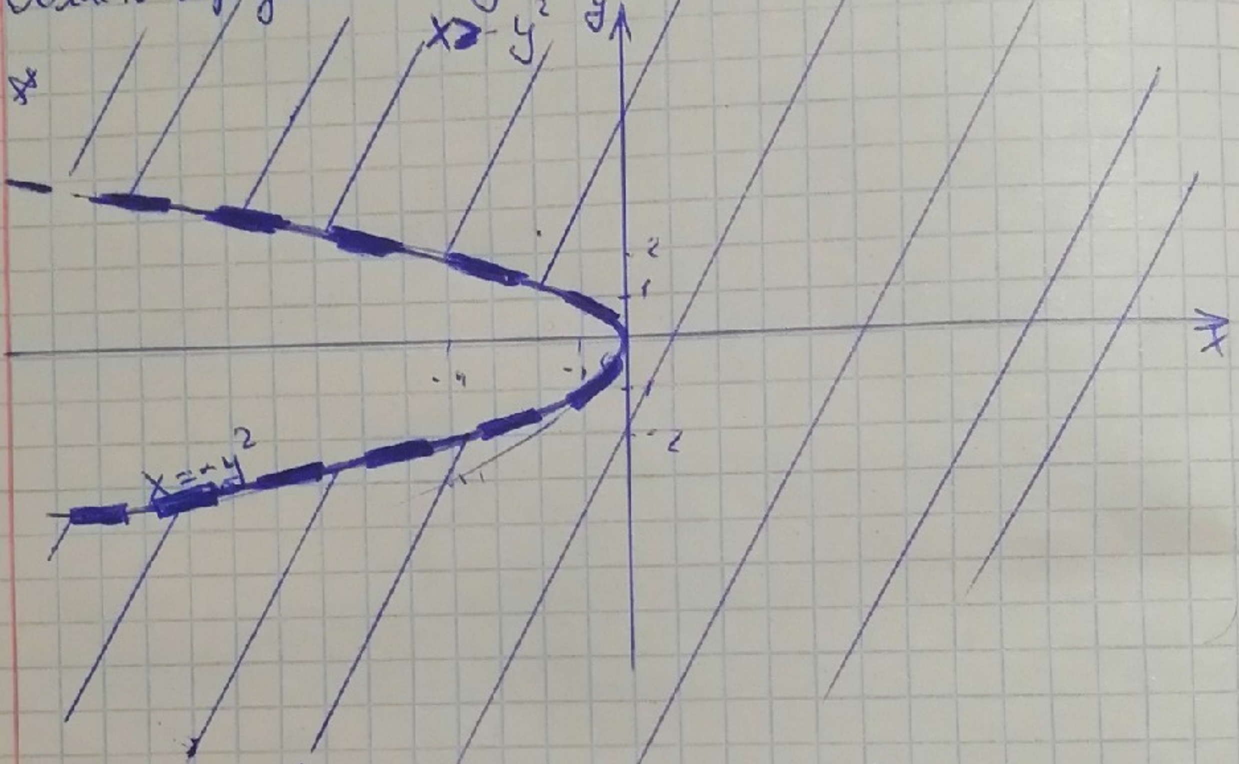
КР по 1АиФНП. Решников В ИУ1-21.

Вариант 13.

