} + C_2$$

$$C_1(x) = \sqrt{\frac{1}{x} + \tilde{C}_2}$$

$$y' = \sqrt{\frac{1}{x} + \tilde{C}_2} \cdot x$$

Исп. Н.У.: $y'(1) = 1$

$$1 = \sqrt{1 + \tilde{C}_2} \Rightarrow \tilde{C}_2 = 0$$

$$y' = \sqrt{x}$$

$$y = \frac{2x^{\frac{3}{2}}}{3} + C_3$$

Исп. Н.У.: $y(1) = \frac{2}{3}$

$$\frac{2}{3} = \frac{2}{3} + C_3 \Rightarrow C_3 = 0$$

$$y = \frac{2x\sqrt{x}}{3} \quad //$$

√/?

N4

$$y'' + \frac{(y')^2}{y} = y'(4+y^2),\ y(0)=1,\ y'(0) = \frac{25}{4}$$

Явно нет x!

$$\begin{cases} y' = p \\ y'' = p\cdot p' \end{cases}$$

$$\cancel{p}\cdot p' + \frac{p^{\cancel{2}}}{y} = \cancel{p}(4+y^2)$$

$$p' + \frac{1}{y}p = 4+y^2$$

$$p = uv,\quad p' = u'v + uv'$$

$$u'v + uv' + \frac{1}{y}uv = 4+y^2$$

$$v\left(u' + \frac{u}{y}\right) + uv' = 4+y^2$$

$$\begin{cases} u' + \frac{u}{y} = 0 \\ uv' = 4+y^2 \end{cases}$$

1) $u' = -\frac{u}{y}$

$\frac{du}{dy} = -\frac{u}{y}$

$\frac{du}{u} = -\frac{dy}{y}$

$\ln|u| = -\ln|y| \cdot \ln c_1$

$u = \frac{c_1}{y}$, пусть $c_1 = 1$

$u = \frac{1}{y}$ - ч. реш.

2) $\frac{1}{y} \cdot v' = 4 + y^2$

$v' = y^3 + 4y$

$dv = (y^3 + 4y)\,dy$

$v = \frac{y^4}{4} + 2y^2 + c_2$

$p = u \cdot v = \frac{1}{y}\left(\frac{y^4}{4} + 2y^2 + c_2\right) = \frac{y^3}{4} + 2y + \frac{c_2}{y}$

$y' = \frac{y^3}{4} + 2y + \frac{c_2}{y}$

Исп. Н.У.: $y = 1, y' = \frac{25}{4}$

$$\frac{25}{4} = \frac{1}{4} + 2 + c_2$$

$$c_2 = 4$$

$$y' = \frac{y^3}{4} + 2y + \frac{4}{y}$$

$$\frac{dy}{dx} = \frac{y^3}{4} + 2y + \frac{4}{y}$$

$$\frac{4y\,dy}{y^4 + 8y^2 + 16} = dx$$

$$2\int \frac{d(y^2+4)}{(y^2+4)^2} = \int dx$$

$$-2 \cdot \frac{1}{y^2+4} = x + c_3$$

Исп. Н.У.: $y(0) = 1$

$$-\frac{2}{5} = c_3$$

$$\frac{2}{y^2+4}+x=\frac{2}{5}$$?

№5

$$y y''+2(y')^2+y(y')^3=0\ , \quad y(0)=1,\ y'(0)=\frac{1}{3}$$

Явно нет x:

$p=y'$

$y''=p\cdot p'$

$$y\cdot p\cdot p'+2p^2+y\cdot p^3=0$$

$$p'+\frac{2}{y}\cdot p=-p^2$$

1) ЛОДУ:

$$p'+\frac{2}{y}p=0$$

$$\frac{dp}{dy}=-2\frac{p}{y}$$

$$\frac{dp}{p}=-2\frac{dy}{y}$$

$$\ln|p|=-2\ln|y|+\ln C_1$$

$$p = \frac{c_1}{y^2}$$

2) Пусть $c_1 = C_1(y)$; $p = \frac{C_1(y)}{y^2}$

$$p' = \frac{C_1'(y)\cdot y^2 - c_1(y)\cdot 2y}{y^4} =$$

$$= \frac{c_1'(y)}{y^2} - \frac{2 c_1(y)}{y^3}$$

$$\frac{c_1'(y)}{y^2} - \cancel{2\frac{c_1(y)}{y^3}} + \cancel{\frac{2c_1(y)}{y^3}} =$$

$$= -\frac{c_1^2(y)}{y^4}$$

$$C_1'(y) = -\frac{c_1^2(y)}{y^2}$$

$$\frac{dc_1(y)}{dy} = -\frac{c_1^2(y)}{y^2}$$

$$\frac{dc_1(y)}{c_1^2(y)} = -\frac{dy}{y^2}$$

$$-\frac{1}{c_1(y)} = \frac{1}{y} + c_2$$

$$c_1(y) = -\frac{y}{1 + c_2 y}$$

$$p = \frac{C_1(y)}{y^2} = -\frac{1}{y(1+C_2 y)}$$

$$y' = -\frac{1}{y(1+C_2 y)}$$

Исп. Н.У.: $y' = \frac{1}{3}, y = 1$

$$\frac{1}{3} = -\frac{1}{1+C_2} \Rightarrow C_2 = -4$$

$$y' = -\frac{1}{y(1-4y)}$$

$$\frac{dy}{dx} = -\frac{1}{y(1-4y)}$$

$$(y-4y^2)\,dy = -dx$$

$$\frac{y^2}{2} - \frac{4y^3}{3} = -x + C_3$$

Исп. Н.У.: $y(0) = 1$

$$\frac{1}{2} - \frac{4}{3} = C_3$$

$$C_3 = -\frac{5}{6}$$

$\frac{y^2}{2} - \frac{4y^3}{3} = -\frac{5}{6} - x$ — общ. инт-л. ?

№6

$$xy'' + y' = \ln x, \quad y(1) = 0, \quad y'(1) = 1$$

Явно нет y!

$$\begin{cases} y' = p \\ y'' = p' \end{cases}$$

$$xp' + p = \ln x$$

$$p' + \frac{1}{x}p = \frac{\ln x}{x}$$

$$p = uv, \quad p' = u'v + uv'$$

$$u'v + uv' + \frac{1}{x}uv = \frac{\ln x}{x}$$

$$v\left(u' + \frac{1}{x}u\right) + uv' = \frac{\ln x}{x}$$

$$\begin{cases} u' + \frac{u}{x} = 0 \\ uv' = \frac{\ln x}{x} \end{cases}$$

$$\frac{du}{dx} = -\frac{u}{x} \Rightarrow \frac{du}{u} = -\frac{dx}{x} \Rightarrow \ln|u| = -\ln|x| + \ln c_1$$

$u = \frac{c_1}{x}$, $c_1 = 1$ - ч. реш.

$$\frac{v'}{x} = \frac{\ln x}{x}$$

$$v' = \ln(x)$$

$$v = \int \ln(x)\,dx$$

$$v = x\ln x - x + C_2$$

$$p = uv = \ln x - 1 + \frac{C_2}{x}$$

$$y' = \ln x - 1 + \frac{C_2}{x}$$

Ust. H.Y.! $y(1)=0,\ y'(1)=1$

$$1 = \cancel{\ln 1} - 1 + C_2$$

$$C_2 = 2$$

$$y' = \ln x - 1 + \frac{2}{x}$$

$$dy = \left(\ln x - 1 + \frac{2}{x}\right)dx$$

$$y = x\ln x - x - x + 2\ln|x| + C_3$$

Ust. H.Y.! $y(1)=0$

$$0 = \cancel{1\ln 1} - 1 - 1 + 2\cancel{\ln 1} + C_3$$

$$C_3 = 2$$

$$y = x\ln x - 2x + 2\ln|x| + 2 \;//$$

?

N7

$$y'' = \frac{y'}{x}\left(1+\ln\frac{y'}{x}\right), \quad y(1)=\frac{1}{2}, \quad y'(1)=1$$

Явно нет y:

$$\begin{cases} y' = p \\ y'' = p' \end{cases}$$

$$p' = \frac{p}{x}\left(1+\ln\frac{p}{x}\right)$$

$$u = \frac{p}{x}, \quad p = ux, \quad p' = u'x + u$$

$$u'x + \cancel{u} = \cancel{u} + u\ln u$$

$$\frac{du}{dx}\cdot x = u\ln u$$

$$\frac{dx}{x} = \frac{du}{u\ln u}$$

$$\ln|C_1 x| = \ln|\ln u|$$

$$C_1 x = \ln u$$

$$C_1 x = \ln\frac{y'}{x}$$

Исп. Н.У.: $y'(1)=1$

$$c_1 = \ln 1$$

$$c_1 = 0$$

$$\ln \frac{y'}{x} = 0$$

$$\frac{y'}{x} = 1$$

$$y' = x$$

$$y = \frac{x^2}{2} + c_2$$

Исп. н.у. : $y(1) = \frac{1}{2}$

$$\frac{1}{2} = \frac{1}{2} + c_2$$

$$c_2 = 0$$

$$y = \frac{x^2}{2} \quad ?$$

№8

$$xy'' - y' = x^2 e^x, \quad y(1) = 0, \quad y'(1) = e$$

Явно нет y:

$$\begin{cases} y' = p \\ y'' = p' \end{cases}$$

$$xp' - p = x^2 e^x$$

$$p' - \frac{1}{x} p = x e^x$$

$$p = uv, \quad p = u'v + uv'$$

$$u'v + uv' - \frac{1}{x} uv = x e^x$$

$$v\left(u' - \frac{1}{x} u\right) + uv' = x e^x$$

$$\begin{cases} u' - \frac{1}{x} u = 0 \\ uv' = x e^x \end{cases}$$

1) $\dfrac{du}{dx} = \dfrac{u}{x}$

$$\ln|u| = \ln|c_1 x|$$

$$u = c_1 x, \quad c_1 = 1$$

2) $\cancel{x} \cdot v' = \cancel{x} e^x$

$$v = e^x + c_2$$

$$p = y' = x e^x + c_2 x$$

Исп. н.у.: $y'(1) = e$

$e = e + c_2$

$c_2 = 0$

$y' = xe^x$

$y = xe^x - e^x + c_3$

Исп. н. у:

$y(1) = 0$

$0 = e - e + c_3$

$c_3 = 0$

$y = xe^x - e^x$?

