Вариант 13. Репников Д ИУ 1-21

а) $7x^2 + 4y^2 + 4xy + 6\sqrt{5}x - 12\sqrt{5}y + 51 = 0$

$A = \begin{pmatrix} 7 & 2 \\ 2 & 4 \end{pmatrix}$ $|A - \lambda E| = 0$

$\begin{vmatrix} 7-\lambda & 2 \\ 2 & 4-\lambda \end{vmatrix} = (7-\lambda)(4-\lambda) - 4 = 0$

$28 - 11\lambda + \lambda^2 - 4 = 0$

$\lambda^2 - 11\lambda + 24 = 0$

$D = 121 - 96 = 25$

$\lambda = \frac{11 \pm 5}{2}$

$\lambda_2 = 8$ $\lambda_1 = 3$

$\underline{\lambda_1 = 3}$

$\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 4 & 2 \\ 0 & 0 \end{pmatrix} \Rightarrow 4x + 2y = 0 \Rightarrow \vec{e}_1 = \frac{1}{\sqrt{5}} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$\underline{\lambda_2 = 8}$

$\begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix} \sim \begin{pmatrix} -1 & 2 \\ 0 & 0 \end{pmatrix} \Rightarrow -x + 2y = 0 \Rightarrow \vec{e}_2 = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ +1 \end{pmatrix}$

$T = \frac{1}{\sqrt{5}} \begin{pmatrix} -1 & 2 \\ 2 & +1 \end{pmatrix}$

$X = T \cdot X'$; $\begin{cases} x = \frac{1}{\sqrt{5}}(-x' + 2y') \\ y = \frac{1}{\sqrt{5}}(2x' + y') \end{cases}$

$3(x')^2+8(y')^2+6\sqrt{5}\cdot\frac{1}{\sqrt{5}}(-x'+2y')-12\sqrt{5}(2x'+y')\frac{1}{\sqrt{5}}+51=$

$=3(x')^2+8(y')^2-6x'+12y'-24x'-12y'+51=$

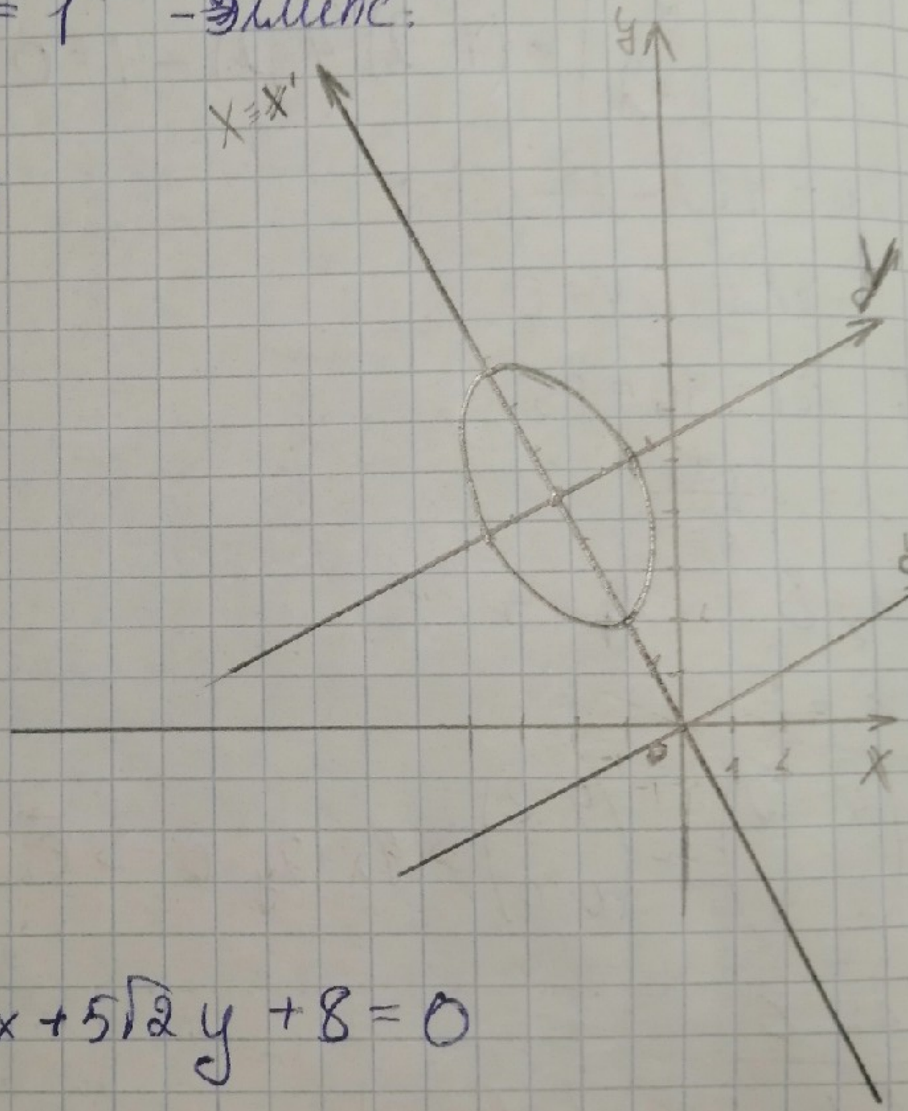
$=3(x')^2+8(y')^2-30x'+51=3(x'-5)^2+8(y')^2-24=0$