1. $z=\ln(x+y^2)$; $A(-3;2)$

Область определения $x+y^2>0$. Часть плоскости вне $x=-y^2$

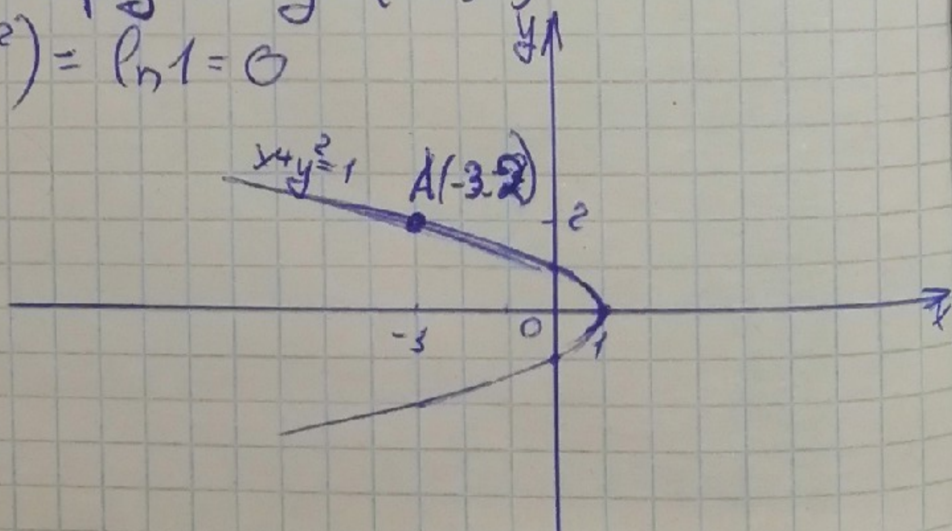


Линии уровня через точку $A(-3;2)$:

$C=\ln(-3+2^2)=\ln 1=0$

$\ln(x+y^2)=0$

$x+y^2=1$



2. $u = y^3 + \sqrt{z^2 - x^2}$; $M(3, 0, -5)$

$\vec{\ell} = 2\vec{i} - \vec{j} + 2\vec{k}$

$$\frac{\partial u}{\partial x} = \left(y^3 + \sqrt{z^2 - x^2}\right)'_x = \frac{1}{2\sqrt{z^2 - x^2}} \cdot (-2x) = -\frac{x}{\sqrt{z^2 - x^2}}$$

$$\left.\frac{\partial u}{\partial x}\right|_M = -\frac{3}{\sqrt{25 - 9}} = -\frac{3}{4}$$

$$\frac{\partial u}{\partial y} = \left(y^3 + \sqrt{z^2 - x^2}\right)'_y = 3y^2$$

$$\left.\frac{\partial u}{\partial y}\right|_M = 0$$

$$\frac{\partial u}{\partial z} = \left(y^3 + \sqrt{z^2 - x^2}\right)'_z = \frac{1}{2\sqrt{z^2 - x^2}} \cdot 2z = \frac{z}{\sqrt{z^2 - x^2}}$$

$$\left.\frac{\partial u}{\partial z}\right|_M = \frac{-5}{\sqrt{25 - 9}} = -\frac{5}{4}$$

$$\text{grad}\, u = \left(\frac{\partial u}{\partial x}; \frac{\partial u}{\partial y}; \frac{\partial u}{\partial z}\right) = \left(-\frac{3}{4}; 0; -\frac{5}{4}\right)$$

$\vec{\ell} = 2\vec{i} - \vec{j} + 2\vec{k}$

$$|\vec{\ell}| = \sqrt{2^2 + (-1)^2 + 2^2} = 3$$

$$\ell_0 = \left(\frac{2}{3}; -\frac{1}{3}; \frac{2}{3}\right)$$

$$\frac{\partial u}{\partial \ell} = \left.\frac{\partial u}{\partial x}\right|_M \cdot \cos\alpha + \left.\frac{\partial u}{\partial y}\right|_M \cdot \cos\beta + \left.\frac{\partial u}{\partial z}\right|_M \cdot \cos\gamma =$$

$$= -\frac{3}{4}\cdot\frac{2}{3} - \frac{1}{3}\cdot 0 + \frac{2}{3}\cdot\left(-\frac{5}{4}\right) = -\frac{1}{2} + 0 - \frac{5}{6} = -\frac{4}{3}$$

3. $$\lim_{\substack{x \to 0 \\ y \to 0}} \cos\left(\frac{1}{x^2} + \frac{1}{y^4}\right)$$