№9

$y'' - (y')^2 \operatorname{tg} y = 2y' \sin y, \quad y\left(\frac{2}{3}\right) = 0, \quad y'\left(\frac{2}{3}\right) = 1$

Явно нет x:

$$\begin{cases} y' = p \\ y'' = p' \cdot p \end{cases}$$

$p' \cdot p - p^2 \operatorname{tg} y = 2p \sin y$

$$p' - p\cdot \operatorname{tg} y = 2\sin y$$

1) ЛОДУ:

$$p' - \operatorname{tg} y\cdot p = 0$$

$$p' = \operatorname{tg} y\cdot p$$

$$\frac{dp}{p} = \operatorname{tg} y\cdot dy$$

$$\ln|p| = \ln\left|\frac{C_1}{\cos y}\right|$$

$$p = \frac{C_1}{\cos y}$$

2) Пусть $C_1 = C_1(y)$

$$p = \frac{C_1(y)}{\cos y}, \quad p' = \frac{C_1'(y)\cos y + C_1(y)\sin y}{\cos^2 y}$$

$$\frac{C_1'(y)}{\cos y} + \cancel{\frac{C_1(y)\sin y}{\cos^2 y}} - \cancel{\frac{C_1(y)}{\cos y}\cdot\frac{\sin y}{\cos y}} = 2\sin y$$

$$\frac{C_1'(y)}{\cos y} = 2\sin y$$

$$C_1'(y) = \sin 2y$$

$$c_1(y) = -\frac{1}{2}\cos 2y + C_2$$

$$p = \frac{C_2 - \frac{\cos 2y}{2}}{\cos y} = y'$$

Исп. Н.У.: $y'\left(\frac{2}{3}\right) = 1$, $y\left(\frac{2}{3}\right) = 0$

$$1 = \frac{C_2 - \frac{1}{2}}{1} \Rightarrow C_2 = \frac{3}{2}$$

$$y' = \frac{3 - \cos 2y}{2\cos y}$$

$$\frac{2\cos y\, dy}{3 - \cos 2y} = dx$$

$$\frac{2\, d(\sin y)}{3 - \cos^2 y + \sin^2 y} = dx$$

$$\frac{\cancel{2} d \sin y}{\cancel{2} + \cancel{2}\sin^2 y} = dx$$

$$x + C_3 = \operatorname{arctg}(\sin y)$$

Исп. Н.У. : $y(\frac{2}{3})=0$

$$\frac{2}{3}+C_3=\operatorname{arctg}0$$

$$C_3=-\frac{2}{3}$$

$$\operatorname{arctg}(\sin y)+\frac{2}{3}=x$$ //

?

N10

$$y'\cdot y^2+y\cdot y''-(y')^2=0\ , \quad y(0)=1,\ y'(0)=2$$

Явно нет x;

$$\begin{cases} y'=p \\ y''=p'\cdot p \end{cases}$$

$$p\cdot y^2+p'p\cdot y-p^2=0$$

$$y^2+p'y-p=0$$

$$p'-\frac{1}{y}p=-y$$

$$p=uv$$

$$u'v+uv'-\frac{1}{y}uv=-y$$

$$v\left(u' - \frac{1}{y}u\right) + uv' = -y$$

$$\begin{cases} u' - \dfrac{u}{y} = 0 \\ uv' = -y \end{cases}$$

1) $u' = \dfrac{u}{y}$

$\dfrac{du}{dy} = \dfrac{u}{y}$

$u = C_1 y = y$ – ч. реш.

2) $y \cdot v' = -y$

$v' = -1$

$v = -y + C_2$

$p = y' = C_2 y - y^2$

Исп. н.у.: $y' = 2, y = 1$

$2 = C_2 - 1$

$C_2 = 3$

$y' = 3y - y^2$

$$\frac{dy}{3y - y^2} = dx$$

$$\frac{1}{3}\int\frac{dy}{y}+\frac{1}{3}\int\frac{dy}{3-y}=\int dx$$

$$\frac{1}{3}\ln|y|+\frac{1}{3}\ln|3-y|=x+C_3$$

Исп. н.у.: $y(0)=1$

$$\cancel{\frac{1}{3}\ln 1}^{\,0}+\frac{1}{3}\ln 2=0+C_3$$

$$C_3=\ln\sqrt[3]{2}$$

$$\frac{1}{3}\ln|y|+\frac{1}{3}\ln|3-y|=x+\ln\sqrt[3]{2}$$ //

?

№11

$$y\cdot y''+2(y')^2=y^2 \quad , y(0)=1, y'(0)=2$$

Явно нет x!

$$\begin{cases} y'=p \\ y''=p'p \end{cases}$$

$$y\cdot p'\cdot p+2p^2=y^2$$

$$p'+\frac{2}{y}\cdot p=\frac{y}{p}$$

$$p = uv$$

$$u'v + uv' + \frac{2uv}{y} = \frac{y}{uv}$$

$$v\left(u' + \frac{2u}{y}\right) + uv' = \frac{y}{uv}$$

1) $u' + \frac{2u}{y} = 0$ 2) $\frac{v'}{y^2} = \frac{y^3}{v}$

$$\frac{du}{u} = -\frac{2dy}{y} \qquad \frac{dv}{dy} \cdot \frac{1}{y^2} = \frac{y^3}{v}$$

$u = \frac{1}{y^2}$ ~ч.реш.

$$v\,dv = y^5 dy$$

$$\frac{v^2}{2} = \frac{y^6}{6} + C_1 \qquad p = uv = y' = \sqrt{\frac{y^2}{3} + \frac{\tilde{C}_1}{y^4}}$$

$$v^2 = \frac{y^6}{3} + \tilde{C}_1$$

Исп. Н.У.: $y = 1,\ y' = 2$

$$v = \sqrt{\frac{y^6}{3} + \tilde{C}_1} \qquad 2 = \sqrt{\frac{1}{3} + \frac{\tilde{C}_1}{1}}$$

$$\tilde{C}_1 = \frac{11}{3}$$

$$y' = \sqrt{\frac{y^2}{3} + \frac{11}{3y^4}}$$

$$y' = \frac{1}{\sqrt{3}\,y^2}\sqrt{\frac{1}{y^4} + 11}$$

$$y' = \frac{1}{y^2\sqrt{3}} \cdot \sqrt{\frac{11y^4+1}{y^4}} = \frac{1}{y^4\sqrt{3}} \sqrt{11y^4+1}$$

$$\frac{dy \cdot y^4\sqrt{3}}{\sqrt{11y^4+1}} = dx$$

$$\frac{\sqrt{3}}{2} \int \frac{d(y^2)}{\sqrt{(y^2\sqrt{11})^2+1}} = \int dx$$

$$\frac{\sqrt{3}}{2} \ln\left| y^2\sqrt{11} + \sqrt{11y^4+1} \right| = x + C_2$$

Нач.Н.У. ; $y(0)=1$

$$\frac{\sqrt{3}}{2} \ln\left| \sqrt{11} + \sqrt{12} \right| = C_2$$

$$\frac{\sqrt{3}}{2} \ln\left| \frac{y^2\sqrt{11} + \sqrt{11y^4+1}}{\sqrt{11}+\sqrt{12}} \right| = x$$

???

№ 12

$$y'' + (y')^2 = \frac{y'}{(e^y+1)^2}, \quad y\left(\frac{1}{2}\right) = 0, \quad y'\left(\frac{1}{2}\right) = -\frac{1}{2}$$

Мбено нет x!

$$\begin{cases} y' = p \\ y'' = p'p \end{cases}$$

$$p' \cdot p + p^2 = \frac{p}{(e^y+1)^2}$$

$$p' + p = \frac{1}{(e^y+1)^2}$$

$$p = uv$$

$$u'v + uv' + uv = \frac{1}{(e^y+1)^2}$$

$$v(u' + u) + uv' = \frac{1}{(e^y+1)^2}$$

$$u' = -u$$

$$\frac{du}{u} = -1dy$$

$$\ln|u| = -y + C_1$$

$u = e^{-y}$ – 2. реш-е

$$e^{-y} \cdot v' = \frac{1}{(e^y+1)^2}$$

$$dv = \frac{d(e^y+1)}{(e^y+1)^2}$$

$$V = -\frac{1}{e^y+1} + C_2$$

$$p = uv = y' = C_2 e^{-y} - \frac{1}{e^{2y}+e^y}$$

Исп. н. у.: $y=0,\ y' = -\frac{1}{2}$

$$-\frac{1}{2} = C_2 - \frac{1}{2}$$

$$C_2 = 0$$

$$y' = -\frac{1}{e^y(e^y+1)}$$

$$(e^{2y}+e^y)\,dy = -dx$$

$$\frac{e^{2y}}{2} + e^y = -x + C_3$$

Исп. н. у: $y\left(\frac{1}{2}\right) = 0$

$$\frac{1}{2}+1=-\frac{1}{2}+C_3$$

$$C_3=2$$

?

$$\frac{e^{2y}}{2}+e^{y}=-x+2 \;//$$

№13

$$y''(2y+3)-2(y')^2=0$$

Явно нет x!