$\frac{(x'-5)^2}{8}+\frac{(y')^2}{3}=1$ – эллипс.

$\frac{X^2}{(\sqrt{8})^2}+\frac{Y^2}{(\sqrt{3})^2}=1$

$\sqrt{8}\approx 2,8=a$

$\sqrt{3}=1,7=b$



б) $x^2+y^2-2xy+\sqrt{2}x+5\sqrt{2}y+8=0$

$A=\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix};$

$\begin{vmatrix}1-\lambda & -1\\ -1 & 1-\lambda\end{vmatrix}=(1-\lambda)^2-1=-2\lambda+\lambda^2=\lambda(\lambda-2)=0$

$\lambda_1=0;\ \lambda_2=2.$

$\lambda_1 = 0$:

$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \Rightarrow x - y = 0 \Rightarrow \vec{e}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\lambda_2 = 2$

$\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} \sim \begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix} \Rightarrow -x - y = 0 \Rightarrow \vec{e}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$T = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \quad \begin{cases} x = \frac{1}{\sqrt{2}}(-x' + y') \\ y = \frac{1}{\sqrt{2}}(x' + y') \end{cases}$

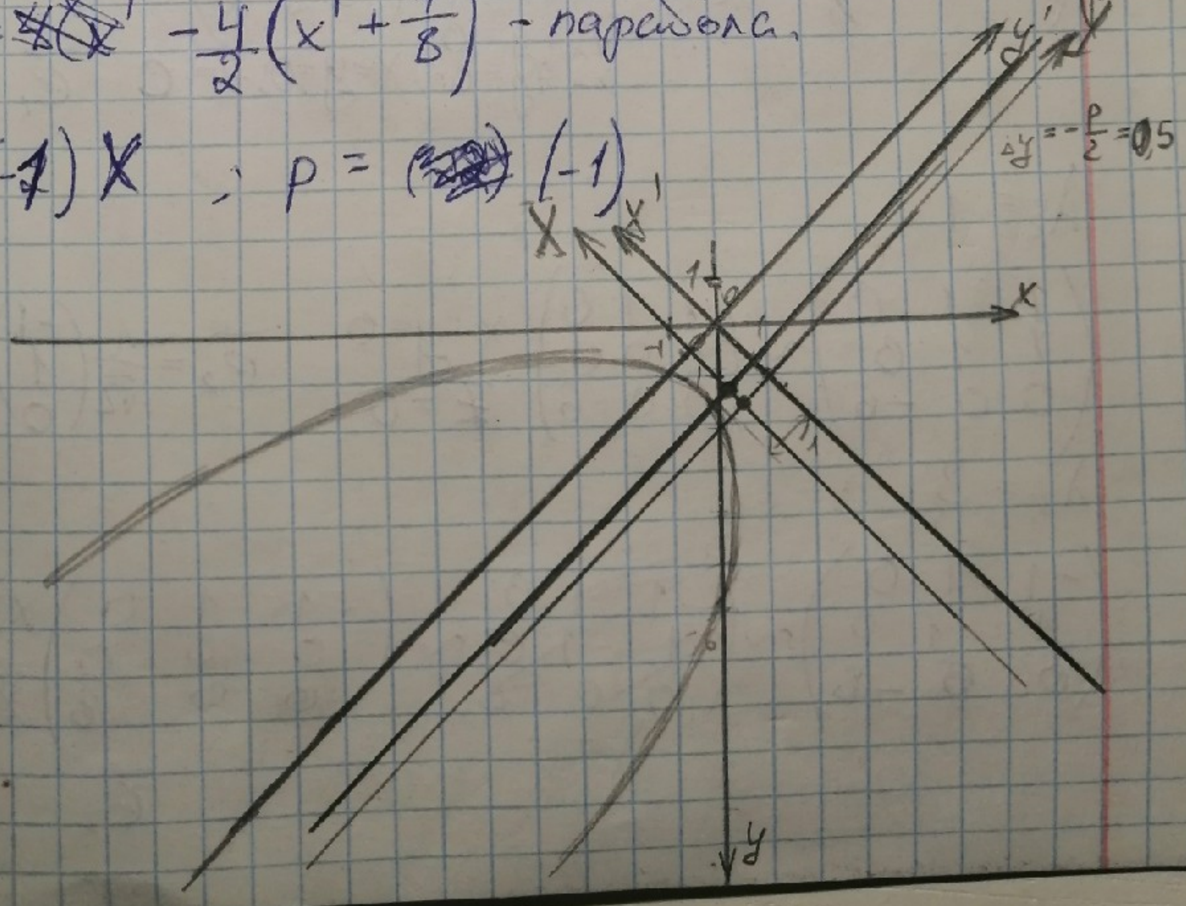
$0 \cdot (x')^2 + 2 \cdot (y')^2 + \sqrt{2} \cdot \frac{1}{\sqrt{2}}(-x' + y') + 5\sqrt{2} \cdot \frac{1}{\sqrt{2}}(x' + y') + 8 =$

$= 2(y')^2 - x' + y' + 5x' + 5y' + 8 =$

$= 2(y')^2 + 6y' + 4x' + 8 = 2\left(y' + \frac{3}{2}\right)^2 + 4x' - \frac{9}{2} + 8 = 0$

$\left(y' + \frac{3}{2}\right)^2 = -\frac{4}{2}\left(x' + \frac{7}{8}\right)$ - парабола.

$Y^2 = 2 \cdot (-1) X \quad ; \quad p = (-1)$



c) $2x^2+2y^2-5z^2+2xy+2\sqrt{2}x-2\sqrt{2}y+10z+74=0$

$$A=\begin{pmatrix}2&1&0\\1&2&0\\0&0&-5\end{pmatrix}$$

$$\begin{vmatrix}2-\lambda&1&0\\1&2-\lambda&0\\0&0&-5-\lambda\end{vmatrix}=(\lambda^2-4\lambda+4)(-5-\lambda)+(5+\lambda)=$$

$$=\cancel{(2-\lambda)^2}=(5+\lambda)(1-\lambda^2+4\lambda-4)=$$

$$=(5+\lambda)(1-(2-\lambda)^2)=0$$

$\lambda_1=-5$; $\lambda_2=1$; $\lambda_3=3$;

