КР ЛАиФНП

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№1. $u = \ln(7 - 2x^2 - 3y^2 - z^2)$; $(1;1;1)$

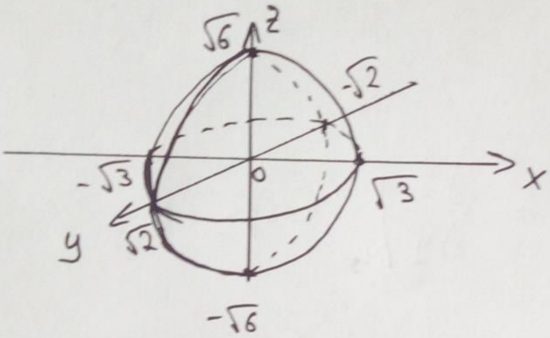
Решение:

$u(x,y,z) = C$ $\ln(7 - 2x^2 - 3y^2 - z^2) = C$

$u(1,1,1) = C$ $\ln(7 - 2 - 3 - 1) = C$ $\Rightarrow C = 0$, тогда

$\ln(7 - 2x^2 - 3y^2 - z^2) = 0 \Rightarrow 7 - 2x^2 - 3y^2 - z^2 = 1$

$$\frac{x^2}{3} + \frac{y^2}{2} + \frac{z^2}{6} = 1$$ – эллипсоид.



№2. $z = \arcsin(xy) + \arccos\left(\frac{x}{y}\right)$; $M_0\left(\frac{1}{2}, 1\right)$; $\bar{l} = -5\bar{i} + 2\bar{j}$

Решение:

$$\frac{\partial z}{\partial x} = \left(\arcsin(xy) + \arccos\left(\frac{x}{y}\right)\right)' = \frac{y}{\sqrt{1 - x^2y^2}} - \frac{1}{y\sqrt{1 - \frac{x^2}{y^2}}} =$$

$$= \frac{y}{\sqrt{1 - x^2y^2}} - \frac{1}{\sqrt{y^2 - x^2}};$$

$$\left.\frac{\partial z}{\partial x}\right|_M = \frac{1}{\sqrt{1 - \frac{1}{4}}} - \frac{1}{\sqrt{1 - \frac{1}{4}}} = 0.$$

$$\frac{\partial z}{\partial y} = \left(\arcsin(xy) + \arccos\left(\frac{x}{y}\right)\right)' = \frac{x}{\sqrt{1-x^2y^2}} + \frac{x}{y^2\sqrt{1-\frac{x^2}{y^2}}} =$$

$$= \frac{x}{\sqrt{1-x^2y^2}} + \frac{x}{y\sqrt{y^2-x^2}}$$

$$\left.\frac{\partial z}{\partial y}\right|_M = \frac{1/2}{\sqrt{1-\frac{1}{4}}} + \frac{1/2}{1\sqrt{1-\frac{1}{4}}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\operatorname{grad} z = \left(\frac{\partial z}{\partial x}; \frac{\partial z}{\partial y}\right) = \left(0; \frac{2\sqrt{3}}{3}\right)$$

$$\bar{l}_0 = \left(\frac{-5}{\sqrt{29}}; \frac{2}{\sqrt{29}}\right)$$

$$\left.\frac{\partial z}{\partial l}\right| = \left.\frac{\partial z}{\partial x}\right|_{M_0} \cdot \cos\alpha + \left.\frac{\partial z}{\partial y}\right|_{M_0} \cdot \cos\beta =$$

$$= 0 \cdot \left(\frac{-5}{\sqrt{29}}\right) + \frac{2\sqrt{3}}{3} \cdot \frac{2}{\sqrt{29}} = \frac{4\sqrt{3}}{3\sqrt{29}}$$

№3.

$$\lim_{\substack{x\to 0\\ y\to 0}} e^{\frac{x}{y}} \; - ?$$

Решение: пусть $y = kx$, тогда

$$\lim_{\substack{x\to 0\\ y\to 0}} e^{\frac{x}{y}} = \lim_{\substack{x\to 0\\ y=kx}} e^{\frac{x}{y}} = \lim_{\substack{x\to 0\\ y=kx}} e^{\frac{1}{k}} = e^{1/k}$$

Т.к. предел зависит от коэф. k, то предела не существует.

№4. $u = f(x(t), y(t), z(t))$, $f = x^2y^3z^5$, $x = t\ln t$,
$y = \ln t$, $z = \frac{1}{\ln t}$; u'_t – ?

Решение:

$$u'_t = f'_x \cdot x'_t + f'_y \cdot y'_t + f'_z \cdot z'_t$$ или

$$u'_t = f'(x(t), y(t), z(t)) = \left(x^2(t)\cdot y^3(t)\cdot z^5(t)\right)' =$$

$$= \left(t^2 \cdot \ln^2 t \cdot \ln^3 t \cdot \left(\frac{1}{\ln t}\right)^5\right)' = (t^2)' = 2t.$$

№5. $x^2+y^2+z^2 = x^3+y^3+z^3$ – ур-е неявно заданной ф-ции $z(x;y)$ в окрестности т. $(1;1;1)$; диф. ф-ции z в т. $(1;1)$ – ? Вычислить приближённо $z(1,1;\ 1,1)$.

Решение:

$$F(x,y,z) = x^2+y^2+z^2-x^3-y^3-z^3$$

$$z'_x = \frac{-F'_x}{F'_z} = -\frac{2x-3x^2}{2z-3z^2}; \quad z'_x\Big|_{(1;1;1)} = -\frac{2-3}{2-3} = -1$$

$$z'_y = -\frac{F'_y}{F'_z} = -\frac{2y-3y^2}{2z-3z^2}; \quad z'_y\Big|_{(1;1;1)} = -\frac{2-3}{2-3} = -1.$$

$$dz = z'_x\,\Delta x + z'_y\,\Delta y = -\Delta x - \Delta y$$

$x = x_0 + \Delta x \Rightarrow x_0 = 1;\ \Delta x = 0,1$

$y = y_0 + \Delta y \Rightarrow y_0 = 1;\ \Delta y = 0,1$

$$z\Big|_{x=1,1;\ y=1,1} \approx z\Big|_{x=1;\ y=1} + dz =$$

$$= 1 - 0,1 - 0,1 = 0,8.$$