$$\begin{cases} y'=p \\ y''=p'\cdot p \end{cases}$$

$$p'\cdot p\,(2y+3)-2p^2=0$$

$$p'(2y+3)-2p=0$$

$$p'-\frac{2}{2y+3}p=0$$

$$p'=\frac{2}{2y+3}p$$

$$\frac{dp}{p}=\frac{d(2y+3)}{2y+3}$$

$$\ln|p| = \ln|2y+3| + \ln C,$$

$$p = 2C_1 y + 3C_1$$

$$y' = 2C_1 y + 3C_1$$

$$\int \frac{dy}{2y+3} = \int C_1 dx$$

$$\ln|2y+3| = 2C_1 x + 2C_2$$

$$y = \frac{1}{2}\left(e^{2(C_1x+C_2)} - 3\right)$$

√

№14

$$xy'' - y' = 2\sqrt{xy'}$$

Явно нет y

$$\begin{cases} p = y \\ p' = y' \end{cases}$$

$$p'x - p = 2\sqrt{px}$$

$$p' - \frac{1}{x}p = 2\sqrt{px}$$

$p = uv$

$$u'v + uv' - \frac{1}{x}uv = 2\sqrt{uvx}$$

$$v\left(u' + \frac{u}{x}\right) + uv' = 2\sqrt{uv \cdot x}$$

1) $u' + \frac{u}{x} = 0$

$$\frac{du}{u} = -\frac{dx}{x}$$

$$u = \frac{1}{x} \text{ - ч.реш.}$$

2) $\frac{v'}{x} = 2\sqrt{v}$

$$\frac{v'}{\sqrt{v}} = 2x$$

$$\frac{dv}{\sqrt{v}} = 2x\,dx$$

$$2\sqrt{v} = x^2 + C_1$$

$$\sqrt{v} = \frac{x^2}{2} + C_1$$

$$v = \left(\frac{x^2}{2} + C_1\right)^2$$

$$y' = p = \frac{1}{x}\left(\frac{x^2}{2} + C_1\right)^2$$

$$y' = \frac{x^3}{4} + xC_1 + \frac{C_1^2}{x}$$

$$y = \frac{x^4}{16} + \frac{C_1x^2}{2} + C_1^2\ln|x| + C_2$$

???

№15

$y \cdot y'' = 1 + (y')^2$, $y(-2) = -1$, $y'(-2) = R$

Явно нет x:

$$\begin{cases} y' = p \\ y'' = p' \cdot p \end{cases}$$

$$y \cdot p' \cdot p = 1 + p^2$$

$$p' - \frac{1}{y} p = \frac{1}{py}$$

1) ЛОДУ:

$$p' = \frac{p}{y}$$

$$p = C_1 y$$

2) $C_1 = C_1(y)$

$$p = C_1(y) \cdot y, \quad p' = C_1'(y) \cdot y + C_1(y)$$

$$C_1'(y) \cdot \cancel{y} + \cancel{C_1(y)} - \cancel{C_1(y)} = \frac{1}{C_1(y) \cdot y^3}$$

$$C_1'(y) = \frac{1}{C_1(y) y^3}$$

$$C_1(y)\, d(C_1(y)) = \frac{dy}{y^3}$$

$$\frac{C_1^2(y)}{2} = -\frac{1}{2y^2} + C_2$$

$$C_1^2(y) = -\frac{1}{y^2} + C_2$$

$$C_1(y) = \sqrt{C_2 - \frac{1}{y^2}}$$

$$p = \sqrt{C_2 - \frac{1}{y^2}} = y'$$

Нач. н.у.: $y' = \sqrt{2}$, $y = -1$

$$\sqrt{2} = \sqrt{C_2 - 1}$$

$$C_2 = 3$$

$$y' = \sqrt{3 - \frac{1}{y^2}}$$

$$\frac{1}{6} \int \frac{d(3y^2 - 1)}{\sqrt{3y^2 - 1}} = \int dx$$

$$\frac{1}{6} \cdot 2\sqrt{3y^2 - 1} = x + C_2$$

Нач. н.у.: $y(-2) = -1$

$$\frac{1}{3}\sqrt{3 - 1} = -2 + C_2$$

$$C_2 = \frac{\sqrt{2}}{3} + 2$$

$$\frac{1}{3}\sqrt{3y^2-1} = x + \frac{\sqrt{2}}{3} + 2 \;//$$

?

№16

$$xy'' - y' = x^2\cos x$$

Явно нет y!

$$\begin{cases} y' = p \\ y'' = p' \end{cases}$$

$$xp' - p = x^2\cos x$$

$$p' - \frac{1}{x}p = x\cos x$$

1) ЛОДУ:

$$\frac{dp}{p} = \frac{dx}{x}$$

$$p = C_1 x$$

2) $C_1 = C_1(x)$

$$p = C_1(x)\,x, \quad p' = C_1'(x)x + C_1(x)$$

$$C_1'(x)\cdot \cancel{x} + \cancel{C_1(x)} - \cancel{C_1(x)} = \cancel{x}\cos x$$

$$C_1'(x) = \cos x$$

$$C_1(x) = \sin x + C_2$$

$$p = (\sin x + C_2)x = y'$$

$$y' = c_2 x + x\sin x$$

$$y = \frac{c_2 x^2}{2} - x\cos x + \sin x + C_1$$

?

№17

$$x^2 y'' + 3xy' - 4(y')^2 = 0, \quad y(1) = -3, \quad y'(1) = 1$$

явно нет y!

$$\begin{cases} p = y' \\ p' = y'' \end{cases}$$

$$p' + \frac{3}{x}\cdot p = \frac{4p^2}{x^2}$$

1) ЛОДУ:

$$\frac{dp}{p} = -\frac{3dx}{x}$$

$$\ln|p| = \ln\left|\frac{C_1}{x^3}\right|$$

$$p = \frac{C_1}{x^3}$$

2) $C_1 = C_1(x)$

$$p = \frac{C_1(x)}{x^3}, \quad p' = \frac{C_1'(x)x^3 - C_1(x)\cdot 3x^2}{x^6} = \frac{C_1'(x)}{x^3} - \frac{3C_1(x)}{x^4}$$

$$\frac{C_1'(x)}{x^3} - \cancel{\frac{3C_1(x)}{x^4}} + \cancel{\frac{3C_1(x)}{x^4}} = \frac{4C_1^2(x)}{x^5}$$

$$C_1'(x) = \frac{4C_1^2(x)}{x^5}$$

$$\frac{d\,C_1(x)}{C_1^2(x)} = 4\frac{dx}{x^5}$$

$$-\frac{1}{C_1(x)} = -\cancel{4}\,\frac{1}{\cancel{4}x^4} + C_2$$

$$\frac{1}{C_1(x)} = \frac{1}{x^4} + C_2$$

$$C_1(x) = \frac{x^4}{C_2x^4+1}$$

$$y' = \frac{x}{C_2x^4+1}$$ — Исп. Н.У.! $y'(1)=1$

$$1 = \frac{1}{C_2+1} \Rightarrow C_2 = 0$$

$$y' = x$$

$$y = \frac{x^2}{2} + C_3$$

$$y(1) = -3$$

$$-3 = \frac{1}{2} + C_3 \Rightarrow C_3 = -3{,}5$$

$$y = \frac{x^2}{2} - \frac{7}{2}$$ //   ?

N18

$$xy'' = y'\ln\frac{y'}{x}$$

Явно нет y:

$$\begin{cases} y' = p \\ y'' = p' \end{cases}$$

$$p' = \frac{p}{x}\ln\frac{p}{x}$$

$$\frac{p}{x} = u, \quad p = ux, \quad p' = u'x + u$$

$$u'x + u = u\ln u$$

$$\frac{du}{dx}\cdot x = u\ln u - u$$

$$\int\frac{du}{u\ln\frac{u}{e}} = \int\frac{dx}{x}$$

$$\int\frac{d\ln\frac{u}{e}}{\ln\frac{u}{e}} = \ln|x| + \ln|C_1|$$

$$\ln\left|\ln\frac{u}{e}\right| = \ln|C_1x|$$

$$\ln|\ln u - 1| = \ln|C_1x|$$

$$u = \frac{p}{x} = \frac{y'}{x}$$

$$\ln u = c_1x+1$$

$$u = e^{c_1x+1}$$

$$p = xe^{c_1x+1}$$

$$y' = xe^{c_1x+1}$$

$$y = \int xe^{c_1x+1}\,dx =$$

$$= \left\{ \begin{matrix} u = x & du = dx \\ dv = e^{c_1x+1} & v = \frac{1}{c_1}e^{c_1x+1} \end{matrix} \right\} =$$

$$= \frac{x}{c_1}e^{c_1x+1} - \frac{1}{c_1}\int e^{c_1x+1}\,dx =$$

$$= \frac{x}{c_1}e^{c_1x+1} - \frac{1}{c_1^2}e^{c_1x+1} + C_2$$

Комплект 5

№1

$$y^{IV}-y''=1-x^2+5xe^x+e^x\sin x$$

1) $y^{IV}-y''=0$

$k^4-k^2=0$

$$\left[\begin{array}{l} k=0,\ r=2 \\ k=\pm1,\ r=1 \end{array}\right.$$

ФСР: $\{e^{0x}, xe^{0x}, e^x, e^{-x}\}$

$y_{oo}=C_1+C_2x+C_3e^x+C_4e^{-x}$

2) $f_1=e^x(\sin x+0\cos x)=\begin{Bmatrix} s=\max\{0,0\}=0 \Rightarrow A_1,B_1 \\ \alpha\pm\beta i=1\pm i,\ r=0 \end{Bmatrix}$

$y_{ч.н.1}=e^x(A_1\cos x+B_1\sin x)$

3) $f_2=e^x\cdot 5x=\begin{Bmatrix} n=1\Rightarrow A_2x+B_2 \\ \alpha=1\Rightarrow r=1 \end{Bmatrix}$

$y_{ч.н.2}=e^x\cdot(A_2x+B_2)\cdot x^1$

4) $f_3=e^{0x}(1-x^2)=\begin{Bmatrix} n=2\Rightarrow A_3x^2+B_3x+\tilde{C}_3 \\ \alpha=0\Rightarrow r=2 \end{Bmatrix}$

$y_{ч.н.3}=e^{0x}(A_3x^2+B_3x+\tilde{C}_3)\cdot x^2$

5) $y_{он.}=C_1+C_2x+C_3e^x+C_4e^{-x}+e^x(A_1\cos x+B_1\sin x)+$
$+e^x(A_2x+B_2)\cdot x+(A_3x^2+B_3x+\tilde{C}_3)x^2$ //