$\lambda_1=-5$

$$\begin{pmatrix}7&1&0\\1&7&0\\0&0&0\end{pmatrix}\sim\begin{pmatrix}7&1&0\\1&7&0\\0&0&0\end{pmatrix}\sim\cancel{\begin{pmatrix}7&1&0\\-48&0&0\\0&0&0\end{pmatrix}}\ \det\begin{pmatrix}7&1&0\\1&7&0\\0&0&0\end{pmatrix}=0$$

$\begin{matrix}7x+y=0\\x+7y=0\end{matrix}$; $x=y=0$; $z=c$ $\quad \vec{e}_1=\begin{pmatrix}0\\0\\1\end{pmatrix}$

$\lambda_2=1$

$$\begin{pmatrix}1&1&0\\1&1&0\\0&0&-6\end{pmatrix}\sim\begin{pmatrix}1&1&0\\0&0&0\\0&0&-6\end{pmatrix}\quad \begin{matrix}x+y=0\\z=0\end{matrix}\quad \vec{e}_2=\frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\\0\end{pmatrix}$$

$\lambda_3=3$

$$\begin{pmatrix}-1&1&0\\1&-1&0\\0&0&-8\end{pmatrix}\sim\cancel{\begin{pmatrix}-1&1&0\\1&-1&0\\0&0&-8\end{pmatrix}}\sim\begin{pmatrix}-1&1&0\\0&0&0\\0&0&-8\end{pmatrix}\quad \begin{matrix}x=y\\z=0\end{matrix}$$

$$\vec{e}_3=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix}$$

$$T=\frac{1}{\sqrt{2}}\begin{pmatrix}0&-1&1\\0&1&1\\\sqrt{2}&0&0\end{pmatrix}\quad\begin{cases}x=(-y'+z')\cdot\frac{1}{\sqrt{2}}\\y=(y'+z')\frac{1}{\sqrt{2}}\\z=\sqrt{2}\cdot x'\frac{1}{\sqrt{2}}\end{cases}$$

$$-5(x')^2+(y')^2+3(z')^2+\left(2\sqrt{2}(-y'+z')-2\sqrt{2}(y'+z')+10\cdot x'\sqrt{2}\right)\cdot\frac{1}{\sqrt{2}}+74=-5(x')^2+(y')^2+3(z')^2-\left(4\sqrt{2}y'+10\sqrt{2}x'\right)\cdot\frac{1}{\sqrt{2}}+74=$$