$y = kx$

$$\lim_{\substack{x \to 0 \\ y = kx}} \cos\left(\frac{1}{x^2} + \frac{1}{k^4 x^4}\right) = \lim_{\substack{x \to 0 \\ y = kx}} \cos \frac{1}{x^2}\left(1 + \frac{1}{x^2 k^2}\right).$$

результат зависит от k, значит предела $\lim\limits_{\substack{x \to 0 \\ y \to 0}} \left(\frac{1}{x^2} + \frac{1}{y^4}\right)$ не сущесвует

4. $u = e^{xy}\sin z$

$$\frac{\partial u}{\partial x} = \left(e^{xy}\sin z\right)'_x = y e^{xy}\cdot \sin z$$

$$\frac{\partial u}{\partial y} = \left(e^{xy}\sin z\right)'_y = x e^{xy}\cdot \sin z$$

$$\frac{\partial u}{\partial z} = \left(e^{xy}\sin z\right)'_z = e^{xy}\cdot \cos z$$

$$\frac{\partial^2 u}{\partial x^2} = \left(y e^{xy}\sin z\right)'_x = y^2 e^{xy}\sin z$$

$$\frac{\partial^2 u}{\partial y^2} = \left(x e^{xy}\cdot \sin z\right)'_y = x^2 e^{xy}\cdot \sin z$$

$$\frac{\partial^2 u}{\partial z^2} = \left(e^{xy}\cdot \cos z\right)'_z = -e^{xy}\sin z$$

$$\frac{\partial^2 u}{\partial x \partial y} = (y e^{xy} \cdot \sin z)'_y = (e^{xy} \cdot yx + e^{xy}) \sin z$$

$$\frac{\partial^2 u}{\partial x \partial z} = (y \cdot e^{xy} \sin z)'_z = y \cdot e^{xy} \cdot \cos z$$

$$\frac{\partial^2 u}{\partial y \partial x} = (x \cdot e^{xy} \cdot \sin z)'_x = (e^{xy} \cdot xy + e^{xy}) \sin z$$

$$\frac{\partial^2 u}{\partial y \partial z} = (x \cdot e^{xy} \cdot \sin z)'_z = e^{xy} \cdot x \cdot \cos z$$

$$\frac{\partial^2 u}{\partial z \partial x} = (e^{xy} \cos z)'_x = y \cdot e^{xy} \cos z$$

$$\frac{\partial^2 u}{\partial z \partial y} = (e^{xy} \cdot \cos z)'_y = x \cdot e^{xy} \cos z$$

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5. $z(x;y)$ - неявная диф. z в т. $B(1;2)$

вокр $A(1;2;-1)$ $z(0,9;2,2)$

$$z^3 + xz + y = 0$$

$$F(x;y;z) = z^3 + xz + y$$

$$z'_{x_0} = -\frac{F'_{x_0}}{F'_{z_0}} = \frac{-z_0}{3z_0^2 + x_0} \qquad z'_{x_0}\Big|_A = \frac{+1}{3\cdot(-1)^2+1} = \frac{1}{4}$$

$$z'_{y_0} = -\frac{F'_{y_0}}{F'_{z_0}} = -\frac{1}{3z_0^2 + x_0} \qquad z'_{y_0}\Big|_A = -\frac{1}{3(-1)^2+1} = -\frac{1}{4}$$

$$dz = z'_x \Delta x + z'_y \Delta y = 0,25 \Delta x - 0,25 \Delta y$$

$z(0,9; 2,2)$.

$x = x_0 + \Delta x \Rightarrow x_0 = 1 ; \Delta x = -0,1$

$y = y_0 + \Delta y \Rightarrow y_0 = 2 ; \Delta y = 0,2$

$z\big|_{x=0,9;\, y=2,2} \approx z\big|_{x=1;\, y=2} + dz$;

$z\big|_{x=0,9;\, y=2,2} \approx -1 - 0,1 \cdot 0,25 + 0,2(-0,25) =$

$= \underline{-1,075}$