№2

$$y'''+9y'=1-3x^2-x\cos 3x+e^x\sin 3x$$

1) $y''' + 9y' = 0$

$k^3 + 9k = 0$

$k(k^2+9) = 0$

$\left[\begin{array}{l} k = 0,\ r=1 \\ k = \pm 3i,\ r=1 \end{array}\right.$

ФСР: $\{e^{0x}, e^{0x}\sin 3x, e^{0x}\cos 3x\}$

$y_{o.o.} = C_1 + C_2\sin 3x + C_3\cos 3x$

2) $f_1 = e^{0x}(1-3x^2) = \left\{\begin{array}{l} n=2 \Rightarrow A_1x^2+B_1x+C_1 \\ \alpha=0 \Rightarrow r=1 \end{array}\right\}$

$y_{ч.н._1} = e^{0x}(A_1x^2+B_1x+C_1)\cdot x^1$

3) $f_2 = e^{0x}(-x\cos 3x + 0\sin 3x) = \left\{\begin{array}{l} s=1 \Rightarrow A_2x+B_2 \\ \alpha\pm\beta i = \pm 3i,\ r=1 \end{array}\right\}$

$y_{ч.н._2} = e^{0x}((A_2x+B_2)\cos 3x + (\tilde{C}_2x+D_2)\sin 3x)\cdot x^1$

4) $f_3 = e^{x}(0\cos 3x + 1\sin 3x) =$

$= \left\{\begin{array}{l} s=0 \Rightarrow A_3 \\ \alpha\pm\beta i = 1\pm 3i \Rightarrow r=1 \end{array}\right\} =$

$= e^x(A_3\cos 3x + B_3\sin 3x)$

5) $y_{о.н.} = C_1 + C_2\sin 3x + C_3\cos 3x + (A_1x^2+B_1x+\tilde{C}_1)\cdot x +$

$+ ((A_2x+B_2)\cos 3x + (\tilde{C}_2x+D_2)\sin 3x)\cdot x + e^x(A_3\cos 3x + B_3\sin 3x)$ //

N3

$y^{IV} + 7y''' + 12y'' = x - x^3 + e^x\sin 3x + (x+2)e^{-4x}$

1) $y^{IV} + 7y''' + 12y'' = 0$

$k^4 + 7k^3 + 12k^2 = 0$

$k^2(k^2+7k+12) = 0$

$\left[\begin{array}{l} k=0,\ r=2 \\ k=-4,\ r=1 \\ k=-3,\ r=1 \end{array}\right.$

$y_{o.o.} = C_1 + C_2x + C_3e^{-4x} + C_4e^{-3x}$

2) $f_1 = e^{0x}(x-x^3) = \left\{\begin{array}{l} n=3 \Rightarrow A_1x^3+\ldots \\ \alpha=0 \Rightarrow r=2 \end{array}\right\}$

$y_{ч.н._1} = e^{0x}(A_1x^3+B_1x^2+C_1x+D_1)\cdot x^2$

3) $f_2 = e^x(0\cos 3x + 1\sin 3x) = \left\{\begin{array}{l} s=0 \Rightarrow A_2 \\ 1\pm 3i \Rightarrow r=0 \end{array}\right\}$

$y_{ч.н._2} = e^x(A_2\cos 3x + B_2\sin 3x)$

4) $f_3 = e^{-4x}(x+2) = \left\{ \begin{matrix} n=1 \Rightarrow A_3x+B_3 \\ \alpha=-4 \Rightarrow r=1 \end{matrix} \right\}$

$y_{ч.н.3} = e^{-4x}(A_3x+B_3)\cdot x$

5) $y_{о.н.} = C_1 + C_2x + C_3\cdot e^{-4x} + C_4e^{-3x} + (A_1x^3+B_1x^2+\tilde{C}_1x+D_1)x^2 +$
$+ e^{x}(A_2\cos3x + B_2\sin3x) + e^{-4x}(A_3x+B_3)\cdot x$ //

N4

$y''' + 2y'' + 5y' = (x-1)e^{-x}\sin2x + 7e^{-x} + x^2 + 3x$

1) $y''' + 2y'' + 5y' = 0$

$k^3 + 2k^2 + 5k = 0$

$k(k^2 + 2k + 5) = 0$

$D = 4 - 20 = -16$

$k = \frac{-2\pm4i}{2} = -1\pm2i$

$\left[\begin{matrix} k=0, \; r=1 \\ k=-1\pm2i, \; r=1 \end{matrix} \right.$

$y_{о.о.} = C_1e^{0x} + e^{-x}(C_2\cos2x + C_3\sin2x)$

2) $f_1 = e^{-x}(0\cos2x + (x-1)\sin2x) = \left\{ \begin{matrix} s=1 \Rightarrow A_1x+B_1 \\ \alpha\pm\beta i = -1\pm2i \Rightarrow r=1 \end{matrix} \right\}$

$y_{ч.н.1} = e^{-x}\left((A_1x+B_1)\cos2x + (\tilde{C}_1x+D_1)\sin2x\right)\cdot x$

3) $f_2 = 7e^{-x} = \left\{ \begin{matrix} n=0 \Rightarrow A_2 \\ \alpha=-1 \Rightarrow r=0 \end{matrix} \right\}$

$y_{ч.н.2} = e^{-x}\cdot A_2$

4) $f_3 = e^{0x}(x^2+3x) = \left\{ \begin{matrix} n=2 \Rightarrow A_3x^2+B_3x+C_3 \\ \alpha=0 \Rightarrow r=1 \end{matrix} \right\}$

$y_{ч.н.3} = e^{0x}(A_3x^2+B_3x+\tilde{C}_3)\cdot x^1$

5) $y_{о.н.} = C_1 + e^{-x}(C_2\cos2x + C_3\sin2x) + e^{-x}\left((A_1x+B_1)\cos2x +\right.$
$\left.+ (\tilde{C}_1x+D_1)\sin2x\right)x + e^{-x}\cdot A_2 + (A_3x^2 + B_3x + \tilde{C}_3)\cdot x$ //

N5

$$y^{IV} - y'' = 1 - x^2 + 5xe^{-x} + e^{x}\cos x$$

1) $y^{IV} - y'' = 0$

$k^4 - k^2 = 0$

$\left[\begin{array}{l} k=0, r=2 \\ r=\pm 1, r=1 \end{array}\right.$

2) $f_1 = e^{0x}(1-x^2) = \left\{\begin{array}{l} n=2 \Rightarrow A_1x^2+\dots \\ \alpha=0 \Rightarrow r=2 \end{array}\right\}$

$y_{ч.н._1} = e^{0x}(A_1x^2 + B_1x + \tilde{C}_1)\cdot x^2$

3) $f_2 = e^{-x}\cdot 5x = \left\{\begin{array}{l} n=1 \Rightarrow A_2x+B_2 \\ \alpha=-1 \Rightarrow r=1 \end{array}\right\}$

$y_{о.о.} = C_1 + C_2x + C_3e^{x} + C_4e^{-x}$ $\quad y_{ч.н._2} = e^{-x}\cdot(A_2x+B_2)\cdot x^1$

4) $f_3 = e^{x}(1\cos x + 0\sin x) = \left\{\begin{array}{l} S=0 \Rightarrow A_3, B_3 \\ \alpha\pm\beta i = 1\pm i \Rightarrow r=0 \end{array}\right\}$

$y_{ч.н._3} = e^{x}(A_3\cos x + B_3\sin x)$

5) $y_{о.н.} = C_1 + C_2x + C_3e^{x} + C_4e^{-x} + (A_1x^2 + B_1x + \tilde{C}_1)\cdot x^2 + e^{-x}\cdot(A_2x+B_2)\cdot x + e^{x}(A_3\cos x + B_3\sin x)$ //

N6

$$y^{IV} + y''' - 2y' = 4e^{x} + x^2 - 3 + \cos x + e^{x}\sin x$$

1) $y^{IV} + y''' - 2y' = 0$

$k^4 + k^3 - 2k = 0$

$k(k^3 + k^2 - 2) = 0$

$\left[\begin{array}{l} k=0 \;, r=1 \\ k=1 \;, r=1 \end{array}\right.$

$y_{оо} = C_1 + C_2e^{x}$

2) $f_1 = 4e^{x} = \left\{\begin{array}{l} n=0 \Rightarrow A_1 \\ \alpha=1 \Rightarrow r=1 \end{array}\right\}$

$y_{ч.н._1} = e^{x}\cdot A_1\cdot x^1$

3) $f_2 = e^{0x}(x^2-3) = \left\{\begin{array}{l} n=2 \Rightarrow A_2x^2+\dots \\ \alpha=0 \Rightarrow r=1 \end{array}\right\}$

$y_{ч.н._2} = e^{0x}(A_2x^2 + B_2x + \tilde{C}_2)\cdot x^1$

4) $f_3 = e^{0x}(1\cos x + 0\sin x) = \left\{\begin{array}{l} S=0 \Rightarrow A_3, B_3 \\ \alpha\pm\beta i = \pm i \Rightarrow r=0 \end{array}\right.$

$y_{2.н._3} = e^{0x}(A_3\cos x + B_3\sin x)$

5) $f_4 = e^x(0\cos x + 1\sin x) = \begin{cases} s=0 \Rightarrow A_4, B_4 \\ \alpha \pm \beta i = 1 \pm i \Rightarrow r = 0 \end{cases}$

$y_{ч.н._4} = e^x(A_4\cos x + B_4\sin x)$

6) $y_{о.н.} = C_1 + C_2e^x + e^x \cdot A_1 \cdot x + (A_2x^2 + B_2x + \tilde{C_2}) \cdot x +$
$+ A_3\cos x + B_3\sin x + e^x(A_4\cos x + B_4\sin x)$ //

N 7

$y^{IV} + 2y'' + y = (x-1)e^{-x} - \cos x - e^x\sin x$

1) $y^{IV} + 2y'' + y = 0$

$k^4 + 2k^2 + 1 = 0$

$(k^2+1)^2 = 0$

$k = \pm i$, $r = 2$

$y_{oo} = e^{0x}(C_1\sin x + C_2x\sin x + C_3\cos x + C_4x\cos x)$

2) $f_1 = (x-1)e^{-x} = \begin{cases} n=1 \Rightarrow A_1x + B_1 \\ \alpha = -1 \Rightarrow r = 0 \end{cases}$