$$\cancel{=-5(x'}$$

$$=-5(x')^2+(y')^2+3(z')^2-4y'+10x'+74=$$

$$=-5(x'-1)^2+(y'-2)^2+3(z')^2+74+5-4=$$

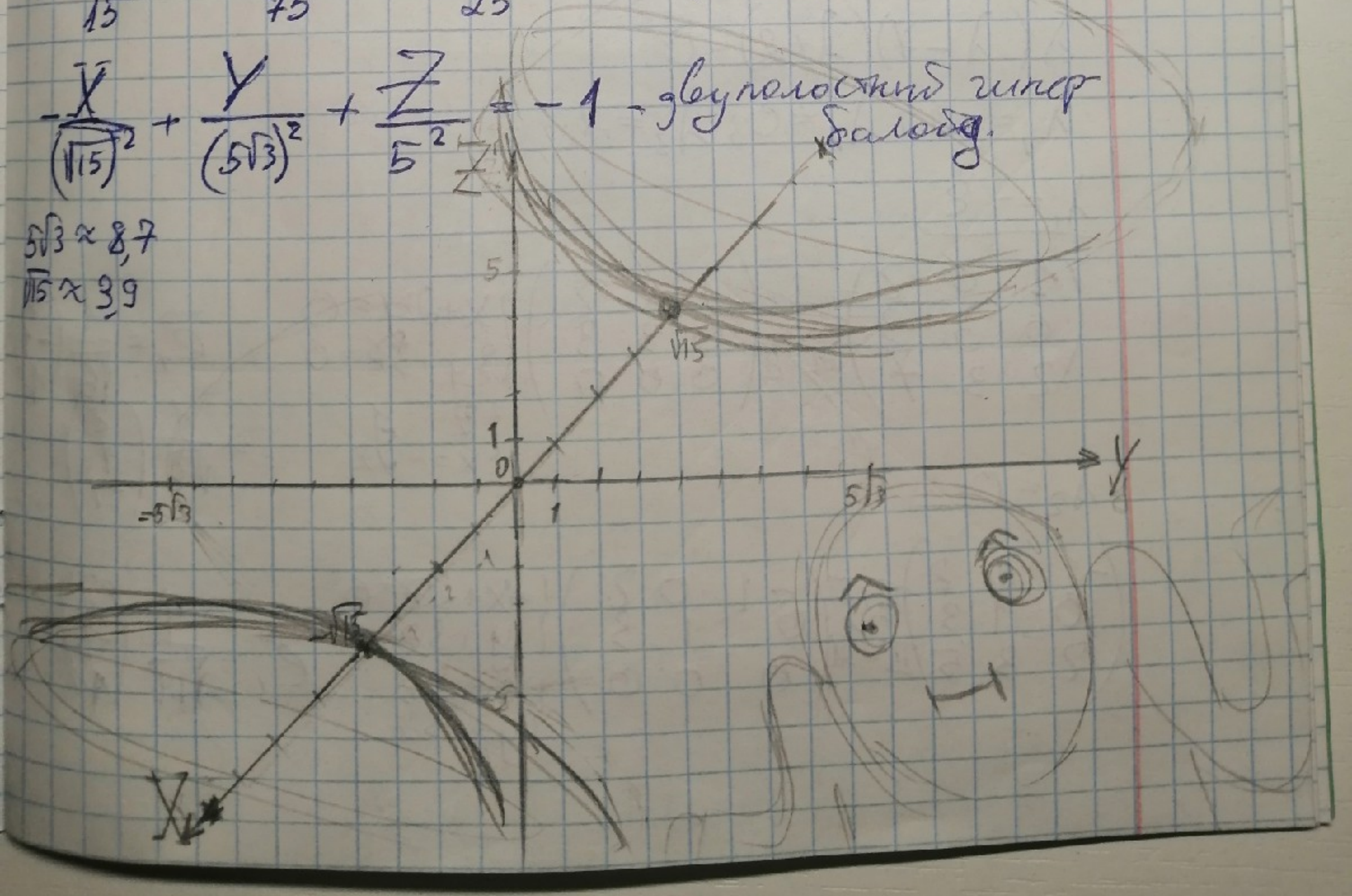
$$=-5(x'-1)^2+(y'-2)^2+3(z')^2+75=0$$

$$-\frac{(x'-1)^2}{15}+\frac{(y'-2)^2}{75}+\frac{(z')^2}{25}=-1$$

$$-\frac{X}{(\sqrt{15})^2}+\frac{Y}{(5\sqrt{3})^2}+\frac{Z}{5^2}=-1$$ – двуполостный гипербалоид.

$5\sqrt{3}\approx 8{,}7$

$\sqrt{15}\approx 3{,}9$



d) $-x^2+y^2+5z^2+4xz+6yz+4\sqrt{14}x-6\sqrt{14}y+$
$+2\sqrt{14}z-28=0$

$$A=\begin{pmatrix}-1 & 0 & 2\\ 0 & 1 & 3\\ 2 & 3 & 5\end{pmatrix}$$

$$\begin{vmatrix}-1-\lambda & 0 & 2\\ 0 & 1-\lambda & 3\\ 2 & 3 & 5-\lambda\end{vmatrix}=(\lambda^2-1)(5-\lambda)-4(1-\lambda)-9(-1-\lambda)=$$