$y_{2.н._2} = e^{-x}(A_1x + B_1)$

3) $f_2 = e^{0x}(-1\cos x + 0\sin x) = \begin{cases} s=0 \Rightarrow A_2, B_2 \\ \pm i \Rightarrow r = 2 \end{cases}$

$y_{2.н._2} = e^{0x}(A_2\cos x + B_2\sin x) \cdot x^2$

4) $f_3 = e^x(-\sin x + 0\cos x) = \begin{cases} s=0 \Rightarrow A_3, B_3 \\ \alpha \pm \beta i = 1 \pm i \Rightarrow r = 0 \end{cases}$

$y_{2.н._3} = e^x(A_3\sin x + B_3\cos x)$

5) $y_{о.н.} = C_1\sin x + C_2\cos x + C_3x\sin x + C_4x\cos x + e^{-x}(A_1x + B_1) +$
$+ (A_2\cos x + B_2\sin x) \cdot x^2 + e^x(A_3\sin x + B_3\cos x)$ //

N 8

$y^{IV}+4y''+4y = x\sin 2x - 7 + x - x^3 + 4e^{2x}\sin x$

1) $y^{IV}+4y''+4y=0$

$k^4+4k^2+4 = 0$

$(k^2+2)^2=0$

$k=\pm i\sqrt{2}, r=2$

$y_{oo} = C_1 \sin x\sqrt{2} + C_2 \cos x\sqrt{2} + C_3 x \sin x\sqrt{2} + C_4 x \cos x\sqrt{2}$

2) $f_1 = e^{0x}(x\sin 2x + 0\cos 2x) = \begin{Bmatrix} S=1 \Rightarrow A_1x+B_1 \\ \alpha\pm\beta i = \pm 2i \Rightarrow r=0 \end{Bmatrix}$

$y_{ч.н._1} = e^{0x}((A_1x+B_1)\cos 2x + (C_1x+D_1)\sin 2x)$

3) $f_2 = -7+x-x^3 = \begin{Bmatrix} n=3 \Rightarrow A_2x^3+\ldots \\ \alpha=0 \Rightarrow r=0 \end{Bmatrix}$

$y_{ч.н._2} = e^{0x}(A_2x^3+B_2x^2+\tilde{C}_2x+D_2)$

4) $f_3 = 4e^{2x}\sin x = \begin{Bmatrix} S=0 \Rightarrow A_3 \\ \alpha\pm\beta i = 2\pm i \Rightarrow r=0 \end{Bmatrix}$

$y_{ч.н._3} = e^{2x}(A_3\cos x + B_3\sin x)$

5) $y_{о.н.} = C_1 \sin x\sqrt{2} + C_2 \cos x\sqrt{2} + C_3 x \sin x\sqrt{2} + C_4 x \cos x\sqrt{2} +$

$+ ((A_1x+B_1)\cos 2x + (C_1x+D_1)\sin 2x) + (A_2x^3+B_2x^2+\tilde{C}_2x+D_2) +$

$+ e^{2x}(A_3\cos x + B_3\sin x)$ //

№9

$$y''' + 8y'' + 16y' = x^3 - 2x + 3e^{-4x} - e^{-4x}\sin 2x$$

1) $y''' + 8y'' + 16y' = 0$

$k^3 + 8k^2 + 16k = 0$

$k(k^2 + 8k + 16) = 0$

$k(k+4)^2 = 0$

$\left[\begin{array}{l} k = 0,\ r = 1 \\ k = -4,\ r = 2 \end{array}\right.$

$y_{oo} = C_1 + C_2 e^{-4x} + C_3 x e^{-4x}$

2) $f_1 = x^3 - 2x = \left\{\begin{array}{l} n = 3 \Rightarrow A_1 x^3 + \dots \\ \alpha = 0 \Rightarrow r = 1 \end{array}\right\}$

$y_{ч.н._1} = (A_1 x^3 + B_1 x^2 + \tilde{C}_1 x + D_2)x$

3) $f_2 = 3e^{-4x} = \left\{\begin{array}{l} n = 0 \Rightarrow A_2 \\ \alpha = -4 \Rightarrow r = 2 \end{array}\right\}$

$y_{ч.н._2} = e^{-4x} \cdot A_2 \cdot x^2$

4) $f_3 = e^{-4x}(0\cos 2x - 1\sin 2x) = \left\{\begin{array}{l} s = 0 \Rightarrow A_3, B_3 \\ \alpha \pm \beta i = -4 \pm 2i \Rightarrow r = 0 \end{array}\right\}$

$y_{ч.н._3} = e^{-4x} \cdot (A_3 \cos 2x + B_3 \sin 2x)$

5) $y_{он.} = C_1 + C_2 e^{-4x} + C_3 x e^{-4x} +$
$+ (A_1 x^3 + B_1 x^2 + \tilde{C}_1 x + D_2)x + e^{-4x} \cdot A_2 \cdot x^2 + e^{-4x} \cdot (A_3 \cos 2x + B_3 \sin 2x)$ //

№10 (далее сокращаю реш-е)

$$y^{IV} + 2y''' + y'' = xe^{-x} + x^3 + 4 - \cos 2x$$

1) $k^4 + 2k^3 + k^2 = 0$

$\left[\begin{array}{l} k = 0,\ r = 2 \\ k = -1,\ r = 1 \end{array}\right.$

$y_{o.o.} = C_1 + C_2 x + C_3 e^{-x}$

2) $y_{ч.н._1} = e^{-x}(A_1 x + B_1)x$

3) $y_{ч.н._2} = (A_2 x^3 + B_2 x^2 + \tilde{C}_3 x + D_3) \cdot x^2$

4) $y_{ч.н._3} = (A_3 \cos 2x + B_3 \sin 2x) \cdot x^2$

5) $y_{о.н.} = y_{oo} + y_{чн_1} + y_{чн_2} + y_{ч.н._3}$ //

N11

$$y''' + 5y'' + 6y' = x - x^2 + (8-x)e^{-2x} - e^{-3x}\sin 3x$$

1) $k^3 + 5k^2 + 6k = 0$

$$\begin{cases} k=0, & r=1 \\ k=-3, & r=1 \\ k=-2, & r=1 \end{cases}$$

$y_{oo} = C_1 + C_2 e^{-3x} + C_3 e^{-2x}$

2) $y_{ч.н._1} = (A_1x^2 + B_1x + \tilde{C}_1)\cdot x$

3) $y_{ч.н._2} = e^{-2x}(A_2x + B_2)\cdot x$

4) $y_{ч.н._3} = e^{-3x}(A_3\cos 3x + B_3\sin 3x)$

5) $y_{о.н.} = y_{oo} + y_{ч.н.1} + y_{ч.н_2} + y_{ч.н_3}$

N12

$$y^{V} + y''' = x^3 + 3 - e^{-x} + x^2\cos x - (x+1)\sin x$$

1) $k^5 + k^3 = 0$

$$\begin{cases} k=0, & r=2 \\ k=\pm i, & r=1 \end{cases}$$

$y_{oo} = C_1 + C_2x + C_3\sin x + C_4\cos x$

2) $y_{ч.н._1} = (A_1x^3 + B_1x^2 + \tilde{C}_1\cdot x + D_1)x^2$

3) $y_{ч.н._2} = e^{-x}\cdot A_2$

4) $y_{ч.н._3} = ((A_3x^2 + B_3x + \tilde{C}_3)\cos x + (D_3x^2 + E_3x + F_3)\cdot\sin x)$

5) $y_{ч.н._4} = ((A_4x + B_4)\cos x + (\tilde{C}_4x + D_4)\sin x)\cdot x$

6) $y_{о.н.} = y_{oo} + y_{чн_1} + y_{чн_2} + y_{чн_3} + y_{чн_4}$

N13

$$y''' - y' = \sin x + xe^x - 1 + x^2\cos x$$

1) $k^3 - k = 0$

$$\begin{cases} k=0, & r=1 \\ k=\pm 1, & r=1 \end{cases}$$

$y_{oo} = C_1 + C_2e^x + C_3e^{-x}$

2) $y_{чн_1} = A_1\sin x + A_2\cos x$

3) $y_{чн_2} = e^x\cdot(A_2x + B_2)\cdot x$

4) $y_{чн._3} = A_3\cdot x$

5) $y_{чн_4} = (A_4x^2 + B_4x + \tilde{C}_3)\cos x + (D_3x^2 + E_3x + F_3)\sin x$

6) $y_{o.н.} = y_{oo} + y_{чн_1} + y_{чн_2} + y_{чн_3} + y_{чн_4}$ //

N14

$$y^{VI} - 8y''' = (1-x^2)e^{2x} + 4x - e^x\cos x + 5\sin x$$

1) $k^6 - 8k^3 = 0$

$\left[\begin{array}{l} k=0,\ r=3 \\ k=2,\ r=1 \end{array}\right.$

$y_{oo} = C_1 + C_2x + C_3x^2 + C_4e^{2x}$

2) $y_{чн_1} = e^{2x}(A_1x^2 + B_1x + \tilde{C}_1)\cdot x$

3) $y_{чн_2} = (A_2x + B_2)\cdot x^3$

4) $y_{чн_3} = e^x(A_3\cos x + B_3\sin x)$

5) $y_{чн_4} = A_4\cos x + B_4\sin x$

6) $y_{o.н.} = y_{oo} + y_{чн_1} + y_{чн_2} + y_{чн_3} + y_{чн_4}$ //

N15

$$y''' - 3y'' + 4y' - 2y = x^4e^x + \underbrace{x^2e^x\cos x - 3e^x\sin x}_{} + e^{-x}\cos 2x$$