$$=-\lambda^3+5\lambda^2+\lambda\cancel{-5}\cancel{-4}+4\lambda+9+9\lambda=$$

$$=-\lambda^3+5\lambda^2+14\lambda=0$$

$\lambda(\lambda^2-5\lambda-14)=0$

$\lambda(\lambda-7)(\lambda+2)=0$

$\lambda_1=-2;\ \lambda_2=0;\ \lambda_3=7$

$\underline{\lambda_1=-2}$

$$\begin{pmatrix}1 & 0 & 2\\ 0 & 3 & 3\\ 2 & 3 & 7\end{pmatrix}\underset{III-II}{\overset{III-2I}{\sim}}\begin{pmatrix}1 & 0 & 2\\ 0 & 3 & 3\\ 0 & 0 & 0\end{pmatrix}\quad\begin{cases}x+2z=0\\ 3y+3z=0\end{cases}\quad\begin{cases}z=-y\\ x=-2z\end{cases}\qquad \vec{e}_1=\frac{1}{\sqrt6}\begin{pmatrix}-2\\-1\\1\end{pmatrix}=\frac{1}{\sqrt6}\begin{pmatrix}2\\1\\-1\end{pmatrix}$$

$\lambda_2=0$

$$\begin{pmatrix}-1 & 0 & 2\\ 0 & 1 & 3\\ 2 & 3 & 5\end{pmatrix}\underset{III-3II}{\overset{III+2I}{\sim}}\begin{pmatrix}-1 & 0 & 2\\ 0 & 1 & 3\\ 0 & 0 & 0\end{pmatrix}\quad\begin{cases}-x+2z=0\\ y+3z=0\end{cases}\quad\begin{cases}x=2z\\ y=-3z\end{cases}\qquad \vec{e}_1=\frac{1}{\sqrt{14}}\begin{pmatrix}2\\-3\\1\end{pmatrix}$$

$-2+\frac{5}{2}+\frac{3}{2}$

$\lambda_3 = 7$

$$\begin{pmatrix} -8 & 0 & 2 \\ 0 & -6 & 3 \\ 2 & 3 & -\frac{3}{2} \end{pmatrix} \sim \begin{pmatrix} -4 & 0 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{matrix} z = 4x \\ z = 2y \end{matrix} \quad \vec{e}_3 = \frac{1}{\sqrt{21}}\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

$$T = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{2}{\sqrt{14}} & \frac{1}{\sqrt{21}} \\ \frac{1}{\sqrt{6}} & -\frac{3}{\sqrt{14}} & \frac{2}{\sqrt{21}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{14}} & \frac{4}{\sqrt{21}} \end{pmatrix} \qquad \begin{cases} x = \frac{2}{\sqrt{6}}x' + \frac{2}{\sqrt{14}}y' + \frac{1}{\sqrt{21}}z' \\ y = \frac{1}{\sqrt{6}}x' - \frac{3}{\sqrt{14}}y' + \frac{2}{\sqrt{21}}z' \\ z = -\frac{1}{\sqrt{6}}x' + \frac{1}{\sqrt{14}}y' + \frac{4}{\sqrt{21}}z' \end{cases}$$

$$-2(x')^2 + 7(z')^2 + \left(8\sqrt{\frac{14}{6}} - 6\sqrt{\frac{14}{6}} - 2\sqrt{\frac{14}{6}}\right)x' +$$