1) $k^3 - 3k^2 + 4k - 2 = 0$

$\left[\begin{array}{l} k=1,\ r=1 \\ k=1\pm i,\ r=1 \end{array}\right.$

$y_{o.o.} = C_1e^x + e^x(C_2\cos x + C_3\sin x)$

2) $y_{ч.н_1} = e^x(A_1x^4 + B_1x^3 + \ldots)\cdot x$

3) $y_{чн_2} = e^x\big((A_2x^2 + B_2x + \tilde{C}_2)\cos x + (D_2x^2 + E_2x + F_2)\sin x\big)\cdot x$

4) $y_{чн_3} = e^{-x}(A_3\cos 2x + B_3\sin 2x)$

5) $y_{o.н} = y_{oo} + y_{чн_1} + y_{чн_2} + y_{чн_3}$ //

N16

$$y^{IV} + 3y''' + 2y'' = x^2e^{-2x} + x^3 + e^{-x}\cos x + 5x + (x-1)e^{-x}$$

1) $k^4+3k^3+2k^2=0$

$\begin{cases} k=0,\ r=2 \\ k=-2,\ r=1 \\ k=-1,\ r=1 \end{cases}$

2) $y_{ч.н._1}=e^{-2x}(A_1x^2+B_1x+\tilde{C}_1)\cdot x$

3) $y_{ч.н._2}=(A_2x^3+B_2x^2+\tilde{C}_2x+D_2)\cdot x^2$

4) $y_{ч.н._3}=e^{-x}(A_3\cos x+B_3\sin x)$

5) $y_{ч.н._4}=e^{-x}(A_4x+B_4)\cdot x$

$y_{oo}=C_1+C_2x+C_3e^{-2x}+C_4e^{-x}$

6) $y_{он}=y_{oo}=y_{чн_1}+y_{чн_2}+y_{чн_3}+y_{чн_4}$

N17

$y^{VI}+2y^{IV}+y''=x^4+1+e^{6x}+x^3\cos x-x\sin x+x^2e^x\sin x$

1) $k^6+2k^4+k^2=0$

$\begin{cases} k=0,\ r=2 \\ k=\pm i,\ r=2 \end{cases}$

2) $y_{ч.н._1}=(A_1x^4+B_1x^3+\ldots)\cdot x^2$

3) $y_{ч.н._2}=A_2\cdot e^{6x}$

4) $y_{чн._3}=\left((A_3x^3+B_3x^2+\tilde{C}_3x+D_3)\cos x+(E_3x^3+F_3x^2+G_3x+H_3)\sin x\right)x^2$

$y_{oo}=C_1+C_2x+C_3\cos x+C_4\sin x+C_5x\cos x+C_6x\sin x$

5) $y_{чн_4}=e^x\left((A_4x^2+B_4x+\tilde{C}_4)\cos x+(D_4x^2+E_4x+F_4)\sin x\right)$

6) $y_{он}=y_{oo}+y_{чн_1}+y_{чн_2}+y_{чн_3}+y_{чн_4}$ //

N18

$y^V-8y^{IV}-9y'''=e^{-x}+5\cos 9x+xe^{9x}+6x-x^3$

1) $k^5-8k^4-9k^3=0$

$\begin{cases} k=0,\ r=3 \end{cases}$

2) $y_{ч.н._1}=e^{-x}\cdot A_1x$

3) $y_{чн_2}=A_2\cos 9x+B_2\sin 9x$

$$\left[\begin{array}{l} k=9,\ r=1 \\ k=1,\ r=1 \end{array}\right.$$

4) $y_{ч.н._3} = e^{9x}\cdot(A_3x+B_3)\cdot x$

5) $y_{ч.н._4} = \left(A_4x^3+B_4x^2+\tilde{C}_4x+D_4\right)\cdot x^3$

$$y_{оо} = C_1+C_2x+C_3x^2+C_4e^{9x}+C_5e^{x}$$

6) $y_{о.н.} = y_{оо}+y_{ч.н._1}+y_{ч.н._2}+y_{ч.н._3}+y_{ч.н._4}$ //

Комплект 6

N1

$$y'' + 2y' + 2y = e^x \sin x$$

1) $k^2 + 2k + 2 = 0$

$D = 4 - 8 = -4$

$k = -1 \pm i, \ r = 1$

ФСР: $e^{-x}(\sin x + \cos x)$

$y_{oo} = e^{-x}(C_1 \cos x + C_2 \sin x)$

2) $y_{ч.н._1} = e^x(A \sin x + B \cos x)$

$$y'_{ч.н._1} = Ae^x \sin x + Ae^x \cos x + Be^x \cos x - Be^x \sin x$$

$$y''_{ч.н._1} = Ae^x \cancel{\sin x} + Ae^x \cos x + Ae^x \cos x - \cancel{Ae^x \sin x} + \cancel{Be^x \cos x} -$$

$$- Be^x \sin x - Be^x \sin x - \cancel{Be^x \cos x} = 2(Ae^x \cos x - Be^x \sin x)$$

$$2(Ae^x \cos x - Be^x \sin x) + 2(Ae^x \sin x + Ae^x \cos x + Be^x \cos x - Be^x \sin x) +$$

$$+ 2e^x(A \sin x + B \cos x) = 2e^x(2A \sin x + 2A \cos x + 2B \cos x - 2B \sin x) =$$

$$= 4e^x(A \sin x + A \cos x + B \cos x - B \sin x) = e^x \sin x$$

$$\begin{cases} 4(A - B) = 1 \\ 4(A + B) = 0 \end{cases} \Rightarrow \begin{cases} -8B = 1 \\ A = -B \end{cases} \Rightarrow \begin{cases} B = -\frac{1}{8} \\ A = \frac{1}{8} \end{cases}$$

$$y_{ч.н.1} = \frac{e^x}{8}(\cos x + \sin x)$$

3) $y_{о.н.} = e^{-x}(C_1\cos x + C_2 \sin x) + \frac{e^x}{8}(\cos x + \sin x)$ //

N 2

$$y'' - 6y' + 13y = 4\cos 3x$$

1) $k^2 - 6k + 13 = 0$

$D = 36 - 52 = -16$

$k = -3 \pm 2i,\ r=1$

$y_{о.о.} = e^{-3x} \cdot (C_1 \cos 2x + C_2 \sin 2x)$

2) $y_{ч.н.} = A\cos 3x + B\sin 3x$

$y'_{ч.н.} = -3A\sin 3x + 3B\cos 3x$

$y''_{ч.н.} = -9A\cos 3x - 9B\sin 3x$

$$-9A\cos 3x - 9B\sin 3x + \underline{18A\sin 3x} - 18B\cos 3x + 13A\cos 3x + 13B\sin 3x = 4\cos 3x$$

$$18A\sin 3x + 4A\cos 3x + 4B\sin 3x - 18B\cos 3x = 4\cos 3x$$

$$\sin 3x(18A + 4B) + \cos 3x(4A - 18B) = 4\cos 3x$$

$$\begin{cases} 18A + 4B = 0 \\ 4A - 18B = 4 \end{cases} \Rightarrow \begin{cases} A = -\frac{2B}{9} \\ \frac{-8B - 162B}{9} = 4 \end{cases} \Rightarrow \begin{cases} A = \frac{4}{85} \\ B = -\frac{18}{85} \end{cases}$$

3) $y_{о.н.} = e^{-3x}(C_1\cos 2x + C_2 \sin 2x) + \frac{4}{85}\cos 3x - \frac{18}{85}\sin 3x$ //

N3

$$y'' - 4y' + 3y = \sin 2x$$

1) $k^2 - 4k + 3 = 0$

$$\left[\begin{array}{l} k = 3,\ r = 1 \\ k = 1,\ r = 1 \end{array}\right.$$

$$y_{oo} = C_1 e^{3x} + C_2 e^{x}$$

2) $y_{ч.н.} = (A\cos 2x + B\sin 2x)\cdot$

$$y_{чн}' = -2A\sin 2x + 2B\cos 2x$$

$$y_{чн}'' = -4A\cos 2x - 4B\sin 2x$$

$$-4A\cos 2x - 4B\sin 2x + 8A\sin 2x - 8B\cos 2x + 3A\cos 2x +$$

$$+ 3B\sin 2x = \sin 2x$$

$$\sin 2x(8A - B) + \cos 2x(-A - 8B) = \sin 2x$$

$$\begin{cases} 8A - B = 1 \\ A + 8B = 0 \end{cases} \Rightarrow \begin{cases} -65B = 1 \\ A = -8B \end{cases} \Rightarrow \begin{cases} A = \frac{8}{65} \\ B = -\frac{1}{65} \end{cases}$$

3) $y_{o.н.} = C_1 e^{3x} + C_2 e^{x} + \frac{8}{65}\cos 2x - \frac{1}{65}\sin 2x$

N4

$$y'' - 3y' = 18x - 10\cos x$$

1) $k^2 - 3k = 0$

$$\left[\begin{array}{l} k = 0,\ r = 1 \\ k = 3,\ r = 1 \end{array}\right.$$

2) $y_{ч.н._1} = (Ax + B)x$

$$y_{чн._1}' = 2Ax + B$$

$$y_{чн._1}'' = 2A$$

$y_{oo} = C_1 + C_2 e^{3x}$

$2A - 6Ax - 3B = 18x$

$$\begin{cases} -6A = 18 \\ 2A - 3B = 0 \end{cases} \Rightarrow \begin{cases} A = -3 \\ B = -2 \end{cases}$$

$y_{ч.н._1} = -(3x+2)x$

3) $y_{ч.н._2} = (A\cos x + B\sin x)$

$y'_{ч.н._2} = -A\sin x + B\cos x$

$y''_{ч.н._2} = -A\cos x - B\sin x$

$-A\cos x - B\sin x + 3A\sin x - 3B\cos x = -10\cos x$

$$\begin{cases} -A - 3B = -10 \\ -B + 3A = 0 \end{cases} \Rightarrow \begin{cases} A + 9A = 10 \\ B = 3A \end{cases} \Rightarrow \begin{cases} A = 1 \\ B = 3 \end{cases}$$

$y_{ч.н._2} = \cos x + 3\sin x$

4) $y_{о.н.} = C_1 + C_2 e^{3x} - (3x+2)x + \cos x + 3\sin x$

N5

$$y'' + y = \frac{1}{\cos x}$$

1) $k^2 + 1 = 0$

$k = \pm i, r = 1$

$y_{oo} = C_1\cos x + C_2\sin x$

2) $C_1 = C_1(x)$, $C_2 = C_2(x)$

$y_{oo} = C_1(x)\cos x + C_2(x)\sin x$

$$\begin{cases} C_1'(x)\cos x + C_2'(x)\sin x = 0 \\ -C_1'(x)\sin x + C_2'(x)\cos x = \frac{1}{\cos x} \end{cases}$$

$$\Delta = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$$

$$\Delta_1 = \begin{vmatrix} 0 & \sin x \\ \frac{1}{\cos x} & \cos x \end{vmatrix} = -\frac{\sin x}{\cos x}$$

$$\Delta_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \frac{1}{\cos x} \end{vmatrix} = 1$$

$$C_1'(x) = -\frac{\sin x}{\cos x} \Rightarrow C_1(x) = +\int \frac{d(\cos x)}{\cos x} = \ln|\cos x| + \tilde{C}_1$$

$$C_2'(x) = 1 \Rightarrow C_2(x) = \int dx = x + \tilde{C}_2$$

3) $y_{он.} = \tilde{C}_1 \cos x + \tilde{C}_2 \sin x + \cos x \ln|\cos x| + x\sin x$ //

N6

$$y'' - 5y' = 1 - 75x^2 + 10\sin 5x$$

1) $k^2 - 5k = 0$

$$\begin{cases} k = 0, r = 1 \\ k = 5, r = 1 \end{cases}$$

$$y_{oo} = C_1 + C_2 e^{5x}$$

2) $y_{чн.1} = (A_1 x^2 + B_1 x + C_1)\,x$

$$y'_{чн.1} = 3A_1x^2 + 2B_1x + C_1$$

$$y''_{чн.1} = 6A_1x + 2B_1$$

$$6A_1x + 2B_1 - 15A_1x^2 - 10B_1x - 5C_1 = 1 - 75x^2$$

$$\begin{cases} -15A_1 = -75 \\ 6A_1 - 10B_1 = 0 \\ 2B_1 - 5C_1 = 1 \end{cases} \Rightarrow \begin{cases} A_1 = 5 \\ B_1 = 3 \\ C_1 = 1 \end{cases} \Rightarrow y_{чн.1} = (5x^2 + 3x + 1)\cdot x$$

3) $y_{чн_2} = A_2 \cos 5x + B_2 \sin 5x$

$y'_{чн_2} = -5A_2 \sin 5x + 5B_2 \cos 5x$

$y''_{чн_2} = -25A_2 \cos 5x - 25B_2 \sin 5x$

$-25A_2 \cos 5x - 25B_2 \sin 5x + 25A_2 \sin 5x - 25B_2 \cos 5x = 10 \sin 5x$

$$\begin{cases} 25A_2 - 25B_2 = 10 \\ -25A_2 - 25B_2 = 0 \end{cases} \Rightarrow \begin{cases} A_2 = \frac{1}{5} \\ B_2 = -\frac{1}{5} \end{cases}$$

$y_{чн_2} = \frac{1}{5}\cos 5x - \frac{1}{5}\sin 5x$

4) $y_{о.н.} = C_1 + C_2 e^{5x} + (5x^2 + 3x + 1) \cdot x + \frac{1}{5}\cos 5x - \frac{1}{5}\sin 5x$ //

№7

$y'' - 8y' + 17y = (2x-13)e^{4x}$

1) $k^2 - 8k + 17 = 0$

$D = 64 - 68 = -4$

$k = 4 \pm i, \; r = 1$

$y_{оо} = e^{4x}(C_1 \sin x + C_2 \cos x)$

2) $y_{чн_1} = e^{4x} \cdot (Ax + B)$

$y'_{чн} = 4Axe^{4x} + 4Be^{4x} + Ae^{4x}$

$y''_{чн} = 8Ae^{4x} + 16Axe^{4x} + 16Be^{4x}$

$8Ae^{4x} + 16Axe^{4x} + 16Be^{4x} - 32Axe^{4x} - 32Be^{4x} - 8Ae^{4x} +$
$+ 17Axe^{4x} + 17Be^{4x} = 2xe^{4x} - 13e^{4x}$

$$\begin{cases} \cancel{8A}+16B-32B-\cancel{8A}+17B=-13 \\ 16A-32A+17A=2 \end{cases} \Rightarrow \begin{cases} A=2 \\ B=-13 \end{cases}$$

$$y_{чн}=e^{4x}(2x-3)$$

$$3)\ y_{он}=e^{4x}(C_1\sin x+C_2\cos x)+e^{4x}(2x-3)$$

№8

$$y''+3y'+2y=\frac{1}{e^x+1}$$

$$1)\ k^2+3k+2=0$$

$$\left[\begin{array}{l} k=-2,\ r=1 \\ k=-1,\ r=1 \end{array}\right.$$

$$y_{о.о.}=C_1e^{-2x}+C_2e^{-x}$$

$$2)\ C_1=C_1(x),\ C_2=C_2(x)$$

$$y_{оо}=C_1(x)\cdot e^{-2x}-C_2(x)\cdot e^{x}$$

$$\begin{cases} C_1'(x)e^{-2x}+C_2'(x)\cdot e^{-x}=0 \\ -C_1'(x)2e^{-2x}-C_2'(x)\cdot e^{x}=\frac{1}{e^x+1} \end{cases}$$

$$-C_1'(x)e^{-2x}=\frac{1}{e^x+1}$$

$$C_1'(x) = -\frac{e^{2x}}{e^x+1}$$

$$C_1(x) = -\int \frac{e^{2x}\,dx}{e^x+1} = -\int \frac{e^x\,d(e^x)}{e^x+1} =$$

$$= -e^x + \ln(e^x+1) + \tilde{C}_1$$

$$C_2'(x) = \frac{e^x}{e^x+1} \Rightarrow C_2(x) = \ln(e^x+1) + \tilde{C}_2$$

3) $y_{o.н.} = -e^{-x} + e^{-2x}\ln(e^x+1) + \tilde{C}_1 e^{-2x} +$
$+ e^{-x}\ln(e^x+1) + \tilde{C}_2 e^{-x}$ //

N9

$$y'' + 9y = \frac{1}{\sin^3 3x}$$

1) $k^2 + 9 = 0$

$k = \pm 3i$

$y_{oo} = C_1 \sin 3x + C_2 \cos 3x$

2) $C_1 = C_1(x)$, $C_2 = C_2(x)$

$$\begin{cases} C_1'(x)\sin 3x + C_2'(x)\cos 3x = 0 \\ 3C_1'(x)\cos 3x - 3C_2'(x)\sin 3x = \frac{1}{\sin^3 3x} \end{cases}$$

$$\Delta = \begin{vmatrix} \sin 3x & \cos 3x \\ 3\cos 3x & -3\sin 3x \end{vmatrix} = -3$$

$$\Delta_1 = \begin{vmatrix} 0 & \cos 3x \\ \frac{1}{\sin^3 3x} & -3\sin 3x \end{vmatrix} = -\frac{\cos 3x}{\sin^3 x}$$

$$\Delta_2 = \begin{vmatrix} \sin 3x & 0 \\ 3\cos 3x & \frac{1}{\sin^3 3x} \end{vmatrix} = \frac{1}{\sin^2 3x}$$

$$C_1'(x) = +\frac{\cos 3x}{3\sin^3 3x} \Rightarrow C_1(x) = +\frac{1}{9}\int \frac{d(\sin 3x)}{\sin^3 3x} =$$

$$= +\frac{1}{18\sin^2 3x} + \tilde{C}_1$$

$$C_2'(x) = -\int \frac{dx}{3\sin^2 3x} = \frac{ctg(3x)}{9} + \tilde{C}_2$$

$$3)\ y_{o.н.} = C_1\sin 3x + C_2\cos 3x + \frac{1}{18\sin 3x} + \tilde{C}_1\sin 3x +$$

$$+ \frac{\cos^2 3x}{9} + \tilde{C}_2\cos 3x //$$

№10

$$y'' - 4y' + 4y = -16x^2 + 4$$

$$1)\ k^2 - 4k + 4 = 0$$

$$(k-2)^2 = 0$$

$$k = 2,\ r = 2$$

$$y_{oo} = C_1 e^{2x} + C_2 x e^{2x}$$

$$2)\ y_{ч_1} = Ax^2 + Bx + C$$

$$y_{ч_1}' = 2Ax + B$$

$$y_{ч_1}'' = 2A$$

$$2A - 8Ax - 4B + 4Ax^2 + 4Bx + 4C = -16x^2 + 4$$

$$\begin{cases} 4A = -16 \\ 4B - 8A = 0 \\ 2A - 4B + 4C = 4 \end{cases} \Rightarrow \begin{cases} A = -4 \\ B = -8 \\ C = -5 \end{cases}$$

$$y_{ч_1} = -4x^2 - 8x - 5$$

3) $y_{\text{он}} = C_1 e^{2x} + C_2 x e^{2x} - 4x^2 - 8x - 5$ //

N11

$$y'' + 4y = 4x + \cos 2x$$

1) $k^2 + 4 = 0$

$k = \pm 2i$

$y_{\text{о.о.}} = C_1 \sin 2x + C_2 \cos 2x$

2) $y_{\text{ч.н.}1} = A_1 x + B_1$

$y''_{\text{чн}1} = 0$

$4A_1 x + 4B_1 = 4x$

$A_1 x + B_1 = x$

$$\begin{cases} A_1 = 1 \\ B_1 = 0 \end{cases}$$

$y_{\text{чн}1} = x$

3) $y_{\text{чн}_2} = (A_2 \cos 2x + B_2 \sin 2x)x$

$y''_{\text{чн}} = -4A_2 \sin 2x + 4B_2 \cos 2x + x(-4A_2 \cos 2x - 4B_2 \sin 2x)$

$-4A_2 \sin 2x + 4B_2 \cos 2x = \cos 2x$

$$\begin{cases} 4B_2 = 1 \\ -4A_2 = 0 \end{cases} \Rightarrow \begin{cases} A_2 = 0 \\ B_2 = \frac{1}{4} \end{cases}$$