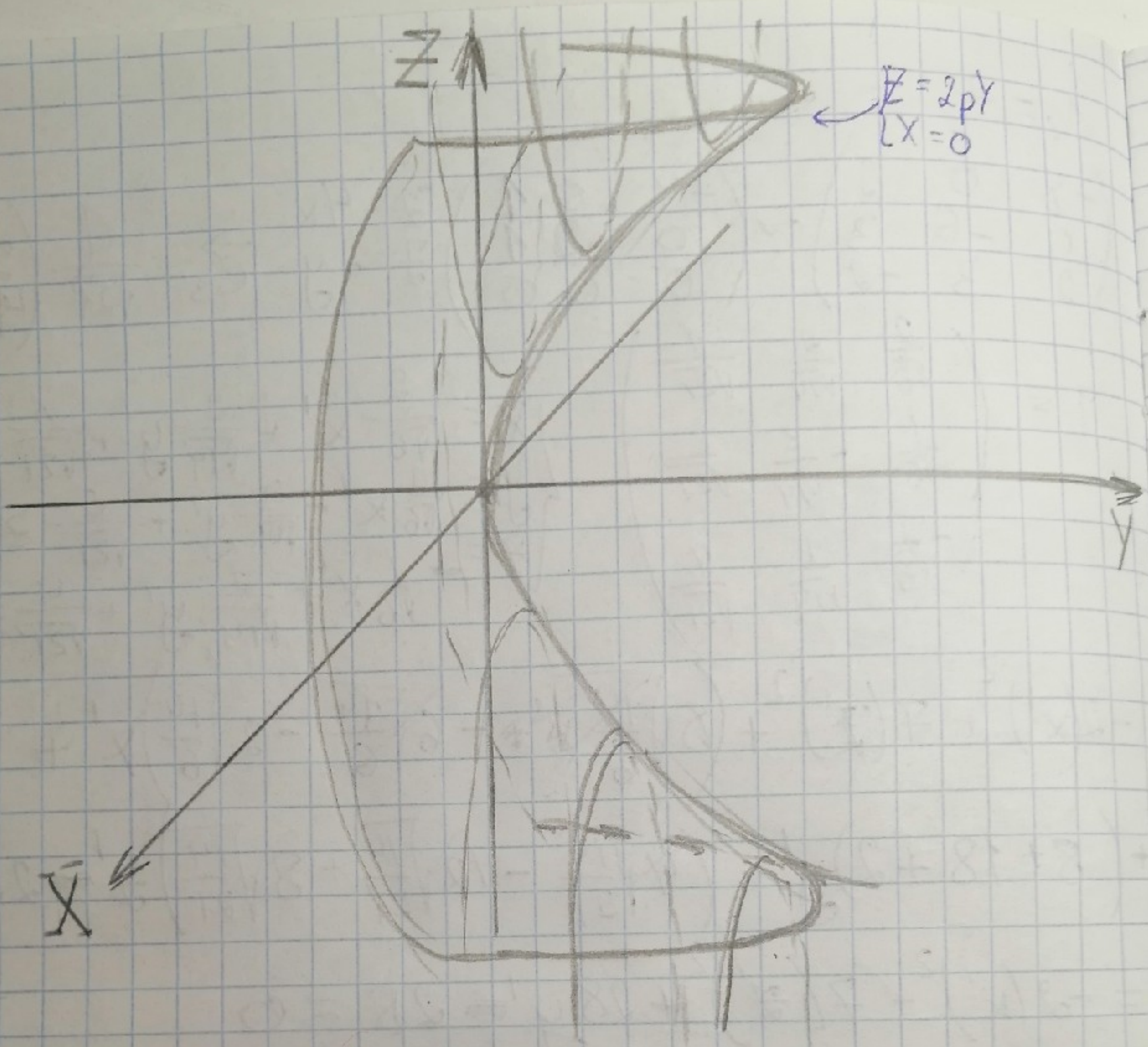
$$+(8+18+2)y' + \left(4\sqrt{\frac{14}{21}} - 12\sqrt{\frac{14}{21}} + 8\sqrt{\frac{14}{21}}\right)z' - 28 =$$

$$= -2(x')^2 + 7(z')^2 + 28y' - 28 = 0$$

$$-\frac{(x')^2}{14} + \frac{(z')^2}{4} = (y'+1)$$

$$-\frac{X^2}{(\sqrt{14})^2} + \frac{Z^2}{(2)^2} = 2Y \qquad \left(Y = \frac{(y'+1)}{2} = \frac{1}{2}(y'+1)\right); \ p = \frac{1}{2}$$

гиперболический параболоид.


Z
Y
X
$\begin{cases} Z = 2pY \\ X = 0 \end{cases}$