$y_{\text{ч.н.}2} = \frac{x}{4} \sin 2x$

4) $y_{\text{он}} = C_1 \sin 2x + C_2 \cos 2x + x + \frac{x}{4} \sin 2x$ //

N12

$$y'' + y = \frac{1}{\cos^3 x}$$

1) $k^2 + 1 = 0$

2) $y_{\text{оо}} = C_2(x) \sin x + C_1(x) \cos x$

$$\begin{cases} C_2'(x) \sin x + C_1'(x) \cos x = 0 \\ C_2'(x) \cos x - C_1'(x) \sin x = \frac{1}{\cos^3 x} \end{cases}$$

$u = \pm i$

$y_{00} = C_2 \sin x + C_1 \cos x$ $\quad \Delta = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = +1$

$\Delta_1 = \begin{vmatrix} 0 & \sin x \\ \frac{1}{\cos^3 x} & \cos x \end{vmatrix} = \frac{\sin x}{\cos^3 x}$

$\Delta_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \frac{1}{\cos^3 x} \end{vmatrix} = \frac{1}{\cos^2 x}$

$C_2'(x) = \frac{1}{\cos^2 x} \Rightarrow C_2(x) = +tg(x) + \tilde{C}_1$

$C_1'(x) = +\frac{\sin x}{\cos^3 x} \Rightarrow C_2(x) = -\frac{1}{2\cos^2 x} + \tilde{C}_2$

3) $y_{o.н.} = \tilde{C}_2 \sin x + \frac{\sin^2 x}{\cos x} - \frac{1}{2\cos x} + \tilde{C}_1 \cdot \cos x$ //

N13

$y'' + y' - 6y = 5e^{2x}$ $\quad y(0) = 0,\ y'(0) = -4$

1) $k^2 + k - 6 = 0$

$(k+3)(k-2) = 0$

$\left[\begin{array}{l} k = -3,\ r = 1 \\ k = 2,\ r = 1 \end{array}\right.$

$y_{00} = C_1 e^{-3x} + C_2 e^{2x}$

2) $y_{ч.н._1} = e^{2x} \cdot A \cdot x$

$y_{ч.н._1}' = 2Ae^{2x}x + Ae^{2x}$

$y_{ч.н._1}'' = 4Ae^{2x}x + 2Ae^{2x} + 2Ae^{2x}$

$\cancel{4Ae^{2x}x} + 4Ae^{2x} + \cancel{2Ae^{2x}x} + Ae^{2x} -$

$- \cancel{6Axe^{2x}} = 5e^{2x}$

$A = 1$

$y_{ч_1} = xe^{2x}$

3) $y_{он} = C_1 e^{-3x} + C_2 e^{2x} + xe^{2x}$

Исп.Н.У.: $y(0) = 0,\ y'(0) = -4$

$$\begin{cases} 0 = C_1 + C_2 \\ -4 = -3C_1 + 2C_2 + 1 \end{cases} \Rightarrow \begin{cases} C_1 = -C_2 \\ C_2 = -1 \end{cases} \Rightarrow \begin{cases} C_1 = 1 \\ C_2 = -1 \end{cases}$$

4) $y = e^{-3x} - e^{2x} + xe^{2x}$ //

N14

$y'' + 2y' + 5y = 4e^{-x} - 5x^2$

1) $k^2 + 2k + 5 = 0$

$D = 4 - 20 = -16$

$k = -1 \pm 2i$

$y_{оо} = e^{-x} \cdot (C_1 \cos 2x + C_2 \sin 2x)$

2) $y_{ч_1} = e^{-x}(A_1)$

$y'_{ч_1} = -e^{-x} A_1$

$y''_{ч_1} = e^{-x} A_1$

$A_1 e^{-x} - 2e^{-x} A_1 + 5A_1 e^{-x} = 4e^{-x}$

$A_1 = 1$

$y_{ч_1} = e^{-x}$

3) $y_{ч_2} = A_2 x^2 + B_2 x + \tilde{C}_2$

$y'_{ч_2} = 2A_2 x + B_2$

$$y''_{чн_2} = 2A_2$$

$$2A_2 + \underbrace{4A_2x} + 2B_2 + \underline{5A_2x^2} + \underbrace{5B_2x} + 5C_2 = -5x^2$$

$$\begin{cases} A_2 = -1 \\ B_2 = \frac{4}{5} \\ C_2 = \frac{2}{25} \end{cases} \qquad y_{чн_2} = -x^2 + \frac{4x}{5} + \frac{2}{25}$$

4) $y_{он} = e^{-x}(C_1\cos 2x + C_2\sin 2x) + e^{-x} - x^2 + \frac{4x}{5} + \frac{2}{25}$ //

N15

$$y'' - 2y' + y = e^x + 1$$

1) $k^2 - 2k + 1 = 0$

$(k-1)^2 = 0$

$k = 1,\ r = 2$

$y_{оо} = C_1e^x + C_2xe^x$

2) $y_{чн_1} = e^x \cdot A \cdot x^2$

$y'_{чн_1} = Ae^xx^2 + 2Axe^x$

$y''_{чн_1} = Ae^xx^2 + 4Axe^x + 2Ae^x$

$$\cancel{Ax^2e^x} + \cancel{4Axe^x} + 2Ae^x - \cancel{2Ax^2e^x} - \cancel{4Axe^x} + \cancel{Ax^2e^x} = e^x$$

$$A = \frac{1}{2} \Rightarrow y_{чн_1} = \frac{1}{2}x^2e^x$$

3) $y_{чн_2} = A_2$

$A_2 = 1$

4) $y_{он} = C_1e^x + C_2xe^x + 1 + \frac{1}{2}x^2e^x$

№66

$$y'' - 2y' + y = (16x-7)e^{5x}$$

1) $k^2 - 2k + 1 = 0$

$(k-1)^2 = 0$

$k = 1,\ r = 2$

$y_{oo} = C_1 e^x + C_2 x e^x$

2) $y_{чн} = (A_1 x + B_1) e^{5x}$

$y'_{чн} = A e^{5x} + 5Axe^{5x} + 5Be^{5x}$

$y''_{чн} = 10Ae^{5x} + 25Axe^{5x} + 25B_1e^{5x}$

$$10Ae^{5x} + 25Axe^{5x} + 25B_1e^{5x} - 2Ae^{5x} - 10Axe^{5x} - 10B_1e^{5x} +$$
$$+ A_1xe^{5x} + B_1e^{5x} = 16xe^{5x} - 7e^{5x}$$

$$\begin{cases} A_1 = 1 \\ B_1 = -\frac{15}{16} \end{cases}$$

$y_{чн} = \left(x - \frac{15}{16}\right)e^{5x}$

3) $y_{он} = C_1e^x + C_2xe^x + \left(x - \frac{15}{16}\right)e^{5x}$

№17

$$y'' - 2y' + y = 10e^x + 6x - 2$$

1) $k^2 - 2k + 1 = 0$

$(k-1)^2 = 0$

$k = 1,\ r = 2$

$y_{oo} = C_1e^x + C_2xe^x$

2) $y_{чн_1} = Ae^x x^2$

$y'_{чн_1} = Ae^x x^2 + 2Axe^x$

$y''_{чн_1} = Ae^x x^2 + 2Axe^x + 2Axe^x + 2Ae^x$

$$Ae^x x^2 + 2Axe^x + 2Axe^x + \underline{2Ae^x} - 2Ae^x x^2 - 4Axe^x + Ae^x x^2 = 10e^x$$

$$A_1 = 5$$

$$y_{zn_1} = 5x^2 e^x$$

3) $y_{zn_2} = A_2 x + B_2$

$$y'_{zn_2} = A_2$$

$$y''_{zn_2} = 0$$

$$0 - 2A_2 + A_2 x + B_2 = 6x - 2$$

$$\begin{cases} A_2 = 6 \\ B_2 = 10 \end{cases} \Rightarrow y_{zn_2} = 6x + 10$$

4) $y_{on} = C_1 e^x + C_2 x e^x + 5x^2 e^x + 6x + 10$ //

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$$y'' + y = \frac{1}{\sqrt{1 + \sin^2 x}}$$

1) $k^2 + 1 = 0$

$k = \pm i$, $r = 1$

$$y_{oo} = C_1 \cos x + C_2 \sin x$$

2) $y_{oo} = C_1(x)\cos x + C_2(x)\sin x$

$$\begin{cases} C_1'(x)\cos x + C_2'(x)\sin x = 0 \\ -C_1'(x)\sin x + C_2'(x)\cos x = 0 \end{cases}$$

$$\Delta = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$$

$$\Delta_1 = \begin{vmatrix} 0 & \sin x \\ \frac{1}{\sqrt{1+\sin^2 x}} & \cos x \end{vmatrix} = -\frac{\sin x}{\sqrt{1+\sin^2 x}}$$

$$\Delta_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \frac{1}{\sqrt{1+\sin^2 x}} \end{vmatrix} = \frac{\cos x}{\sqrt{1+\sin^2 x}}$$

$$C_1'(x) = -\frac{\sin x}{\sqrt{1+\sin^2 x}}$$

$$C_1(x) = -\int \frac{\sin x\,dx}{\sqrt{1+\sin^2 x}} = \int \frac{d\cos x}{\sqrt{2-\cos^2 x}} =$$

$$= \arcsin \frac{\cos x}{\sqrt{2}} + \tilde{C}_1$$

$$C_2(x) = \int \frac{\cos x\,dx}{\sqrt{1+\sin^2 x}} = \int \frac{d\sin x}{\sqrt{1+\sin^2 x}} =$$

$$= \ln\left|\sin x + \sqrt{1+\sin^2 x}\right| + \tilde{C}_2$$

3) $$y_{\text{o.н.}} = \tilde{C}_1 \cos x + \tilde{C}_2 \sin x + \cos x \arcsin \frac{\cos x}{\sqrt{2}} +$$

$$+ \sin x \cdot \ln\left|\sin x + \sqrt{1+\sin^2 x}\right|$$